2024

M.Sc. 2nd Semester Examination

APPLIED MATHEMATICS

PAPER : MTM-201

(Fluid Mechanics)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.

A. Answer any **four** questions : 2×4=8

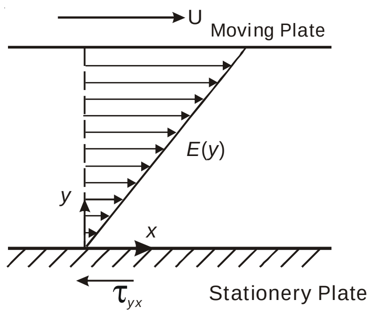
1. What are uniform flow and non-uniform flow? Discuss briefly.

2. For the Gulf Stream, velocity $U \sim 1\ \text{m/s}$, depth $L \sim 100\ \text{km}$ and viscosity $v \sim 10^{-6}\ \text{m}^2/s$. Calculate the Reynolds number.

3. Define the Newtonian and Non-Newtonian fluids with the help of relation between stress and strain.

4. What are mechanically similar flows? Discuss based on the definition of Reynolds number.

5. Let us consider a viscous flow inside the following channel.



Channel walls are impermeable. Write the boundary condition for tangential and normal components of velocity at both top and bottom walls.

6. Derive the hydrostatic equation and hence estimate the pressure experienced by a fish at a depth of 80 m.

B. Answer any **four** questions : 4×4=16

7. Draw an infinitesimally small element and show all the surface forces acting along the *y*-direction on the element. Finally, find the net surface and body forces acting on that element.

14. Consider steady, laminar, fully developed flow between two parallel plates separated by a distance 2H. The fluid is driven between the plates by an applied pressure gradient in the *x*-direction. It is assumed that conduction in the *y*-direction is much greater than conduction in *x*-direction.

(i) Determine the fully developed velocity distribution of the fluid as a function of the mean velocity.

(ii) Determine the fully developed temperature distribution as a function of the surface and mean temperatures. 5+3

15. *(i)* Define Rossby Number and discuss the cases of its low and high values.

(ii) Derive the *x*-component of Reynolds Averaged Navier-Stokes (RANS) equations in terms of eddy viscosity. 2+6

16. *(i)* Write reason for the use of scaling or ordering terms in the fluid mechanics.

(ii) Write the assumptions of boundary layer theory. Based on the above assumptions, derive the set of governing equations for the boundary layer flow along a flat plate. 2+6

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8. Derive the expression for the substantial derivative of temperature (*T*) and hence discuss its physical significance. Identify the local as well as convective derivative parts. Also derive the above substantial derivative using the chain rule.

9. Write the set of governing equations for the boundary layer flow along a flat plate and also write the proper boundary conditions for the above set of equations. Show that the *x*-component of the momentum equation applied at the edge of the boundary layer reduces to the Bernoulli equation. Finally write the governing equations for outside the Boundary layer.

10. Make the two-dimensional unsteady *x*-momentum equation

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

into non-dimensional form (in terms of Reynolds number $Re=\frac{UL}{v}$) with the help of characteristics time, length and velocity as L/U, L and U, respectively and symbols have their usual meanings.

11. For a typical horizontal length scale (L) of 500 km, horizontal speeds (U) are of the order of 0·15 ms^{-1} and a vertical scale length (*H*) of 1000 m. Estimate a typical vertical speed (*W*).

12. An incompressible velocity fields is given by $u=a(x^2-y^2)$, $v=-2axy$ and $w=0$. Determine under what conditions it is a solution to the Navier-Stokes momentum equation for the case of without any body forces. Assuming that these conditions are met, determine the resulting pressure distribution.

C. Answer *any* **two** questions : 8×2=16

13. *(i)* State the first law of thermodynamics. Show the energy fluxes associated with an infinitesimally small moving fluid element along the *x* direction and hence derive the non-conservation form of the energy equation.

(ii) Discuss briefly the similarity/dissimilarity between the *x*-momentum and simplified form of energy equations for the steady-state, two-dimensional flow of an incompressible fluid with constant properties. 6+2