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Hydromagnetic Effect on the Three Dimensional Flow Past a Vertical Porous Plate

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ABSTRACT

The effect of magnetic field on the three dimensional flow of viscous incompressible fluid past a porous vertical plate subjected to periodic suction velocity distribution has been analyzed. Appromimate solution for velocity and temperature fields have been obtained by using perturbation technique. It is found that main fluid velocity decreases with increase in magnetic parameter while the magnetude of the cross velocity decreases near the plate but increases away from the plate with increase in magnetic parameter. It is also found that the shear stress due to main flow decreases with increase in Prandtl number. Furthermore, the heat transfer is enhanced due to increasing the values of the Prandtl number.

Keywords : Three-dimensional, Injection, Suction, Transverse sinusoidal.

1. Introduction

The problem of laminar flow control plays and important role in many engineering applications, particularly in the fields of aeronautical engineering to reduce drag and hence the vehicle power requirement by substantial amount. The transition from laminar to turbulent flow was first examined by O, Reynolds in a pipe flow. Later Prandtl shown experimentally that the boundary layer also can be both laminar or turbulent and that process of separation and thus the drag problems are controlled by this transition. The effect of different arrangements and configurations of the suction holes and slits on the drag has been studied extensively. An analysis of such flows find applications in ion propulsion, electromagnetic pumps, MHD power generators, controlled fusion etc. The method of "cooling of the wall" in controlling the laminar flow together with the application of suction has become important

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and has received the attention of scholars. Singh et at. [1] studied the effect on wall shear stress and heat transfer of the flow caused by the periodic suction when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the directions of free stream. Singh et al. [2] investigated the three dimensional fluctuating flow and heat transfer by applying the transverse sinusoidal suction velocity distribution fluctuating with time at the porous plate. Singh [3] studied the effects of magnetic field on the three dimensional flow of a viscous, incompressible fluid past a porous plate by applying transverse sinusoidal suction. Singh [4] analyzed the effect of magnetic forld on the oscillatory flow past a porous plate by applying transverse simusoidal suction. Also singh [5] studied the flow of viscous incompressible fluid past a porous plate with transverse sinusoidal suction in the presence of viscous dissipative heat. Recently Guria and Jana [6] considered the unsteady three dimensional flow of viscous incompressible fluid past a vertical porous plate. The aim of this paper is to study the three dimensional flow past a vertical porous plate subjected to a periodic suction velocity distribution in the presence of a uniform mangetic field.

2. Formulation of the Problem

Consider the flow of viscous, incompressible, electrically conducting fluid past along a semi infinite vertical porous plate. The x^*-axis is chosen along the vertical plate that is the direction of the flow, y^*-axis is perpendicular to the plate and z^*-axis is perpendicular to the x^*y^*-plane. A uniform magnetic field B_0 is imposed in the y^*-direction. The plate is subjected to a periodic suction velocity distribution of the form

$$v^* = -V_0\left[1+\in \cos\frac{u\infty z^*}{\nu}\right], \tag{2.1}$$

where $\in$ (<<1) is the amplitude of the suction velocity.

Denoting velocity components u^*, v^*, w^* in the directions x^*-, y^*-, z^*-axes repectively, the flow is governed by the following equations

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \tag{2.2}$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\nu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)+g\beta\left(T-T_\infty\right)-\frac{\sigma B_0^2 u^*}{\rho}, \tag{2.3}$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right), \tag{2.4}$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\nu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\sigma B_0^2 w^*}{\rho}, \tag{2.5}$$

$$v^* \frac{\partial T}{\partial y^*} + w^* \frac{\partial T}{\partial z^*} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^{*2}} + \frac{\partial^2 T}{\partial z^{*2}}\right),$$

where ρ is the density, p^* is the fluid pressure, g is the acceleration due to gravity, β is the coefficient of thermal expansion β, k is the coefficient of heat conduction, c_p is the specific heat at constant pressure, σ is the electrical conduction.
The boundary conditions of the problem are

$$u^* = 0, v^* = -V_0\left[1 + \in \cos\frac{u\infty z^*}{\nu}\right], w^* = 0,\ T = T_\infty \ at\ y^* = 0$$

$$u^* = 0, v^* = -V_0,\ w^* = 0,\ p^* = p_\infty \ at\ y^* \to \infty \tag{2.7}$$

