Tourism Development, Environment and Agricultural Sector: A Theoretical Analysis

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Abstract

We develop a small, open economy with a two-sector general equilibrium model with three factors of production: skilled labour, land, and capital. The two sectors are the agricultural sector and the tourism sector. Agricultural production is also dependent on the environmental quality of the economy. We show that tourism development raises the output of tourism-related products, degrades environmental quality and reduces the production of agricultural products.

Keywords: *Tourism Development, Environmental Degradation, Agricultural Sector*
JEL classification: *Z32, O13*

1. Introduction

Tourism development stimulates travellers to explore nature and societies, discover cultures, interact with values, and experience new traditions and events while satisfying a person's aesthetic needs. Tourism development attracts tourists to a particular destination to develop and sustain a tourism industry, boosting the local economy, especially where agriculture and industries are not so developed. To make tourism the backbone of an economy, environmental sustainability is the most conscious effort to conserve socio-cultural heritage and preserve natural resources to protect environmental ecosystems. Environment sustainability can be reflected in clean and green natural landscaping and thriving biodiversity, which motivate tourists and boost the willingness of the local community to welcome visitors. According to the World Tourism Organization (UNWTO), tourism is one of the fastest-growing industries; international tourism receipts reached USD 1.4 trillion in 2023, according to preliminary estimates, about 93% of the USD 1.5 trillion earned by destinations in 2019(precovid-19 pandemic). Substantial development in tourism is likely to trickle environmental pollution along with its positive effects on employment, wealth creation, and the economy. The pollution at tourist destinations may include air emissions, noise, solid waste, littering, etc. In addition, an uncontrolled and overcrowded tourist area has substantial adverse effects on the quality of the environment. It results in the over-consumption of natural resources, degradation of service quality and natural resources, and a rapid increase in wastage and pollution. Furthermore, tourism arrivals beyond capacity bring problems, such as soil erosion, attrition of natural resources, accumulation of waste and air pollution, endangering biodiversity, decomposition of socio-cultural habitats, and virginity of land. Tourism development leads to deforestation,

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which causes landslides in hills, floods in sea shores and river banks, and hampers the natural growth of forest animals. Sun and Walsh (1998), Gossling (2000, 2002), Jim (2000), Maddison (2001) etc., supply empirical evidence related to these problems[14]. Cole and Bayfield (1993) have shown that tourism activity reduces agricultural productivity. Biological effects include causing damage to the agricultural sector in the United States due to tourism development, as Hawes (1992) shows. Liddle (1988) shows that tourism development lowers agricultural productivity in Australia. Torres (2003) concluded contention that state-driven Planned Tourism Development in Cancun failed to stimulate agricultural development in Quintana Roo. This represents both a lost opportunity for local agriculture and a degradation of tourism benefits. So, empirical studies show a strong relationship between tourism development and fall in agricultural productivity. A few papers deal with the problem of tourism development and environmental pollution. **Copeland (1991)** analyzes tourism development by forming a static short-run general equilibrium model of a small open economy. Though this literature doesn't include environmental degradation as a critical element of analyzing tourism development, this was a pioneering attempt to discuss and solve an essential area of research in terms of Economics. **Yabuuchi (2015)** develops a static model where tourism is expected to increase employment and improve national welfare in labour-surplus developing economies. However, tourism promotion degrades and depletes the environment due to the pollution generated by the production of tourism goods and the increase in the number of tourists. Thus, together, tourism promotion coupled with efforts towards environmental protection (in the form of a pollution tax) generates complicated economic effects. Other noted papers include **Hazari and Kaur (1995),** which examines the impact of tourism on the welfare of the domestic residents and the relative price of the non-traded goods when this commodity is produced by a monopoly and consumed by tourists and residents. It established that in the presence of monopoly production of non-traded goods (with or without foreign ownership) and services, an expansion of tourism may result in a decline in the welfare of domestic residents. **Hazari and Sgro (2004)** provided a systematic treatment of a two-commodity, two-factor general equilibrium model of an open economy used in several accurate models of trade and growth. **Hazari and Nowak(2003)** present a static model that captures the interdependence between tourism and the rest of the economy, particularly agriculture and manufacturing. The critical result is that the tourist boom may 'immiserize' the residents. Beladi (2009) addressed the issues of pollution taxes, the environment and welfare for a small open economy which has a monopoly over the trade of goods consumed by foreign tourists, and the export of tourism services improves terms of trade. However, this expansion may also induce more excellent production of non-traded goods, which causes environmental damage. However, none of the above papers have discussed the impact of environmental pollution on agricultural productivity.

This paper develops a small, open economy with a two-sector general equilibrium model with three factors of production: skilled labour, land, and capital. The two sectors are the agricultural sector and the tourism sector. The paper shows the effect of tourism development on the production of tourism goods, agricultural goods, and environmental quality. A rise in

[14] **Kwakwa et al. (2022)** used the time-series data from 1971 to 2017 and applied regression analysis and a variance decomposition analysis and shown that the country's agricultural development is adversely affected by aggregate carbon emission. Further, industrial development and emissions from transport sector, industrial sector and other sectors adversely affect Ghana's agriculture development. The contribution of carbon emission together with other explanatory variables to the changes in agricultural development generally increases over the period.

foreigners' income causes tourism development, which raises the production of tourism-related products, degrades environmental quality, and reduces the production of agricultural products. In contrast, an increase in the price of agricultural goods increases the output of tourism goods, decreases the production of agricultural goods and degrades the environment more when the price of agricultural goods is less than the rise in the price of tourism goods.

The rest of the paper is organized as follows. In section 2, we have formulated the model, and in section 3, comparative statics and propositions are derived. Concluding remarks are made in Section 4.

2. The model

We consider a small open developing economy with two sectors namely, the non-traded (tourism) goods sector and the agricultural goods sector. The economy uses three factors of production, which are labour (L), physical capital (K) and land (N). The tourism sector (T) uses labour and physical capital to produced non traded tourism goods. The agricultural sector uses labour and land to produce exportable agricultural goods. Expenses on imports are financed by revenue earned from tourism. Domestic consumers also demand for this domestic tourism services. There is no exclusive exportable good in this economy. Price ratio of tourism sector and agricultural sector represent the terms of trade. Labour is perfectly mobile in two sectors but physical capital and land is specific to tourism and agricultural sector respectively. Price of the traded agricultural good is exogenously specified; and seller's effective price of this good is treated as a policy parameter. Total demand and total supply of domestic tourism service determine its price at market clearing level. This tourism is a normal good having negative relationship its demand and own price and positive relationship between its demand and own income as well as income of the rest of world. The rise in income of the rest of the world causes foreign tourists to visit more to the host country; and this rise is defined as tourism development in this model. Domestic consumers also consume tourism service along with the traded good. So the demand for tourism service is also affected by the increase in national income of the host country which is distributed as additional factor income to domestic consumers. Production function of each of these two sectors obeys all standard neo-classical properties including constant returns to scale. Factor endowments of the host country are exogenously given in the static model. We assume that only tourism services cause's pollution emission to make the environment worse. Environmental quality of that place is degraded due to expansion of tourism in that place. Hence, the product efficiency of the agricultural sector decreases. Factor prices in each of these two sectors are perfectly flexible and this flexibility ensures full employment of all these factors. All markets are competitive; and the representative firm in each of these two sectors maximizes profit.

We follow the following notations:

a_{KT}: Capital output ratio in tourism sector.
a_{UT}: labor output ratio in tourism sector .
a_{N2}: land output ratio in agricultural sector.
a_{U2}: labour output ratio in agricultural sector.
P_i: Effective producer's price of the product produced by ith sector for i=T, 2.
W: Wage rate of labor.
r : rate of return on capital.
D_T: Demand function for commodity T.
Y: Total factor income/output of the home country.
X_i: Amount of output produced by ith sector for i=T, 2
$\overline{K}$: Endowment of capital given exogenously.

