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A Survey on Interval-valued Fuzzy Graphs

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ABSTRACT

Interval-valued fuzzy graphs (IVFGs) belonging to the fuzzy graphs (FGs) family, have good capabilities when faced with problems that cannot be expressed by FGs. The notion of an IVFG is a new mathematical attitude to model the ambiguity and uncertainty in decision-making issues. IVFGs are well-articulated, useful and practical tools to manage the uncertainty preoccupied in all real-life difficulties where not sure facts, figures and explorations are explained. In this paper, the concepts of neighbourly irregular interval-valued fuzzy graphs, neighbourly totally irregular interval-valued fuzzy graphs, and highly irregular interval-valued fuzzy graphs are introduced and several examples are presented.

Keywords: Fuzzy set, fuzzy graph, interval-valued fuzzy graph, neighbourly irregular, highly irregular.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Since many parameters in real-world networks are specifically tied to the concept of regularity, this concept has become one of the most widely used concepts in graph theory. However, the regularity concept in FG is so important because of the confrontation with uncertain and ambiguous topics. This concept becomes more interesting when we know that we are dealing with an FG called IVFG. This led us to examine the regularity concept in IVFG. In 1975, Zadeh [30] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets [31] in which the values of the membership degrees are intervals of numbers instead of numbers. Interval-valued fuzzy sets provide a more adequate description of uncertainty than traditional fuzzy sets. It is therefore important to use interval-valued fuzzy sets in the application, such as fuzzy control. In 1975, Rosenfeld [13] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffman [9] in 1973. The fuzzy relation between fuzzy sets was also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogues of several graph theoretical concepts. Bhattacharya [4] gave some remarks on fuzzy graphs. Mordeson and Peng [11] introduced some operations on fuzzy graphs. The complement of a fuzzy graph was defined by Mordeson [10]. Bhutani and Rosenfeld introduced the concept of M-strong fuzzy graphs in [5] and studied some of their properties. Shannon and Atanassov [25] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Hongmei and Lianhua defined an interval-valued graph in [8]. Recently Akram introduced the concepts of bipolar fuzzy graphs and interval-valued fuzzy graphs in [1, 2, 3]. Pal and Rashmanlou [12] studied irregular interval-valued fuzzy graphs. Also, they defined antipodal interval-valued fuzzy graphs [14], balanced interval-valued fuzzy graphs [15] and a study on bipolar fuzzy graphs [16]. Rashmanlou and Yong Bae Jun investigated complete interval-valued fuzzy graphs [17]. Samanta and Pal defined fuzzy tolerance graphs [20], fuzzy threshold graphs [24], fuzzy planar graphs [21], fuzzy k-competition graphs and p-competition fuzzy graphs [22], and irregular bipolar fuzzy graphs [23]. Manr researchers studied new results in fuzzy graphs [6, 7, 18, 19, 26, 27, 28, 29].

In this paper, we present the concepts of neighbourly irregular interval–valued fuzzy graphs, neighbourly totally irregular interval–valued fuzzy graphs, highly irregular interval–valued fuzzy graphs, and highly totally irregular interval–valued fuzzy graphs are introduced and investigated. A necessary and sufficient condition under which neighbourly irregular and highly irregular interval-valued fuzzy graphs are equivalent is discussed.

2. Preliminaries

By a graph, we mean a pair $G^* = (V, E)$, where V is the set and E is a relation on V. The elements of V are vertices of G^* and the elements of E are edges of G^*. We write to mean $(x, y) \in E$, and if $e = xy \in E$, we say x and y are adjacent. Formally, given a graph $G^* = (V, E)$, two vertices $x, y \in V$ are said to be neighbours or adjacent nodes, if $xy \in E$. The number of edges, the cardinality of E, is called the size of the graph and denoted by $|E|$. The number of vertices, the cardinality of V, is called the order of the graph and denoted by $|V|$.

The neighbourhood of a vertex v in a graph G^* is the induced subgraph of G^* consisting of all vertices. The neighbourhood is often denoted $N(v)$. The degree deg(v) of vertex v is the number of edges incident on V or equivalently, $\deg(v) = |N(v)|$. The set of neighbours called a (open) neighbourhood $N(v)$ for a vertex v in a graph G^*, consists of all vertices adjacent to v but not including v, that is $N(v) = \{u \in V \mid uv \in E\}$. When v is also included, it is called a closed neighbour- hood $N[v]$, that is $N[v] = N(v) \cup \{v\}$. A regular graph is a graph where each vertex has some number of neighbours, i.e. all the vertices have the same closed neighbourhood degree. The interval–valued fuzzy set A in V is defined by

$$A = \{(x, [\mu_{A^-(x)}, \mu_{A^+(x)}]) | x \in V\},$$

where $\mu_{A^-(x)}$ and $\mu_{A^+(x)}$ are fuzzy subsets of V such that $\mu_{A^-(x)} \le \mu_{A^+(x)}$ for all $x \in V$. If $G^* = (V, E)$ is a graph, then by an interval-valued fuzzy relation B on a set E we mean an interval- valued fuzzy set such that

$$\mu_{B^-(xy)} \le \min\left(\mu_{A^-(x)}, \mu_{A^-(y)}\right),\ \mu_{B^+(xy)} \le \min\left(\mu_{A^+(x)}, \mu_{A^+(y)}\right),$$

for all $xy \in E$.