Introducing the non-dimensional variables

$$y = \frac{u_\infty y^*}{\nu},\ z = \frac{u_\infty z^*}{\infty},\ p = \frac{p^*}{pu_\infty^2},\ u = \frac{u^*}{u_\infty},\ v = \frac{v^*}{u_\infty},\ \theta = \frac{(T - T_\infty)}{T_w - T_\infty}, \tag{2.8}$$

equations (2.2)-(2.6) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2.9}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + G_r\theta - Mu, \tag{2.10}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right), \tag{2.11}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - Mw, \tag{2.12}$$

$$v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = -\frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \tag{2.13}$$

where $G_r = \dfrac{g\beta(T_w - T_\infty)}{u_\infty^3}$ is the Grashof number, $M = \dfrac{\sigma B_0^2 v}{\rho u_\infty^2}$ is the magnetic parameter

and $P_r = \dfrac{\rho\nu c_p}{K}$ is the Prandtl number, T_∞ is the temperature outside the boundary layer and

p_∞ is the pressure outside the boundary layer.

The corresponding boundary conditions (2.7) become

$$u=0,\ v=-S\left[1+\in \cos(\pi z)\right],\ w=0, \theta=1 \text{ at } y=o$$

$$u=0,\ v=-S,\ w=0, \theta=0 \text{ at } y\to\infty \tag{2.14}$$

S = $V_0/u_¥$, the suction parameter.

3. Solution of the Problem

To sovle (2.9)-(2.13), we assume the solution of the following form :

$$u=u_0+\in u_1+\in^2 u_2+...,$$

$$v=v_0+\in v_1+\in^2 v_2+...,$$

$$w=w_0+\in w_1+\in^2 w_2+...,$$

$$p=p_0+\in p_1+\in^2 p_2+..., \tag{3.15}$$

$$\theta=\theta_0+\in \theta_1+\in^2 \theta_2+...,$$

Substituting (3.15) in (2.9)-(2.13), comparing the term free from $\in$ and the coefficient of $\in$ from both sides, and neglecting those of $\in^2$, the term free from $\in$ is

$$v_0'=0, \tag{3.16}$$

$$u_0''+Su_0'+G_r\theta_0-Mu_0=0, \tag{3.17}$$

$$\theta_0''+SP_r\theta_0'=0, \tag{3.18}$$

where the primes denote differentiation with respect to y.

The corresponding boundary contitions are

$$u_0=0,\ v_0=-S,\ \theta_0=1 \text{ at } y=0, \text{ and } u_0=0,\ v_0=-S,\ \theta_0=0 \text{ at } y\to\infty. \tag{3.19}$$

The solution of equations (3.16)-(3.18) subject to the boundary conditions (3.19) are

$$v_0(y)=-S,\ u_0(y)=\frac{-G_r}{S^2P_r(P_r-1.0)-M}\left(e^{-SP_ry-e^{-\lambda_1 y}}\right), \tag{3.20}$$

$$\theta_0(y)=e^{-SP_ry}, \tag{3.21}$$

where $$\lambda_1=\frac{1}{2}\left[S+\sqrt{S^2+4M}\right]$$

The term depending on $\in$ is

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0, \tag{3.22}$$

$$v_1 \frac{\partial u_0}{\partial y} - S \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + G_r \theta_1 - M u_1, \tag{3.23}$$

$$-S \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial v_1}{\partial z^2} \right), \tag{3.24}$$

$$-S \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - M w_1, \tag{3.25}$$

$$v_1 \frac{\partial \theta_0}{\partial y} - S \frac{\partial \theta_1}{\partial y} = \frac{1}{P_r} \left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right). \tag{3.26}$$

The corresponding boundary conditions are

$$u_1 = 0,\ v_1 = -S\cos(\pi z),\ w_1 = 0,\ \theta_1 = 0 \text{ at } y = 0$$

$$u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0 \text{ at } y \to \infty$$