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$\overline{N}$: Endowment of land given exogenously.
$\overline{L}$: Endowment of labour given exogenously.
Y_F: Income of rest of world.
θ_{ij}: Distributive share of jth input in ith sector for j=L, K, N and i=T, 2 .
λ_{ji}=Proportion of jth input involved in ith sector for j=L, K, N and i=T, 2, .
S_{ji}^{h}= The elasticity of factor out put coefficient of jth factor in hth sector with respect to price of ith factor, for i,j=L,K,N and h=T,2. For example $S_{LK}^{T}=(\frac{r}{a_{LT}})(\frac{\partial a_{LT}}{\partial r}).S_{jj}^{h} < 0, S_{ji}^{h} > 0$; i≠j
The production functions of the tourism and agricultural sectors are given by:

$$X_T = F^1(L_T, K) \quad (1);$$

and

$$X_2 = eF^2(L_2, N) \quad (2).$$

F^1 And F^2 are production functions of tourism service and agricultural good respectively. These production functions are homogenous and strictly concave. Marginal product of each factor is positive and diminishing.
e, in the production function of the agricultural sector is expressed as follows.

$$e = \frac{\overline{E} - \beta X_T}{\overline{E}} \quad (3);$$

where e represents the environmental quality of the economy. $e = 1$ specifies best condition in environment. Decrease in e means degradation in environmental quality. $\overline{E}$ represents the environment endowment without any pollution in the economy. β is the amount of pollution which the tourism sector generates for producing one unit of tourism good. Here we assume that the tourism activity pollutes the environment[15].
Equations (2) and (3) simultaneously describe the relationship between the agricultural and the tourism sector. Due to tourism sector environment is polluted and that is represented by the term 'e ' in equation (3). 'e' is also present in equation (2) which is production function of agricultural sector. If there is no pollution then $e = 1$. Due to pollution value of 'e' is less than unity and productivity of the agricultural sector is reduced. Our empirical relationship presented in Introduction section justified this kind of relationship between tourism sector and agricultural sector.
We consider the markets are perfectly competitive. So the profit maximising conditions are

$$P_T = a_{UT}W + a_{KT}r \quad (4);$$

$$P_2 = a_{U2}W + a_{N2}R \quad (5);$$

The market clearing conditions of the three production factors: labour, capital and land are given respectively as follows:

$$a_{UT}X_T + a_{U2}X_2 = \overline{L} \quad (6);$$

$$a_{KT}X_T = \overline{K} \quad (7);$$

$$a_{N2}X_2 = \overline{N} \quad (8);$$

Where $\overline{L}$, $\overline{K}$ and $\overline{N}$ represent the endowment of labour, capital and land respectively.
Demand supply equality in the market for tourism goods is given by

$$D_T(P_T, Y, Y_F) = X_T \quad (9);$$

, with $\frac{\partial D_T}{\partial P_T}.\frac{P_T}{D_T} = e_{P_T} < 0, \frac{\partial D_T}{\partial Y}.\frac{Y}{D_T} = e_Y > 0, \frac{\partial D_T}{\partial Y_F}.\frac{Y_F}{D_T} = e_{Y_F} > 0$
The output equation is given as

$$Y = w\overline{L} + R\overline{N} + r\overline{K} \quad (10).$$

[15] It can be mentioned here that agricultural production also pollutes environment. But, here in this study our focus is on the impact of tourism pollution on the agricultural production. So we ignore the pollution arises from agricultural production.

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In our model P_2 is exogenously given and P_T is endogenously determined by equating domestic demand and supply. There are eight unknowns in the model: e, w, r, R, P_T, X_T, X_2 and Y. Parameters of this system are $\bar{L}$, $\bar{K}$, Y_F and $\bar{N}$. There are eight independent equations from equation (3) to equation (10) with eight unknowns, so the system is solvable. The production structure does not possess the decomposition property and so factor prices cannot be solved independent of factor endowments.

The working of the general equilibrium model is described as follows. Given P_2, $\bar{L}$, $\bar{K}$ and $\bar{N}$, we solve for w, r, R, X_2 and X_T simultaneously in terms of P_T, from equations (4) to (8). Putting the values of w, r, R, X_2 and X_T in equation (10) we get Y in terms of P_T. Putting Y and X_T in equation (9) we solve P_T in terms of Y_F . We get e from equation (3).

Completely differentiating equation (3) to equation (10) and using equation (2) and equation (3), we get

$$\theta_{UT}\widehat{W} + \theta_{KT}\hat{r} = \widehat{P_T} \qquad (11);$$

$$\theta_{U2}\widehat{W} + \theta_{N2}\widehat{R} + \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} = \widehat{P_2} \qquad (12);$$

$$e\hat{e} = -(1-e)\widehat{X_T} \qquad (13);$$

$$\widehat{X_T} = -(S^T_{KU}\widehat{w} + S^T_{KK}\hat{r}) \qquad (14);$$

$$\widehat{X_2} = -(S^T_{NU}\widehat{w} + S^T_{NN}\widehat{R}) \qquad (15);$$

$$\lambda_{UT}\widehat{X_T} + \lambda_{U2}\widehat{X_2} = \lambda_{UT}\widehat{a_{UT}} - \lambda_{U2}\widehat{a_{U2}} = A_1\widehat{w} + A_2\hat{r} + A_3\widehat{R} \qquad (16);$$

Where $A_1 = -\left(\lambda_{UT}S^T_{UU} + \lambda_{U2}S^2_{UU}\right) > 0$; $A_2 = -\lambda_{UT}S^T_{UK} < 0$; $A_3 = -\lambda_{U2}S^2_{UN} < 0$

$$\widehat{X}_T = e_{PT}\widehat{P_T} + e_Y\widehat{Y} + e_{Y_F}\widehat{Y_F} \qquad (17);$$

Where, $e_{PT} < 0, e_Y > 0, e_{Y_F} > 0$

$$\widehat{Y} = \phi_1\widehat{W} + \phi_2\widehat{R} + \phi_3\hat{r} \qquad (18);$$

Where $\phi_1 = \frac{WL}{Y}, \phi_2 = \frac{RN}{Y}, \phi_3 = \frac{Kr}{Y}$

3. Comparative static exercise:

3.1. Effect of change in tourism development:

In this section of the paper, we examine the consequences of an exogenous increase in foreigner's income (Y_F) on w, r, R, X_T and X_2 .

Using Cramer's rule From (11),(12),(14),(15) and (16) we get $\widehat{W}$, $\hat{r}$, $\widehat{R}$, $\widehat{X}_T$ and $\widehat{X_2}$ in terms of $\widehat{P_T}$ only, since $\widehat{P_2} = 0$. Using (17) and (18) in $\widehat{W}$,$\hat{r}$, $\widehat{R}$, $\widehat{X}_T$ and $\widehat{X_2}$(all function of $\widehat{P_T}$) we finally obtain.

$$\widehat{W^*}^{16} = (A_4 + \rho)A_{10}\widehat{Y_F} \qquad (19);$$

Where $A_4 > 0;\ \rho < 0; A_{10} > 0$

$$\widehat{r^*}^{17} = (A_5 + \rho)A_{10}\widehat{Y_F} \qquad (20);$$

Where $A_5 > 0;\ \rho < 0; A_{10} > 0$

$$\widehat{R^*}^{18} = (A_6 + \rho)A_{10}\widehat{Y_F} \qquad (21);$$

Where $A_6 < 0;\ \rho < 0; A_{10} > 0$

$$\widehat{X_T}^{*19} = A_7A_{10}\widehat{Y_F} \qquad (22);$$

Where $A_7 > 0;\ A_{10} > 0$

[16] Equation (19) is derived in Appendix A.
[17] Equation (20) is derived in Appendix A.
[18] Equation (21) is derived in Appendix A.
[19] Equation (22) is derived in Appendix A. .

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$\widehat{X_2}^{*}$[20] $= A_8 A_{10} \widehat{Y_F}$ (23);
Where $A_8 < 0;\ A_{10} > 0$
$\hat{e}^{*}$[21] $= -\frac{(1-e)}{e} A_{10} \widehat{Y_F}$ (24);
where $A_{10} > 0$.
Summing equations (19)-(24), we propose the following.