3. Some results in Interval-valued fuzzy graphs

Definition 3.1. By an interval-valued fuzzy graph of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A = [\mu_{A^-}, \mu_{A^+}]$ is an interval-valued fuzzy set on V and $B = [\mu_{B^-}, \mu_{B^+}]$ is an interval–valued fuzzy relation on E such that

$$\mu_{B^-(xy)} \le \min\left(\mu_{A^-(x)}, \mu_{A^-(y)}\right)$$

$$\mu_{B^+(xy)} \le \min\left(\mu_{A^+(x)}, \mu_{A^+(y)}\right).$$

Throughout in this paper, G^* is a crisp graph, and G is an interval – valued fuzzy graph.

Definition 3.2. The number of vertices, the cardinality of V, is called the order of an interval-valued fuzzy graph $G = (A, B)$ and denoted by $|V|$ (or $O(G)$), and defined by

$$O(G) = |V| = \sum_{x \in V} \frac{1 + \mu_{A^-(x)} + \mu_{A^+(x)}}{2}.$$

The number of edges, the cardinality of E, is called the size of an interval–valued fuzzy graph $G = (A, B)$ and is denoted by $|E|$ (or $S(G)$), and defined by

$$S(G) = |E| = \sum_{xy \in E} \frac{1 + \mu_{B^-(xy)} + \mu_{B^+(xy)}}{2}.$$

Definition 3.3. Let G be an interval – valued fuzzy graph. The neighbourhood of a vertex x in G is defined by $N(x) = \left(N_\mu(x), N_\nu(x)\right)$, where

$N_\mu(x) = \left\{y \in V : \mu_{B^-(xy)} \le \min\left(\mu_{A^-(x)}, \mu_{A^-(y)}\right)\right\}$ and

$N_\nu(x) = \left\{y \in V : \mu_{B^+(xy)} \le \min\left(\mu_{A^+(x)}, \mu_{A^+(y)}\right)\right\}$.

Definition 3.4. Let G be an interval – valued fuzzy graph. The neighbourhood degrees of vertex x is G is defined by $\deg(x) = \left(\deg_\mu(x), \deg_\nu(x)\right)$, where

$\deg_{\mu}(x) = \sum_{y \in N(x)} \mu_{A^-(y)}$ and $\deg_{\nu}(x) = \sum_{y \in N(x)} \mu_{A^+(y)}$.

Notice that

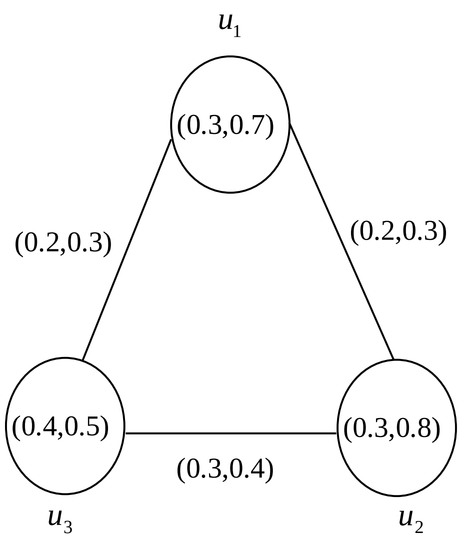
$\mu_{B^-(xy)} > \circ, \mu_{B^+(xy)} > \circ$ for all $xy \in E$, and $\mu_{B^-(xy)} = \mu_{B^+(xy)} = \circ$ for all $xy \notin E$

Definition 3.5. Let G be an interval-valued fuzzy graph on G^*. If there is a vertex that is adjacent to vertices with distinct neighbourhood degrees, then G is called an irregular interval-valued fuzzy graph. That is, $\deg(x) \neq n$ foa all $x \in V$.

Example 3.6. Consider a graph $G^* = (V, E)$ such that $V = \{u_1, u_2, u_3\}$, $E = \{u_1u_2, u_2u_3, u_3u_1\}$. Let A be an interval-valued fuzzy subset of V and let B be an interval-valued fuzzy subset of $E \subseteq V \times V$ defined by

	u_1	u_2	u_3
μ_{A^-}	0.3	0.3	0.4
μ_{A^+}	0.7	0.8	0.5

	$u_1\, u_2$	$u_2\, u_3$	$u_3\, u_1$
μ_{B^-}	0.2	0.3	0.2
μ_{B^+}	0.3	0.4	0.3



By routine computations, we have $\deg(u_1) = (0.7, 1.3)$, $\deg(u_2) = (0.7, 1.2)$ and $\deg(u_3) = (0.6, 1.5)$. It is clear that G is an irregular interval– valued fuzzy graph.