Equations (3.22)-(3.26) are the linear partial differential equations describing the three-dimensional flow. Now we assume velocity components, pressure, and temperature in the following form

$$u_1(y,z) = u_{11}(y)\cos(\pi z),\ v_1(y,z) = v_{11}(y)\cos(\pi z),$$

$$w_1(y,z) = -\frac{1}{\pi} v'_{11}(y)\sin(\pi z),\ p_1(y,z) = p_{11}(y)\cos(\pi z), \tag{3.28}$$

$$\theta_1(y,z) = \theta_{11}(y)\cos(\pi z),$$

Substituting (3.28) in (3.22)-(3.26), we obtain the following set of differential equations

$$v''_{11} + S v'_{11} - \pi^2 v_{11} = p'_{11}, \tag{3.29}$$

$$v'''_{11} + S v''_{11} - \left(\pi^2 + M\right) v'_{11} = \pi^2 p_{11}, \tag{3.30}$$

$$\theta''_{11} + S P_r \theta'_{11} - \pi^2 \theta_{11} = -S P_r^2 v_{11} e^{-S P_r y}, \tag{3.31}$$

$$u''_{11} + S u'_{11} - \left(\pi^2 + M\right) u'_{11} = v_{11} u'_0 - G_r \theta_{11}. \tag{3.32}$$

Using (3.28), boundary conditions (3.27) become

$$u_{11} = 0,\ v_{11} = -S,\ w_1 = 0,\ \theta_{11} = 0 \text{ at } y = 0,$$
$$u_{11} = 0,\ v_{11} = -S,\ w_{11} = 0,\ \theta_{11} = 0 \text{ at } y \to \infty. \tag{3.33}$$

Solving (3.29)-(3.32), under the boundary conditions (3.33), we get

$$v_1(y,z)=\frac{S}{(\alpha-\beta)}\left[\beta e^{\alpha y}-\alpha e^{-\beta y}\right]\cos\pi z, \tag{3.34}$$

$$w_1(y,z)=\frac{-\alpha\beta S}{\pi(\alpha-\beta)}\left[e^{-\beta y}-e^{-\alpha y}\right]\sin\pi z, \tag{3.35}$$

$$p_1(y,z)=\frac{\alpha\beta S}{\pi^2(\alpha-\beta)}\left[\{\beta(\lambda_2-S)-M\}e^{-\beta y}-\{\alpha(\lambda_1-S)-M\}e^{\alpha y}\right]\cos\pi z, \tag{3.36}$$

$$\theta_1(y,z)=\frac{S^2P_r^2}{(\alpha-\beta)\alpha\beta}\left[\frac{\alpha^2}{\lambda_2+SP_r}\left(e^{-(\beta+SP_r)y}-e^{-\lambda 3y}\right)\right.$$

$$\left.-\frac{\beta^2}{(\lambda_1+SP_r)}\left(e^{-(\alpha+SP_r)y}-e^{-\lambda 3y}\right)\right]\cos\pi z, \tag{3.37}$$

$$u_1(y,z)=\frac{1}{(\alpha-\beta)}\left[C_1\left\{e^{\lambda_3 y}-e^{-\lambda_4 y}\right\}+C_2\left\{e^{-(\alpha+SP_r)y}-e^{-\lambda_4 y}\right\}+C_3\left\{e^{-(\beta+SP_r)y}-e^{-\lambda_4 y}\right\}\right.$$

$$\left.+C_4\left\{e^{-(\alpha+\lambda_1)}-e^{-\lambda_4 y}\right\}+C_5\left\{e^{-(\beta+\lambda_1)y}-e^{-\lambda_4 y}\right\}\right]\cos\pi z, \tag{3.38}$$

where

$$\lambda_2=\frac{1}{2}\left[S-\sqrt{S^2+4M}\right],$$

$$\lambda_3=\frac{1}{2}\left[SP_r-\sqrt{S^2P_r^2+4\pi^2}\right],\ \lambda_4=\frac{1}{2}\left[S+\sqrt{S^2+4\left(\pi^2+M\right)}\right],$$

$$\alpha=\frac{1}{2}\left[\lambda_1+\sqrt{\lambda_1^2+4\pi^2}\right],\ \ \beta=\frac{1}{2}\left[\lambda_2+\sqrt{\lambda_2^2+4\pi^2}\right].$$

$$A=\frac{S^2P_r^2\alpha}{\beta(\alpha-\beta)(SP_r+\lambda_2)}\quad B=\frac{S^2P_r^2\beta}{\beta(\alpha-\beta)(SP_r+\lambda_1)}$$

$$C=\frac{-G_r}{S^2P_r(P_r-10.0)-M}\quad C_1=\frac{G_r(A-B)}{S\lambda_3(P_r-1.0)-M}$$