Proposition 1: *Tourism development causes a rise in the production of tourism related production, degrades environmental quality and reduces the production of agricultural products.*

We explain proposition 1 intuitively as follows. An increase in foreigner's income (Y_F) causes an increase in the demand for tourism services. This raises the price of tourism service. So return to specific factor capital rises along with return to mobile factor labour. Rise in wage rate lowers the return to specific factor land in the agricultural sector. In this specific factor model, rise in wage rate of mobile factor labour is always less than rise in rental rate of specific factor capital. So, on the one hand, wage to rental rate falls in tourism sector which raises the supply of tourism service; on the other hand wage to return to land rises which lowers the supply of agricultural product. Rise in production of tourism service degrade the quality of environment. This fall in environmental quality have some secondary impact on all factor prices. As all factor prices changes in same magnitude due to this secondary impact, so this makes relative wage rental ratio and relative wage to return to labour ratio unchanged. So there is no secondary change also in the supply of tourism services or production of agricultural sector.

3.2. Effect of change in price of agricultural good (P_2):

In this section of the paper, we examine the consequences of an exogenous increase in the price of agricultural good (P_2) on w, r, R, X_T and X_2 .
Using Cramer's rule From (11),(12),(14),(15) and (16) we get $\widehat{W}$, $\hat{r}$, $\widehat{R}$, $\widehat{X}_T$ and $\widehat{X_2}$ in terms of $\widehat{P_T}$ and $\widehat{P_2}$ with $\widehat{Y}_F = 0$. Using (17) and (18) in $\widehat{W}$,$\hat{r}$, $\widehat{R}$, $\widehat{X}_T$ and $\widehat{X_2}$we finally obtain.
$\widehat{W}^{*}$[22] $= (A_4 + \rho + A_4^{'}) A_{10} \widehat{P_2}$ (19.1);
where $A_4 > 0;\ \rho < 0; A_4^{'} > 0; A_{10} > 0$.
$\hat{r}^{*}$[23] $= (A_5 + \rho + A_5^{'}) A_{10} \widehat{P_2}$ (20.1);
where $A_5 > 0;\ \rho < 0; A_{10} > 0; A_5^{'} < 0$.
$\widehat{R}^{*}$[24] $= (A_6 + \rho + A_6^{'}) A_{10} \widehat{P_2}$ (21.1);
where $A_6 < 0;\ \rho < 0; A_{10} > 0; A_6^{'} > 0$.
$\widehat{X_T}^{*}$[25] $= (A_{10}^{'} - 1) A_7 \widehat{P_2} > 0$ (22.1);
where $A_7 > 0; A_7^{'} > 0;\ A_{10} > 0$.
$\widehat{X_2}^{*}$[26] $= (A_8 + A_8^{'}) A_{10} \widehat{P_2} > 0$ (23.1);

[20] Equation (23) is derived in Appendix A.
[21] Equation (24) is derived in Appendix A.

[22] Equation (19.1) is derived in Appendix A.1.
[23] Equation (20.1) is derived in Appendix A.1.
[24] Equation (21.1) is derived in Appendix A.1.
[25] Equation (22.1) is derived in Appendix A.1.

[26] Equation (23.1) is derived in Appendix A.1.

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where $A_8 > 0; A_8' > 0;\ A_{10} > 0.$ $\hat{e}^{*}$[27] $= -\frac{(1-e)}{e} A_{10}\hat{P_2}; A_{10} > 0$ (24.1).

Summing equations (19.1)-(24.1), we propose the following.

Proposition 2: *An increase in the price of agricultural goods increases the production of tourism goods, decreases the production of agricultural goods and degrades environment more when the rise in the price of agricultural goods is less than the rise in tourism goods.*

We explain proposition 2 intuitively as follows. An increase in price of agricultural goods raises return to its factors, i.e., wage rate and land rental rate. Though, as land is specific to agricultural sector its return rises more than return to labour. Rise in return to land lowers the return to capital. Rise in wage rate and land rental rate outweigh the effect of fall in rental rate on national income. Thus national income rises due rise is price of agricultural goods. Also production of tourism goods falls initially due to rise in price of agricultural goods. All these raise the price of tourism goods. Now if this rise in price of tourism goods is lower (higher) than rise in price of agricultural goods then finally there is fall (rise) in tourism services and improvement (degradation) in environmental quality in the economy. The change in environmental quality has some secondary impact on all variables. But, just like previous case due to equal opposite forces they are cancel out.

4. Conclusion

This paper develops a small open economy two sector general equilibrium model with three factors of production: skilled labour, land and capital. Two sectors are the agricultural sector and the tourism sector. There are two production functions of the agricultural and tourism sectors, respectively. Agricultural production is dependent on the environment and the economy. The paper shows the effect of tourism development on the production of tourism goods, agricultural goods, and environmental quality. Tourism development due to an increase in foreigners' income causes a rise in the production of tourism-related products, degrades environmental quality and reduces the production of agricultural products. An increase in the price of agricultural goods increases the output of tourism goods, decreases the production of agricultural goods and degrades the environment more when the rise in the price of agricultural goods is less than the rise in the price of tourism goods. So our study shows that tourism development measured by rise in foreigner's income is always accompanied with environmental pollution and degrades the agricultural production. We get this result empirically from literature as mentioned in the Introduction section. The present study gives the theoretical justification of the same and bridges the gap between theory and empirics. Also rise in price of agricultural good raise production of tourism output and eventually pollute environment. Last result cannot be obtained in a standard Heckscher-Ohlin trade set up. Our study has important policy implications. Tourism development that is accompanied by environmental pollution negatively impacts agricultural production. The government should introduce policies like environmental pollution abatement by taxing the tourism sector to protect the agricultural sector. However, our model only introduces some essential aspects of reality. The problem of the imperfection of markets is not considered here. Also, we consider a static model where environmental quality and capital do not accumulate over time. In reality, they change over time. We plan to do further research to remove the significant problems above.

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[27] Equation (24.1) is derived in Appendix A.1.

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Appendix A

Completely differentiating equations (4)-(8), we get

$\theta_{UT}\widehat{w} + \theta_{KT}\widehat{R} = \widehat{P_T}$ (A.1);

$\theta_{U2}\widehat{w} + \theta_{N2}\widehat{R} + \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)}\widehat{X_T} = \widehat{P_2} = 0$ (A.2);

$\widehat{X_T} = -(S^T_{KU}\widehat{w} + S^T_{KK}\hat{r})$ (A.3);

$\widehat{X_2} = -(S^T_{NU}\widehat{w} + S^T_{NN}\widehat{R})$ (A.4);

$\lambda_{UT}\widehat{X_T} + \lambda_{U2}\widehat{X_2} = \lambda_{UT}\widehat{a_{UT}} - \lambda_{U2}\widehat{a_{U2}} = A_1\widehat{w} + A_2\hat{r} + A_3\widehat{R}$ (A.5);

where,

$A_1 = -(\lambda_{UT}S^T_{UU} + \lambda_{U2}S^2_{UU}) > 0$; $A_2 = -\lambda_{UT}S^T_{UK} < 0$; $A_3 = -\lambda_{U2}S^2_{UN} < 0$.

Derivation of equation (A.2)

From equation (2), we get

$X_2 = eF^2(L_2, N)$

$\Rightarrow 1 = \frac{e}{X_2}F^2(L_{U2}, N) = eF^2(a_{U2}, a_{N2})$

By complete differentiation we get

$\Rightarrow 0 = F^2(a_{U2}, a_{N2})de + e\{\frac{\partial F^2}{\partial a_{U2}}da_{U2} + \frac{\partial F^2}{\partial a_{N2}}da_{N2}\}$

$\Rightarrow 0 = F^2(a_{U2}, a_{N2})de + e\{wda_{U2} + Rda_{N2}\}$ Since $\frac{\partial F^2}{\partial a_{U2}} = w$ and $\frac{\partial F^2}{\partial a_{N2}} = R$

$\Rightarrow 0 = F^2(a_{U2}, a_{N2})\frac{de}{e} + \{wda_{U2} + Rda_{N2}\}$

$\Rightarrow wda_{U2} + Rda_{N2} = -\frac{\hat{e}}{e}$, Since $F^2(a_{U2}, a_{N2}) = \frac{1}{e}$

Completely differentiating equation (5), we get

$\widehat{P_2} = \theta_{U2}\widehat{W} + \theta_{N2}\widehat{R} + wda_{U2} + Rda_{N2}$

$\Rightarrow \widehat{P_2} = \theta_{U2}\widehat{W} + \theta_{N2}\widehat{R} - \frac{\hat{e}}{eP_2}$

Now Totally differentiating equation (3), we get

$e\hat{e} = -(1-e)\widehat{X_T}$

$\Rightarrow \hat{e} = -\frac{(1-e)}{e}\widehat{X_T}$ (A.6).

