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M. Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

PAPER – MTM-403(Unit-1&2)(Old)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – MTM-403

UNIT – I

(*Magneto Hydro-Dynamics*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) Describe the working principle of MHD power generator.

(*Turn Over*)

(*b*) Define magnetic Reynolds number and explain its significance.

(*c*) Write, the statement of Alfvén's theorem.

(*d*) Explain the term 'Lorentz force'.

2. Answer any *two* questions : 4×2

(*a*) Write down the basic equations of mageto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.

(*b*) Find the equations of motion of a conducting fluid in the context of MHD flow.

(*c*) Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis.

(*d*) Define the terms Alfvén's velocity and Alfvén's waves. Hence, derive the speed of propagation is $\sqrt{C^2 + V_A^2}$ for magneto hydrodynamic wave, where symbols have their usual meaning.

3. Answer any *one* question : 8×1

(*a*) A Viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = -L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$ (upper). Assume that a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field. 8

(*b*) (*i*) Find the rate of change of magnetic energy in magneto-hydrodynamic. 3

(*ii*) Write down the mathematical formulation of the mageto-hydrodynamic flow past a porous plate and derive its velocity expressions. 5

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UNIT – II

(*Stochastic Process and Regression*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) Discuss how a Markov Chain can be represented as a graph.

(*b*) Show that the bivariate correlation coefficients r_{12}, r_{13} and r_{23} must satisfy the inequality $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23} \leq 1$.

(*c*) Define transient and persistent states. When a persistent state is called null-persistent ?

(*d*) State Galton-Watson branching process.

2. Answer any *two* questions : 4×2

(*a*) Suppose that the probability of a dry day following a rainy day is $\frac{2}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$ and t.p.m.

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

If June 2 is a dry day then find the probability that June 4 and June 6 are dry days.

(*b*) Let $\{X_n, n \geq 1\}$ be a Markov chain having state space S={1,2,3,4} and transition *matrix*

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Identify the states as transient, persistent, or ergodic.

(*c*) Find the differential-difference equation for the birth and death process.

(*d*) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let X_n, $n \geq 1$ be the digit leaving the n^{th} stage of the system and X_0 be the digit entering the first stage (leaving the 0^{th} stage). At each stage, there is a constant probability q that the digits which enter will be transmitted unchanged (i.e. the digit will remain unchanged when it leaves), and probability

p otherwise (i.e. the digit changes when it leaves), $p+q=1$. Find the one-step transition matrix P, and n-step transition matrix P^n. Also Find P^n when $n \to \infty$.

3. Answer any *one* question : 8 × 1

(*a*) Let $\{X_n, n \geq 0\}$ be a branching process.

Show that if $m = E(X_1) = \sum_{k=0}^{\infty} kp_k$ and $\sigma^2 = Var(X_1)$, then $E(X_n) = m^n$ and

$$Var(X_n) = \begin{cases} \frac{m^{n-1}(m^n - 1)}{m-1}\sigma^2 & if \quad m \neq 1 \\ n\sigma^2 & if \quad m = 1 \end{cases}.$$

(*b*) Prove that $r_{1.2.3...p} = \left(1 - \frac{|R|}{R_{11}}\right)^{\frac{1}{2}}$, where the symbols have their usual meaning.

[Internal Assessment — 10 Marks]

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