$$C_2=\left[G_rB-\frac{S^2\,\mathrm{Pr}\,\beta C}{\alpha-\beta}\right]\Big/\left\{(SP_r+\alpha)(SP_r+\alpha-S)-\left(\pi^2+M\right)\right\},$$

$$C_3 = \left[-G_r A + \frac{S^2 \Pr \alpha C}{\alpha - \beta} \right] \Big/ \left\{ (SP_r + \beta)(SP_r + \beta - S) - \left(\pi^2 + M \right) \right\},$$

$$C_4 = \frac{SC\lambda_1 \beta}{\alpha - \beta} \Big/ \left\{ (\alpha + \lambda_1)(\alpha + \lambda_1 - S) - \left(\pi^2 + M \right) \right\},$$

$$C_5 = \frac{-SC\lambda_1 \alpha}{\alpha - \beta} \Big/ \left\{ (\beta + \lambda_1)(\beta + \lambda_1 - S) - \left(\pi^2 + M \right) \right\},$$

4. Result and Discussion

We have presented the non-dimensional velocity u for Gr = 5.0, Pr = 0.72, S = 1.0, $\in$ = 0.2 Fig. 1 and cross velocity w for S = 1.0, z = 0.5, $\in$ = 0.2 in Fig. 2 against y for different values of magnetic parameter M. It is shown that the main flow velocity is significantly affected with magnetic parameter. It is observed that main fluid velocity u decreases with increase in magnetic parameter while the magnitude of the cross velocity *w* decreases near the plate and increases away from the plate with increase in magnetic parameter. This is due to the fact the magnetic field exerts a retarding force on the free convective flow.

One of the important characteristic of the problem is the shear stress. The shear stress components due to main flow and cross flow direction can be calculated as

$$\tau_x = u_0'(0) + \in u_1'(0),$$

$$= C(\lambda_1 - SP_r)$$

$$+ \frac{\in}{(\alpha - \beta)} \left[-C_6 \lambda_4 - C_1 \lambda_1 - C_2 (\alpha + SP_r) - C_3 (\beta + SP_r) - C_4 (\alpha + \lambda_1) - C_5 (\beta + \lambda_1) \right] \cos \pi z \tag{4.40}$$

$$\tau_z = \in w_1'(0),$$

$$= -\frac{\in \alpha \beta S}{\pi} \sin \pi z \tag{4.41}$$

The shear stresses due to main flow are obtained for different values of magnetic parameter and suction parameter and for G_r = 5.0, P_r = 0.72, z = 0.0, $\in$ = 0.2. It is clear from Table 1 that the shear stress due to main flow decreases with increase in either suction parameter or magnetic parameter. Table 2 shows the computed values of the shear stress due to cross flow for different values of S and M and for G_r = 5.0, P_r = 0.72. It is clear that the magnitude of the shear stress due to cross flow decreases with increase in magnetic parameter.

Table 1
Shear stress component due to main flow for G_r = 5.9, S = 0.9, z = 0.2.

M	τ_x			
	$P_r = 0.5$	$P_r = 0.71$	$P_r = 1.0$	$P_r = 2.0$
4.0	2.417526	2.204472	1.960588	1.361472
5.0	2.177563	2.004038	1.802101	1.302868
6.0	1.997334	1.850867	1.678216	1.246100
7.0	1.855545	1.728778	1.577792	1.94944

Table 2
Shear stress component due to cross flow for S = 0.9, z = 0.2.

M	4.0	5.0	6.0	7.0
τ_x	– 0.381100	–0.380520	–0.379960	–0.379419

The temperature profile has been plotted in Fig. 3 against y for different values of Prandtl number and for S = 1.0, M = 10.0, z = 0.2, $\in$ = 0.2. It is observed that the temperature decreases with increase in Prandtl number. This shows that the thermal boundary layer thickness decreases with increase in Prdndtl number. The effect of the Prandtl number is very important in temperature profile.