Now, $(1-e) = 1 - \frac{\bar{E}-\beta X_T}{\bar{E}} = \frac{\beta X_T}{\bar{E}}$

Therefore $\frac{(1-e)}{e} = -\frac{\beta X_T}{\bar{E}-\beta X_T}$

So, $-\frac{\hat{e}}{eP_2} = \frac{\beta X_T}{\bar{E}-\beta X_T}\widehat{X_T}$

We can write the equation system as

$$\begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S^T_{KU} & S^T_{KK} & 0 & 1 & 0 \\ S^2_{NU} & 0 & S^2_{NN} & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} \begin{bmatrix} \widehat{W} \\ \hat{r} \\ \widehat{R} \\ \widehat{X_T} \\ \widehat{X_2} \end{bmatrix} = \begin{bmatrix} 0 \\ \widehat{P_2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using equations (A.1)-(A.5) , we have

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$$\widehat{W} = \begin{bmatrix} 0 & \theta_{KT} & 0 & 0 & 0 \\ \widehat{P}_2 & 0 & \theta_{N2} & \dfrac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ 0 & S^T_{KK} & 0 & 1 & 0 \\ 0 & 0 & S^2_{NN} & 0 & 1 \\ 0 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} / |D|$$

$$\text{where,} |D| = \begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S^T_{KU} & S^T_{KK} & 0 & 1 & 0 \\ S^2_{NU} & 0 & S^2_{NN} & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix}$$

Therefore,

$$\widehat{W} = \frac{-\widehat{P_T}}{|D|}\theta_{N2}\{-S^T_{KK}\lambda_{UT} - A_2\} + \widehat{P_T}\frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S^T_{KK}\{\lambda_{U2}S^2_{NN} + A_3\}$$

$$\Rightarrow \widehat{W} = A_4\widehat{P_T} + \boldsymbol{\rho}\widehat{P_T} \qquad \text{(A.7);}$$

where $A_4 = \frac{-\theta_{N2}\{-S^T_{KK}\lambda_{UT}-A_2\}}{|D|}$>0 since $|D| < 0, A_2 < 0, S^T_{KK} < 0$.

And $\rho = \frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S^T_{KK}\{\lambda_{U2}S^2_{NN} + A_3\}$<0 since $|D| < 0, A_3 < 0, S^T_{KK} < 0$

Similarly,

$$\hat{r} = \begin{bmatrix} \theta_{UT} & 0 & 0 & 0 & 0 \\ \theta_{U2} & \widehat{P}_2 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S^T_{KU} & 0 & 0 & 1 & 0 \\ S^2_{NU} & 0 & S^2_{NN} & 0 & 1 \\ -A_1 & 0 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} / |D|$$

$$\text{where } |D| = \begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S^T_{KU} & S^T_{KK} & 0 & 1 & 0 \\ S^2_{NU} & 0 & S^2_{NN} & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix}$$

Therefore,

$$\hat{r} = \frac{-\widehat{P}_T S^T_{KU}\theta_{N2}\lambda_{UT} + \widehat{P}_T\theta_{U2}(A_3 + S^2_{NN}\lambda_{U2}) - \widehat{P}_T\theta_{N2}(A_1 + S^2_{NU}\lambda_{U2})}{|D|} + \widehat{P}_T\frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S^T_{KK}\{\lambda_{U2}S^2_{NN} + A_3\}$$

$$\Rightarrow \hat{r} = A_5\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T \qquad \text{(A.8).}$$

where, $A_5 = \frac{-\widehat{P}_T S^T_{KU}\theta_{N2}\lambda_{UT} + \widehat{P}_T\theta_{U2}(A_3+S^2_{NN}\lambda_{U2}) - \widehat{P}_T\theta_{N2}(A_1+S^2_{NU}\lambda_{U2})}{|D|}$>0

since $|D| < 0, A_1 > 0, S^2_{NN} < 0, S^T_{KU} >, S^2_{NU} > 0$

$\rho = \frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S^T_{KK}\{\lambda_{U2}S^2_{NN} + A_3\}$<0 since $|D| < 0, A_3 < 0, S^T_{KK} < 0$.

$$\widehat{R}=\begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \widehat{P}_2 & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & 0 & 0 & 1 \\ -A_1 & -A_2 & 0 & \lambda_{UT} & \lambda_{U2} \end{bmatrix}/|D|$$

$$\Rightarrow \widehat{R} = \frac{\theta_{U2}}{|D|}\widehat{P}_T\{-S_{KK}^T\lambda_{UT} - A_2\} + \widehat{P}_T\frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S_{KK}^T\{\lambda_{U2}S_{NN}^2 + A_3\}$$

$$\Rightarrow \widehat{R} = A_6\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T \qquad \text{(A.9);}$$

where $A_6 = \frac{\theta_{U2}}{|D|}\widehat{P}_T\{-S_{KK}^T\lambda_{UT} - A_2\}<0$,since $A_2 < 0, |D| < 0$,

and $\boldsymbol{\rho} = \frac{\beta X_T}{|D|eP_2(\bar{E}-\beta X_T)}S_{KK}^T\{\lambda_{U2}S_{NN}^2 + A_3\}<0$.

Now, from equation (A.3), we get

$$\widehat{X_T} = -(S_{KU}^T\widehat{W} + S_{KK}^T\hat{r})$$

$$\Rightarrow \widehat{X_T} = -S_{KK}^T(\hat{r} - \widehat{W})$$

Putting equations (A.6) and (A.7), we get

$$\widehat{X_T} = -S_{KK}^T\left(A_5\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T - A_4\widehat{P}_T - \boldsymbol{\rho}\widehat{P}_T\right) = -S_{KK}^T\left(A_5\widehat{P}_T - A_4\widehat{P}_T\right)$$

Or, $\widehat{X_T} = -S_{KK}^T\widehat{P}_T(A_5 - A_4)\widehat{P}_T$

So using the property of $S_{ii}^h + S_{ij}^h = 0$ and $\theta_{ih+}\theta_{jh} = 1$

Where h=T,2 and i,j=L,K,N

$$(A_5 - A_4) = \frac{-\widehat{P}_T S_{KU}^T\theta_{N2}\lambda_{UT} + \widehat{P}_T\theta_{U2}(A_3 + S_{NN}^2\lambda_{U2}) - \widehat{P}_T\theta_{N2}(A_1 + S_{NU}^2\lambda_{U2})}{|D|} + \frac{\theta_{N2}\{-S_{KK}^T\lambda_{UT} - A_2\}}{|D|}$$

Putting the values of A_1, A_2 and A_3 we get

$$(A_5 - A_4) = \frac{\hat{p}_T(-\lambda_{UT}\theta_{N2}S_{KU}^T + \lambda_{U2}\theta_{U2}S_{NN}^2 - \lambda_{U2}\theta_{U2}S_{UN}^2 - \lambda_{U2}\theta_{N2}S_{NU}^2 + \lambda_{UT}\theta_{N2}S_{UU}^T + \lambda_{U2}\theta_{N2}S_{UU}^2 - \lambda_{UT}\theta_{N2}S_{KK}^T + \lambda_{UT}\theta_{N2}S_{UK}^T)}{|D|}$$

$$\Rightarrow (A_5 - A_4) = \frac{\{-\lambda_{UT}\theta_{N2}(S_{KU}^T + S_{KK}^T) + \lambda_{U2}\theta_{U2}S_{NN}^2 - \lambda_{U2}\theta_{U2}S_{UN}^2 - \lambda_{U2}\theta_{N2}S_{NU}^2 + \lambda_{UT}\theta_{N2}(S_{UU}^T + +S_{UK}^T) + \lambda_{U2}\theta_{N2}S_{UU}^2\}}{|D|}\widehat{P}_T$$

$$\Rightarrow (A_5 - A_4) = \frac{\{\lambda_{U2}\theta_{U2}S_{NN}^2 + \lambda_{U2}\theta_{N2}S_{UU}^2 - \lambda_{U2}\theta_{U2}S_{UN}^2 - \lambda_{U2}\theta_{N2}S_{NU}^2\}}{|D|}\widehat{P}_T,$$

Since $(S_{KU}^T + S_{KK}^T) = 0$ and $(S_{UU}^T + +S_{UK}^T) = 0$

$\Rightarrow (A_5 - A_4) = \frac{\{\lambda_{U2}(\theta_{N2}S_{UU}^2 + \theta_{U2}S_{UU}^2) + \{\lambda_{U2}(\theta_{N2}S_{NN}^2 + \theta_{U2}S_{NN}^2)\}}{|D|}\widehat{P}_T$ Since, $S_{UU}^2 + S_{UN}^2 = 0$ and $S_{UU}^2 + S_{UN}^2 = 0$ and $S_{NU}^2 + S_{NN}^2 = 0$.

Using the condition $\theta_{N2} + \theta_{U2} = 1$

We get $(A_5 - A_4) = \frac{\lambda_{U2}(S_{UU}^2 + S_{NN}^2)}{|D|}>$ since $|D| < 0, \lambda_{U2}(S_{UU}^2 + S_{NN}^2) < 0$

Therefore $\widehat{X_T} > 0$

So we can write $\widehat{X_T} = A_7\widehat{P}_T$ (A.10);

where $A_7 = -S_{KK}^2(A_5 - A_4) > 0$

Similarly from equation (A.4), we get

$$\widehat{X_2} = -(S_{NU}^T\widehat{W} + S_{NN}^T\widehat{R})$$

$$\Rightarrow \widehat{X_2} = -S_{NN}^2\left(\widehat{R} - \widehat{W}\right)$$

Putting equations (A.6) and (A.8), we have

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$\widehat{X_2} = -S^2_{NN}(A_6 - A_4)\widehat{P_T}$

where, $A_6 - A_4 = \frac{\theta_{U2}}{|D|}\widehat{P_T}\{-S^T_{KK}\lambda_{UT} - A_2\} + \widehat{P_T}\frac{\theta_{N2}\{-S^T_{KK}\lambda_{UT}-A_2\}}{|D|}$.

Or, $A_6 - A_4 = \frac{\theta_{U2}+\theta_{N2}}{|D|}\widehat{P_T}\{-S^T_{KK}\lambda_{UT} - A_2\}<0$

Since $S^T_{KK} < 0$, $A_2 < 0$, $|D| < 0$

We get $(A_6 - A_4) < 0$ Since numerator> and $|D| < 0$

Therefore,$\widehat{X_2} < 0$

So we can write $\widehat{X_2} = A_8\widehat{P_T}$ (A.11);

Where, $A_8 = -S^2_{NN}(A_6 - A_4) < 0$.

Totally differentiating equation(10), we get

$\widehat{Y} = \frac{WL}{Y}\widehat{W} + \frac{RN}{Y}\widehat{R} + \frac{Kr}{Y}\widehat{r}$

$\Rightarrow\widehat{Y} = \phi_1\widehat{W} + \phi_2\widehat{R} + \phi_3\widehat{r}$ (A.12);

where $\phi_1 = \frac{WL}{Y}$, $\phi_2 = \frac{RN}{Y}$, $\phi_3 = \frac{Kr}{Y}$.

Putting (A.6),(A.7) and (A.8) in (A.11) we get

$\widehat{Y} = A_9\widehat{P_T}$ (A.13);

where $A_9 = \{\phi_1(A_4 + \boldsymbol{\rho}) + \phi_2(A_6 + \boldsymbol{\rho}) + \phi_3(A_5 + \boldsymbol{\rho})\}$.

Totally differentiating equation(9), we get

$\widehat{X}_T = e_{PT}\widehat{P_T} + e_Y\widehat{Y} + e_{Y_F}\widehat{Y_F}$ (A.14).

$e_{PT} < 0, e_Y > 0, e_{Y_F} > 0$

Now, putting (A.9) and (A.12) in (A.13) we get

$e_{PT}\widehat{P_T} + e_YA_9\widehat{P_T} - A_7\widehat{P_T} = -e_{Y_F}\widehat{Y_F}$

$\Rightarrow\widehat{P_T} = A_{10}\widehat{Y_F}$ (A.15);

Where $A_{10} = \frac{-e_{Y_F}\widehat{Y_F}}{e_{PT}+e_YA_9-A_7} > 0$.

Stability condition implies Excess demand <0.

Therefore $e_{PT} + e_YA_9 - A_7 < 0$ this implies $\widehat{P_T} > 0$

So putting (A.15) in (A.7),(A.8),(A.9), (A.10), (A.11) and (A.6) we get

$\widehat{W} = (A_4 + \boldsymbol{\rho})A_{10}\widehat{Y_F}$ (A.16).

Equation(A.16) is same as equation (19) in the main paper

$\widehat{r} = (A_5 + \boldsymbol{\rho})A_{10}\widehat{Y_F}$ (A.17).

Equation(A.17) is same as equation (20) in the main paper

$\widehat{R} = (A_6 + \boldsymbol{\rho})A_{10}\widehat{Y_F}$ (A.18).

Equation(A.18) is same as equation (21) in the main paper

$\widehat{X_T} = A_7A_{10}\widehat{Y_F}$ (A.19).

Equation(A.19) is same as equation (22) in the main paper

$\widehat{X_2} = A_8A_{10}\widehat{Y_F}$ (A.20).

Equation(A.20) is same as equation (23) in the main paper

$\widehat{e} = -\frac{(1-e)}{e}A_{10}\widehat{Y_F}$ (A.21).

Equation (A.21) is same as equation (24) in the main paper.

Relationship between $\frac{\widehat{W}}{\widehat{P_T}}$ and $\frac{\widehat{r}}{\widehat{P_T}}$:

Now, $\frac{\widehat{r}}{\widehat{P_T}} = A_5 + \boldsymbol{\rho}$

$\Rightarrow\frac{\widehat{r}}{\widehat{p_t}} = \frac{-S^T_{KU}\theta_{N2}\lambda_{UT}\theta_{U2}(A_3+S^2_{NN}\lambda_{U2})+\theta_{U2}(A_3+S^2_{NN}\lambda_{U2})-\theta_{N2}(A_1+S^2_{NU}\lambda_{U2})}{|D|} + \frac{\beta X_T}{|D|eP_2(\overline{E}-\beta X_T)}S^T_{KK}\{\lambda_{U2}S^2_{NN} + A_3\}$

Using the property of $S^h_{ii} + S^h_{ij} = 0$, we get

$\frac{\hat{r}}{\hat{p_t}} = \frac{\theta_{U2}(A_3 - S_{NU}^2\lambda_{U2}) - \theta_{U2}(A_3 + S_{NN}^2\lambda_{U2}) - \theta_{N2}A_1 - \theta_{N2}S_{NU}^2\lambda_{U2}}{|D|} + \boldsymbol{\rho}$

where $\boldsymbol{\rho} = \frac{\beta X_T}{|D| e P_2(\bar{E} - \beta X_T)} S_{KK}^T\{\lambda_{U2}S_{NN}^2 + A_3\}$.

$\Rightarrow \frac{\hat{r}}{\hat{p_T}} = \frac{-\lambda_{U2}S_{NU}^2 + \theta_{U2}A_3 - \theta_{N2}A_1 - S_{KU}^T\lambda_{UT}\theta_{N2}}{|D|} + \rho$, Since $\theta_{U2} + \theta_{N2} = 1$

$\Rightarrow \frac{\hat{r}}{\hat{p_T}} = \frac{-\lambda_{U2}S_{NU}^2 - \lambda_{U2}S_{UN}^2 + \theta_{N2}\lambda_{UT}S_{NU}^T - S_{KU}^T\lambda_{UT}\theta_{N2}}{|D|} + \rho$ (Putting the values of A_3 and A_1 and using $\theta_{N2} + \theta_{U2} = 1$ and $S_{ij}^h + S_{ii}^h = 0$)

$\Rightarrow \frac{\hat{r}}{\hat{p_T}} = \frac{-\lambda_{U2}(S_{NU}^2 + S_{UN}^2)}{|D|} + \frac{\hat{W}}{\hat{p_T}}$

Now $\frac{-\lambda_{U2}(S_{NU}^2 + S_{UN}^2)}{|D|} > as|D| < 0$

Therefore $\frac{\hat{r}}{\hat{p_T}} > \frac{\hat{W}}{\hat{p_T}}$.