The rate of heat transfer from the wall to the fluid can be calculated as

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\theta_0'(0) - \in \theta_1'(0),$$

$$= SPr - \in \frac{S^2 Pr^2}{(\alpha - \beta)\alpha\beta}\left[\frac{\alpha^2}{(\lambda_2 + SPr)}\{\lambda_3 - (\beta + SPr)\} - \frac{\beta^2}{(\lambda_1 + SPr)}\{\lambda_3 - (\alpha + SPr)\}\right]. \quad (4.42)$$

The heat transfer coefficient in terms of Nusselt number is shown in Fig. 4 against suction parameter for different values of Prandtl number and for M = 10.0, $\in$ = 0.02. It is seen that the heat transfer increases with increase in Prndtl number Pr. This is due to the fact that a fluid having larger Prandtl number possesses a larger heat capacity and hence enhances the heat transfer. Thus, the fluid with larger Prandtl numbers will perform more efficiently the cooling of the heated plate.

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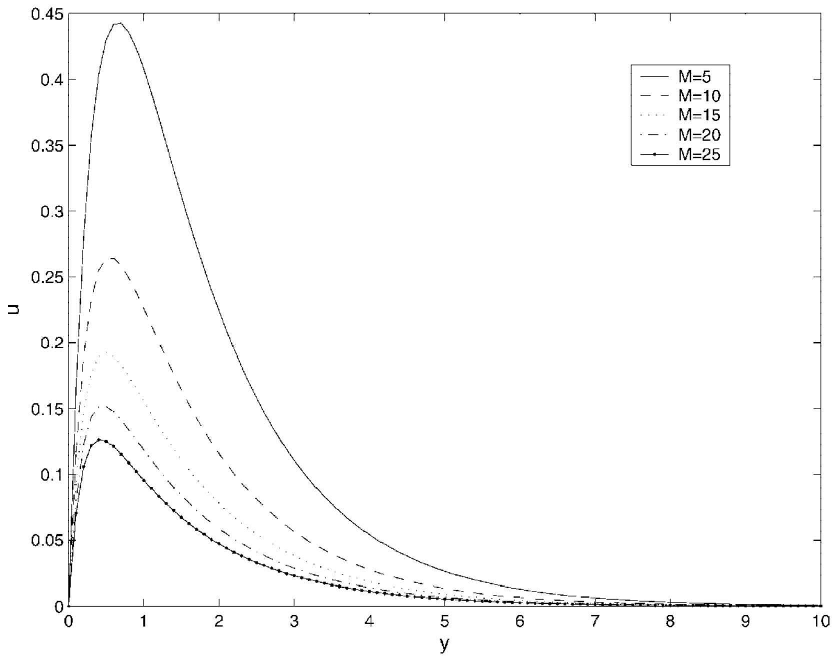

Fig.1 Main flow velocity u for Gr = 5.0, S = 1.0, Pr = 0.72, z = 0.0, ∈ = 0.2

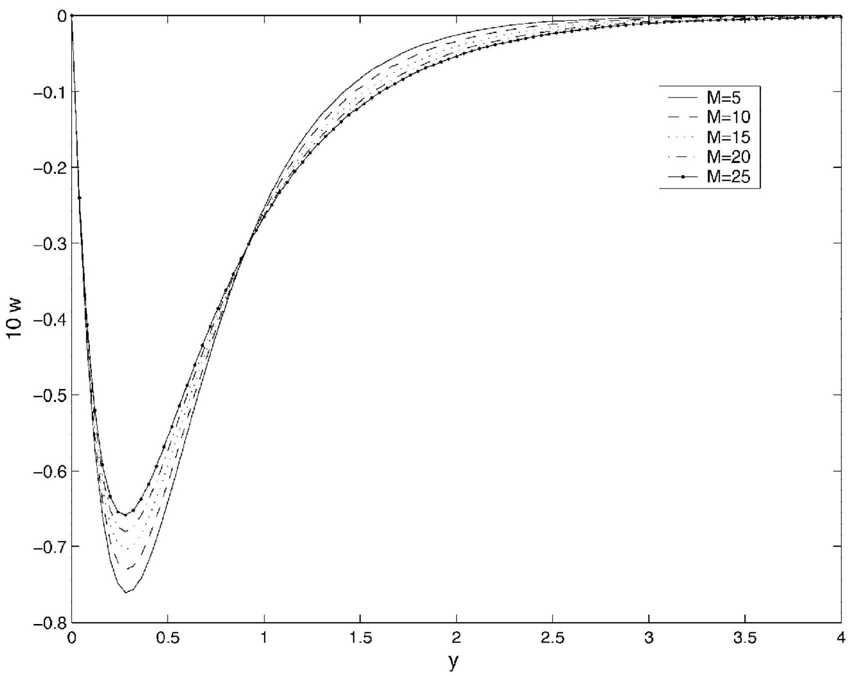

Fig.2 Cross velocity 10 × w for S = 1.0, Pr = 0.72, z = 0.5, ∈ = 0.2

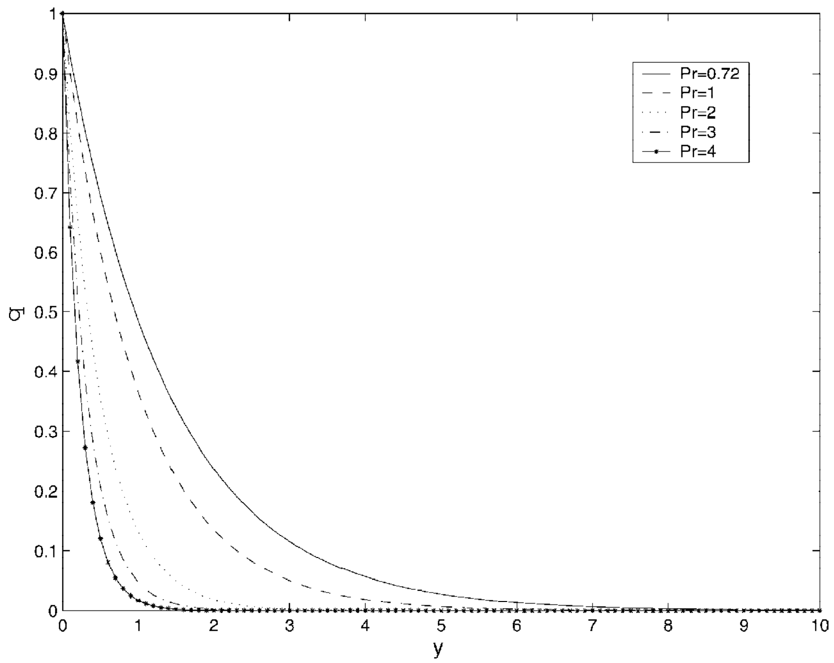

Fig.3 Temperature Profile θ for S = 1.0, M = 10.0, z = 0.2, ∈ = 0.2

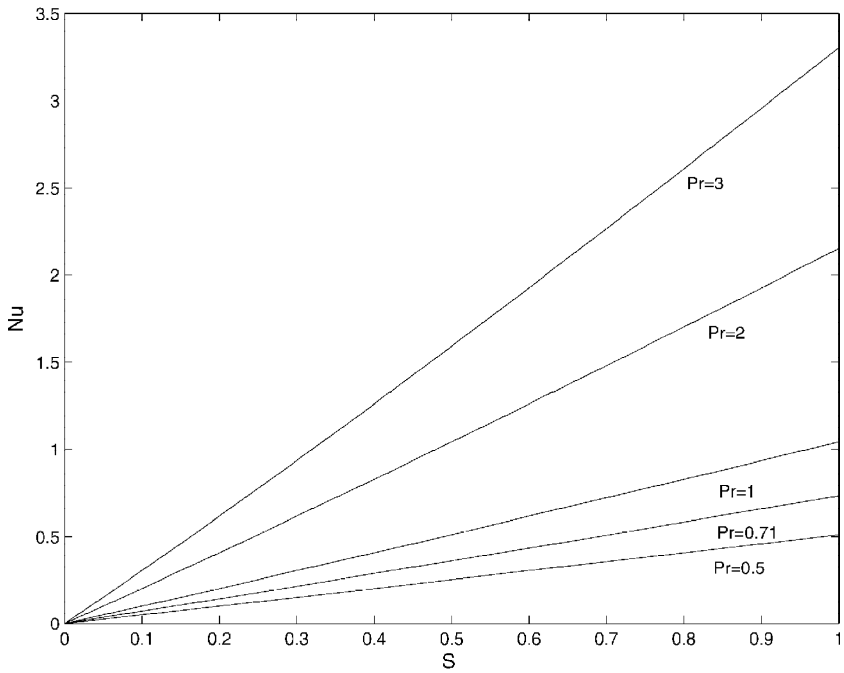

Fig.4 Rage of heat transfer at the plate for M = 10.0, z = 0.2, ∈ = 0.2