Relationship between $\frac{\hat{W}}{\hat{P_T}}$ and $\frac{\hat{R}}{\hat{p_T}}$:

Now, $\frac{\hat{R}}{\hat{P_T}} = A_6 + \boldsymbol{\rho}$

$\Rightarrow \frac{\hat{R}}{\hat{P_T}} = \frac{\theta_{U2}}{|D|}\{-S_{KK}^T\lambda_{UT} - A_2\} + \boldsymbol{\rho}$

$\Rightarrow \frac{\hat{R}}{\hat{P_T}} = \frac{\theta_{N2}S_{KK}^T\lambda_{UT} + \theta_{N2}A_2 - A_2 - S_{KK}^T\lambda_{UT}}{|D|} + \boldsymbol{\rho}$

Putting A_2 and using $S_{ii}^h + S_{ij}^h = 0$, we get

$\frac{\hat{R}}{\hat{P_T}} = \frac{\hat{\mathbf{W}}}{\hat{\mathbf{P_T}}} + \frac{\lambda_{UT}(S_{UK}^T - S_{KK}^T)}{|D|}$.

Appendix A.1

Completely differentiating equations (4)-(8), we get

$\theta_{UT}\hat{w} + \theta_{KT}\hat{R} = \hat{P_T}$ (A.1.1);

$\theta_{U2}\hat{w} + \theta_{N2}\hat{R} + \frac{\beta X_T}{eP_2(\bar{E} - \beta X_T)} = \hat{P_2}$ (A.2.1);

$\hat{X_T} = -(S_{KU}^T\hat{w} + S_{KK}^T\hat{r})$ (A.3.1);

$\hat{X_2} = -(S_{NU}^T\hat{w} + S_{NN}^T\hat{R})$ (A.4.1);

$\lambda_{UT}\hat{X_T} + \lambda_{U2}\hat{X_2} = \lambda_{UT}\hat{a_{UT}} - \lambda_{U2}\hat{a_{U2}} = A_1\hat{w} + A_2\hat{r} + A_3\hat{R}$ (A.5.1).;

where $A_1 = -(\lambda_{UT}S_{UU}^T + \lambda_{U2}S_{UU}^2) > 0$; $A_2 = -\lambda_{UT}S_{UK}^T < 0$; $A_3 = -\lambda_{U2}S_{UN}^2 < 0$

Derivation of equation (A.2.1)

From Equation (2) we get

$X_2 = eF^2(L_2, N)$

$\Rightarrow 1 = \frac{e}{X_2}F^2(L_{U2}, N) = eF^2(a_{U2}, a_{N2})$

By complete differentiation we get

$0 = F^2(a_{U2}, a_{N2})de + e\{\frac{\partial F^2}{\partial a_{U2}}da_{U2} + \frac{\partial F^2}{\partial a_{N2}}da_{N2}\}$

$\Rightarrow 0 = F^2(a_{U2}, a_{N2})de + e\{wda_{U2} + Rda_{N2}\}$ Since $\frac{\partial F^2}{\partial a_{U2}} = w$ and $\frac{\partial F^2}{\partial a_{N2}} = R$

$\Rightarrow 0 = F^2(a_{U2}, a_{N2})\frac{de}{e} + \{wda_{U2} + Rda_{N2}\}$

$\Rightarrow wda_{U2} + Rda_{N2} = -\frac{\hat{e}}{e}$, Since $F^2(a_{U2}, a_{N2}) = \frac{1}{e}$

Completely differentiating Equation(5) we get

$\widehat{P_2} = \theta_{U2}\widehat{W} + \theta_{N2}\widehat{R} + wda_{U2} + Rda_{N2}$

$\Rightarrow \widehat{P_2} = \theta_{U2}\widehat{W} + \theta_{N2}\widehat{R} - \frac{\hat{e}}{eP_2}$

Now Totally differentiating Equation(3.1) we get

$e\hat{e} = -(1-e)\widehat{X_T}$

$\Rightarrow \hat{e} = -\frac{(1-e)}{e}\widehat{X_T}$ (A.6.1).

Now, $(1-e) = 1 - \frac{\bar{E}-\beta X_T}{\bar{E}} = \frac{\beta X_T}{\bar{E}}$

Therefore $\frac{(1-e)}{e} = -\frac{\beta X_T}{\bar{E}-\beta X_T}$

So, $-\frac{\hat{e}}{eP_2} = \frac{\beta X_T}{\bar{E}-\beta X_T}\widehat{X_T}$

We can write the equation system as

$$\begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & S_{NN}^2 & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} \begin{bmatrix} \widehat{W} \\ \hat{r} \\ \widehat{R} \\ \widehat{X_T} \\ \widehat{X_2} \end{bmatrix} = \begin{bmatrix} \widehat{P_T} \\ \widehat{P_2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using equations (A.1.1)- (A.5.1), we get

$$\widehat{W} = \begin{bmatrix} \widehat{P_T} & \theta_{KT} & 0 & 0 & 0 \\ \widehat{P_2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ 0 & S_{KK}^T & 0 & 1 & 0 \\ 0 & 0 & S_{NN}^2 & 0 & 1 \\ 0 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} / |D|$$

$$\text{Where, } |D| = \begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & S_{NN}^2 & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} < 0$$

$\Rightarrow \widehat{W} = \frac{\{\theta_{N2}(\lambda_{UT}S_{KK}^T + A_2) + aS_{KK}^T(\lambda_{U2}S_{NN}^2 + A_3)\}}{|D|}\widehat{P_T} + \frac{\theta_{KT}(\lambda_{U2}S_{NN}^2 + A_3)}{|D|}\widehat{P_2}.$

$\Rightarrow \widehat{W} = A_4\widehat{P_T} + \boldsymbol{\rho}\widehat{P_T} + A_4'\widehat{P_2}.$ (A.7.1)

where, $A_4 = \frac{\theta_{N2}(\lambda_{UT}S_{KK}^T + A_2)}{|D|} > 0$; $A_4' = \frac{\theta_{KT}(\lambda_{U2}S_{NN}^2 + A_3)}{|D|} > 0$; $\rho = \frac{aS_{KK}^T(\lambda_{U2}S_{NN}^2 + A_3)}{|D|} < 0$;

$|D| < 0, A_3 < 0, S_{KK}^T < 0$; $a = \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)}$.

Similarly,

$$\hat{r} = \begin{bmatrix} \theta_{UT} & \widehat{P_T} & 0 & 0 & 0 \\ \theta_{U2} & \widehat{P_2} & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & 0 & 0 & 1 & 0 \\ S_{NU}^2 & 0 & S_{NN}^2 & 0 & 1 \\ -A_1 & 0 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} / |D|$$

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where $|D| = \begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & S_{NN}^2 & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix}$

$\Rightarrow \hat{r} = \frac{\{\lambda_{U2}(S_{UU}^2+S_{NN}^2)+\lambda_{UT}(S_{UU}^T+S_{UU}^T)\}}{|D|}\widehat{P}_T + \frac{aS_{KK}^T(\lambda_{U2}S_{NN}^2+A_3)}{|D|}\widehat{P}_T - \frac{(\lambda_{U2}S_{NN}^2+A_3)\theta_{UT}}{|D|}\widehat{P}_2.$

$\Rightarrow \hat{r} = A_5\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T + A_5'\widehat{P}_2$ (A.8.1);

where$A_5 = \frac{\{\lambda_{U2}(S_{UU}^2+S_{NN}^2)+\lambda_{UT}(S_{KK}^T+S_{UU}^T)\}}{|D|} > 0; \frac{aS_{KK}^T(\lambda_{U2}S_{NN}^2+A_3)}{|D|} = \rho < 0$; $|D| < 0, A_3 < 0, S_{KK}^T < 0$; $A_5' = \frac{-\theta_{UT}(\lambda_{U2}S_{NN}^2+A_3)}{|D|} < 0$; $a = \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)}$.

$\widehat{R} = \begin{bmatrix} \theta_{UT} & \theta_{KT} & \widehat{P}_T & 0 & 0 \\ \theta_{U2} & 0 & \widehat{P}_2 & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & 0 & 0 & 1 \\ -A_1 & -A_2 & 0 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} / |D|$

where, $|D| = \begin{bmatrix} \theta_{UT} & \theta_{KT} & 0 & 0 & 0 \\ \theta_{U2} & 0 & \theta_{N2} & \frac{\beta X_T}{eP_2(\bar{E}-\beta X_T)} & 0 \\ S_{KU}^T & S_{KK}^T & 0 & 1 & 0 \\ S_{NU}^2 & 0 & S_{NN}^2 & 0 & 1 \\ -A_1 & -A_2 & -A_3 & \lambda_{UT} & \lambda_{U2} \end{bmatrix} < 0.$

$\Rightarrow \widehat{R} = \frac{\{-\theta_{U2}(\lambda_{UT}S_{KK}^T+A_2)+aS_{KK}^T(\lambda_{U2}S_{NN}^2+A_3)\}}{|D|}\widehat{P}_T + \frac{\lambda_{UT}(S_{KK}^T+S_{UU}^T)+\lambda_{U2}\theta_{KT}(S_{NN}^2+S_{UU}^2)}{|D|}\widehat{P}_2$

$\Rightarrow \widehat{R} = A_6\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T + A_6'\widehat{P}_2$ (A.9.1);

where $A_6 = \frac{-\theta_{U2}(\lambda_{UT}S_{KK}^T+A_2)}{|D|} < 0; \rho = \frac{aS_{KK}^T(\lambda_{U2}S_{NN}^2+A_3)}{|D|} < 0$; $A_6' = \frac{\lambda_{UT}(S_{KK}^T+S_{UU}^T)+\lambda_{U2}\theta_{KT}(S_{NN}^2+S_{UU}^2)}{|D|} > 0.$

$$\widehat{X_T} = -(S_{KU}^T\hat{w} + S_{KK}^T\hat{r})$$

$\Rightarrow \widehat{X_T} = -S_{KK}^T(\hat{r} - \hat{w})$

$\Rightarrow \widehat{X_T} = -S_{KK}^T(A_5\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T + A_5'\widehat{P}_T - A_4\widehat{P}_T - \boldsymbol{\rho}\widehat{P}_T - A_4'\widehat{P}_T)$

$\Rightarrow \widehat{X_T} = -S_{KK}^T\{(A_5\widehat{P}_T - A_4\widehat{P}_T) + (A_5'\widehat{P}_2 - A_4'\widehat{P}_2)\}$

Putting the value of A_1, A_2A_5, A_4A_3 we get

$\Rightarrow \widehat{X_T} = -S_{KK}^T\{(\theta_{U2}\lambda_{U2}S_{NN}^2 - \theta_{U2}\lambda_{U2}S_{UN}^2 - \theta_{N2}\lambda_{UT}S_{KU}^T - \theta_{N2}\lambda_{U2}S_{NU}^2 + \theta_{N2}\lambda_{UT}S_{UU}^T + \theta_{N2}\lambda_{U2}S_{UU}^T)/|D| + (A_5' - A_4')\widehat{P}_2\}$

Using $\theta_{N2} + \theta_{U2} = 1$ and $S_{ii}^h + S_{ij}^h = 0$, where $h = T, 2$ and i,j=L,K

We get $\widehat{X_T} = -S_{KK}^T\{(\frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)+\lambda_{UT}\theta_{N2}(S_{KK}^T+S_{UU}^T)-\theta_{N2}(\lambda_{UT}S_{KK}^T-\lambda_{UT}S_{UK}^T}{|D|})+(A_5' - A_4')\widehat{P}_2\}$

$\Rightarrow \widehat{X_T} = -S_{KK}^T\{\frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)\widehat{P}_T}{|D|} - \frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)}{|D|}\widehat{P}_2\}$

$\Rightarrow \widehat{X_T} = A_7\widehat{P}_T + A_7'\widehat{P}_2$ (A.10.1);

where, $A_7 = -S_{KK}^T\frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)}{|D|} > 0; A_7' = -S_{KK}^T\{-\frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)}{|D|}\} < 0.$

Now, $\frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)}{|D|} > 0$ Since $\lambda_{U2}(S_{NN}^2 + S_{UU}^2) < 0$ and $|D| < 0$

$\Rightarrow A'_7 = -A_7$
$\Rightarrow \widehat{X_T} = A_7(\widehat{P}_T - \widehat{P}_2)$
$\Rightarrow \widehat{X_T}$ depends on $(\widehat{P}_T - \widehat{P}_2)$
Similarly,
$\widehat{X_2} = -(S^T_{NU}\widehat{w} + S^T_{NN}\widehat{R})$
$\Rightarrow \widehat{X_2} = -S^T_{NN}(\widehat{R} - \widehat{w})$
$\Rightarrow \widehat{X_2} = -S^T_{NN}(A_6\widehat{P}_T + \boldsymbol{\rho}\widehat{P}_T + A'_6\widehat{P}_T - A_4\widehat{P}_T - \boldsymbol{\rho}\widehat{P}_T - A'_4\widehat{P}_T)$
$\Rightarrow \widehat{X_2} = -S^T_{NN}\{(A_6\widehat{P}_T - A_4\widehat{P}_T) + (A'_6\widehat{P}_2 - A'_4\widehat{P}_2)\}$
Putting the value of $A_1 A_2 A'_6, A'_4, A_3$ and using the property we get $\theta_{N2} + \theta_{U2} = 1$ and $S^h_{ii} + S^h_{ij} = 0$, where $h = T, 2$ and i,j=L,K
$\Rightarrow \widehat{X_2} = -S^T_{NN}\{-\frac{\lambda_{UT}(S^T_{KK}+S^T_{UU})}{|D|}\widehat{P}_T + \frac{\lambda_{UT}(S^T_{KK}+S^T_{UU})}{|D|}\widehat{P}_2\}$
$\Rightarrow \widehat{X_2} = A_8\widehat{P}_T + A'_8\widehat{P}_2$ (A.11.1);
where $A'_8 = -S^T_{NN}\frac{\lambda_{UT}(S^T_{KK}+S^T_{UU})}{|D|} > 0; A_8 = -S^T_{NN}\left\{-\frac{\lambda_{UT}(S^T_{KK}+S^T_{UU})}{|D|}\right\} < 0;$
$|D| < 0$.
$\Rightarrow A_8 = -A'_8$
$\Rightarrow \widehat{X_2} = A_8(\widehat{P}_T - \widehat{P}_2)$
$\Rightarrow \widehat{X_2}$ depends on $(\widehat{P}_T - \widehat{P}_2)$
Totally differentiating Equation(10.1) we get
$$\widehat{Y} = \frac{WL}{Y}\widehat{W} + \frac{RN}{Y}\widehat{R} + \frac{Kr}{Y}\widehat{r}$$
$\Rightarrow \widehat{Y} = \phi_1\widehat{W} + \phi_2\widehat{R} + \phi_3\widehat{r}$ (A.12.1);
where $\phi_1 = \frac{WL}{Y}, \phi_2 = \frac{RN}{Y}, \phi_3 = \frac{Kr}{Y}$.
Putting (A.7.1),(A.8.1) and (A.9.1) in (A.12.1) we get
$\widehat{Y} = A_9\widehat{P_T} + A'_9\widehat{P_2}$ (A.13.1);
Where, $A_9 = \{\phi_1(A_4 + \boldsymbol{\rho}) + \phi_2(A_6 + \boldsymbol{\rho}) + \phi_3(A_5 + \boldsymbol{\rho})\}$
and $A'_9 = (\phi_1 A'_4 + \phi_2 A'_5 + \phi_3 A'_6)\widehat{P}_2$.
$\Rightarrow A_9 - \boldsymbol{\rho} = \phi_1 A_4 + \phi_2 A_6 + \phi_3 A_5$ Since $\phi_1 + \phi_2 + \phi_3 = 1$
Now putting A_4 and A_6 in $A_9 - \boldsymbol{\rho}$ above we get
$A_9 - \boldsymbol{\rho} = \frac{WL}{Y}\frac{\theta_{N2}}{|D|}\lambda_{UT}(S^T_{KK} + S^T_{UU}) - \frac{RN}{Y}\frac{\theta_{U2}\lambda_{UT}}{|D|}(S^T_{KK} + S^T_{UU}) + \phi_3 A_5$
$\Rightarrow A_9 - \boldsymbol{\rho} = \frac{1}{|D|Y}\lambda_{UT}(S^T_{KK} + S^T_{UU})(WL\frac{a_{N2}}{P_2}R - RN\frac{a_{U2}}{P_2}W) + \phi_3 A_5$
$\Rightarrow A_9 - \boldsymbol{\rho} = \frac{1}{|D|Y}\lambda_{UT}(S^T_{KK} + S^T_{UU})\frac{WR}{P_2}(a_{N2}L - a_{U2}N) + \phi_3 A_5$
$\Rightarrow A_9 - \boldsymbol{\rho} = \frac{1}{|D|Y}\lambda_{UT}(S^T_{KK} + S^T_{UU})\frac{WR}{X_2 P_2}(L - a_{U2}X_2) + \phi_3 A_5$
Now $(L - a_{U2}X_2) > 0$ and $A_5 > 0$
$\Rightarrow A_9 > 0$
Now putting A'_4 and A'_5 in A'_9 above we get
$\phi_1 A'_4 + \phi_2 A'_6 + \phi_3 A'_5$
$\Rightarrow \frac{WL}{Y}\frac{\theta_{KT}}{|D|}(\lambda_{U2}S^2_{NN} + A_3) - \frac{rK}{Y}\frac{\theta_{UT}}{|D|}(\lambda_{U2}S^2_{NN} + A_3) + \phi_2 A'_6$
$\Rightarrow \frac{\lambda_{U2}S^2_{NN}+A_3}{|D|}\frac{1}{Y}(WL\theta_{KT} - rK\theta_{UT}) + \phi_2 A'_6$
$\Rightarrow \frac{\lambda_{U2}S^2_{NN}+A_3}{|D|}\frac{1}{Y}(WL\frac{a_{KT}r}{P_T} - rK\frac{a_{UT}W}{P_T}) + \phi_2 A'_6$
$\Rightarrow \frac{\lambda_{U2}S^2_{NN}+A_3}{|D|}\frac{1}{Y}\frac{Wr}{P_T}(La_{KT} - Ka_{UT}) + \phi_2 A'_6$

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$\Rightarrow \frac{\lambda_{U2}S_{NN}^2+A_3}{|D|}\frac{1}{Y}\frac{Wr}{P_T}\frac{1}{X_T}(La_{KT}X_T - Ka_{UT}X_T) + \phi_2 A'_6$

$\Rightarrow \frac{\lambda_{U2}S_{NN}^2+A_3}{|D|}\frac{1}{Y}\frac{Wr}{P_T}\frac{K}{X_T}(L - a_{UT}X_T) + \phi_2 A'_6$

Now $(L - a_{UT}X_T) > 0$ and $A'_6 > 0$

$\Rightarrow A'_9 > 0$

Now $A_4 - A_6 = \frac{\lambda_{UT}(S_{KK}^T+S_{UU}^T)}{|D|}$ and $A'_4 - A'_5 = \frac{\lambda_{U2}(S_{NN}^2+S_{UU}^2)}{|D|}$

So,$A_5 = (A'_4 - A'_5) + \frac{\lambda_{UT}\theta_{N2}(S_{KK}^T+S_{UU}^T)}{|D|} = (A'_4 - A'_5 + A_4)$ and

$$A'_6 = (A_4 - A_6) + \frac{\lambda_{U2}\theta_{KT}(S_{UU}^2 + S_{NN}^2)}{|D|} = (A_4 - A_6 + A'_4)$$

$\Rightarrow A'_5 + A_5 = A'_4 + A_4$ and $A'_6 + A_6 = A'_4 + A_4$

$\Rightarrow A'_5 + A_5 = A'_4 + A_4 = A'_6 + A_6 = b(\text{let})$

Since $A'_4 + A_4 > 0 \Rightarrow b > 0$

Now $A_9 + A'_9 = (b+\boldsymbol{\rho})(\phi_1 + \phi_2 + \phi_3) = A_9 + A'_9 = b + \boldsymbol{\rho}$

$\Rightarrow A'_9 = -(A_9 - \boldsymbol{\rho}) + b$

Putting the above relation in (A.13.1), we get

$\widehat{Y} = A_9(\widehat{P_T} - \widehat{P_2}) + (b + \boldsymbol{\rho})\widehat{P_2}$ (A.15.1).

We know that $A_9 = (\phi_1 A_4 + \phi_2 A_6 + \phi_3 A_5) + \boldsymbol{\rho}$

$\Rightarrow A_9 - \boldsymbol{\rho} = (\phi_1 A_4 + \phi_2 A_6 + \phi_3 A_5) > 0$

Now since we have proved that $A'_9 > 0 \Rightarrow b > (A_9 - \boldsymbol{\rho})$

Totally differentiating equation (9.1), we get

$\widehat{X_T} = e_{PT}\widehat{P_T} + e_Y\widehat{Y} + e_{Y_F}\widehat{Y_F}$ (A.16.1).

$e_{PT} < 0, e_Y > 0, e_{Y_F} > 0$

Now $\widehat{Y_F} = 0$

Now, putting (A.10.1) and (A.13.1) in (A.14.1), we get

$e_{PT}\widehat{P_T} + e_Y(A_9\widehat{P_T} + A'_9\widehat{P_2}) = A_7\widehat{P_T} - A_7\widehat{P_2}$

$\Rightarrow \{e_{PT} + e_Y A_9 - A_7\}\widehat{P_T} = \{A'_7 - e_Y A'_9\}\widehat{P_2}$

$\Rightarrow \widehat{P_T} = \frac{\{A'_7 - e_Y A'_9\}}{\{e_{PT} + e_Y A_9 - A_7\}}\widehat{P_2}$

Using the relation that $A'_9 = -(A_9 - \boldsymbol{\rho}) + b$, we get

$$\widehat{P_T} = \frac{\{e_Y A_9 - e_Y(b + \rho) - A_7\}}{\{e_{PT} + e_Y A_9 - A_7\}}\widehat{P_2}$$

$\Rightarrow \widehat{P_T} = A'_{10}\widehat{P_2}$ Where $A'_{10} > 0$ (A.17.1);

where $A'_{10} = \frac{\{e_Y A_9 - e_Y(b+\rho) - A_7\}}{\{e_{PT} + e_Y A_9 - A_7\}} > 0.$

Stability condition implies Excess demand <0.

$A'_{10} > 1$ if $e_Y(b + \rho) > -e_{PT}$

Therefore $e_{PT} + e_Y A_9 - A_7 < 0$,$A'_7 < 0 A'_9 > 0$ is negative So numerator is also negative.

So putting (A.17.1) in (A.7.1),(A.8.1),(A.9.1), (A.10.1), (A.11.1) and (A.13.1) we get

$\widehat{W^*} = \{A'_{10}(A_4 + \boldsymbol{\rho}) + A'_4\}\widehat{P_2}$ (A.18.1).

Equation (A.18.1) is same as equation (19.1) in the main paper.

$\widehat{r^*} = \{A'_{10}(A_5 + \boldsymbol{\rho}) + A'_5\}\widehat{P_2}$ (A.19.1).

Equation (A.19.1) is same as equation (20.1) in the main paper.

$\widehat{R^*} = (A_6 + \boldsymbol{\rho} + A'_6)A'_{10}\widehat{P_2}$ (A.20.1).

Equation (A.20.1) is same as equation (21.1) in the main paper.

$\widehat{X_T}^* = A_7(A'_{10} - 1)\widehat{P_2}$ is ambiguous where $A_7 = -A'_7$ and $A_7 > 0$ (A.21.1).

Equation (A.21.1) is same as equation (22.1) in the main paper

$\widehat{X_2}^* = A_8(A'_{10} - 1)\widehat{P}_2$ is ambiguous. Where $A_8 = -A'_8$ and $A_8 < 0$ (A.22.1).
Equation (A.22.1) is same as equation (23.1) in the main paper.
$\hat{e} = -\frac{(1-e)}{e} A'_{10}\widehat{P}_2$ (A.23.1).
Equation (A.21) is same as equation (24.1) in the main paper.
$\widehat{Y} = (A_9 A'_{10} + A'_9)\widehat{P}_2$. Where, $A_9 > 0$; $A'_9 > 0$; $A'_{10} > 0$ (A.24.1).

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