Definition 3.7. Let G be an interval–valued fuzzy graph. The closed neighbour-hood degree of a vertex x is defined by $\deg[x] = (\deg_\mu[x], \deg_\nu[x])$, where

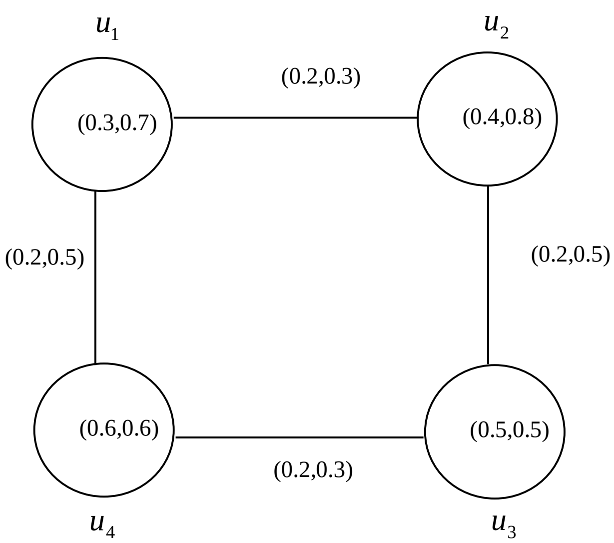
$\deg_\mu[x] = \deg_\mu(x) + \mu_{A^-}(x), \deg_\nu[x] = \deg_\nu(x) + \mu_{A^+}(x).$

If there is a vertex which is adjacent to vertices with distinct closed neighbourhood degrees, then G is called a totally irregular interval-valued fuzzy graph.

Definition 3.8. A connected interval-valued fuzzy graph G is said to be a neighbourly irregular interval-valued fuzzy graph if every two adjacent vertices of G have distinct open neighbourhood degrees.

Example 3.9. Consider an interval–valued fuzzy graph G such that

$V = \{u_1, u_2, u_3, u_4\},\ E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}.$

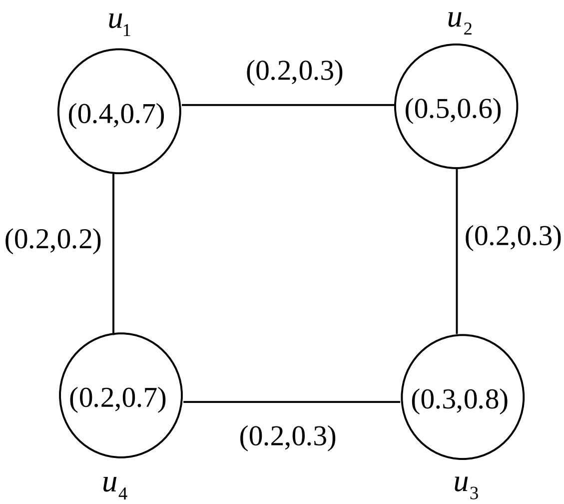


By routine computations, we have $\deg(u_1) = (1,1.4), \deg(u_2) = (0.8,1.2)$,

$\deg(u_3) = (1,1.4)$ and $\deg(u_4) = (0.8,1.2)$. It is clear from calculations that G is a neighbourly irregular interval– valued fuzzy graph.

Definition 3.10. A connected interval – valued fuzzy graph G is said to be a neighbourly totally irregular interval–valued fuzzy graph if every two adjacent vertices of G have distinct closed neighbourhood degree.

Example 3.11. Consider an interval–valued fuzzy graph G such that $V=\{u_1,u_2,u_3,u_4\}$, $E=\{u_1u_2,u_2u_3,u_3u_4,u_4u_1\}$.



By routine computations, we have

$$\deg[u_1]=(1.1,2),\deg[u_2]=(1.2,2.1),\deg[u_3]=(1,2.1)$$

and $\deg[u_4]=(0.9,2.2)$. It is easy to see that G is a neighbourly totally irregular interval–valued fuzzy graph.

Definition 3.12. Let G be a connected interval-valued fuzzy graph. G is called a highly irregular interval-valued fuzzy graph if every vertex of G is adjacent to vertices with distinct neighbourhood degrees.

Remark 3.13. A highly irregular interval–valued fuzzy graph may not be a neighbourly irregular interval-valued fuzzy graph. There is no relation between highly irregular interval–valued fuzzy graphs and neighbourly irregular interval-valued fuzzy graphs. We explain this concept with the following example.

4. Conclusions

Considering the precision, elasticity, and compatibility in a system, interval-valued models outweigh the other FGs. The interval-valued fuzzy graph concept generally has a large variety of applications in different areas such as computer science, operation research, topology, and natural networks. In this paper, the concepts of neighbourly irregular interval-valued fuzzy graphs, neighbourly totally irregular interval-valued fuzzy graphs, highly irregular interval-valued fuzzy graphs and highly totally irregular interval-valued fuzzy graphs are introduced and investigated. A necessary and sufficient condition under which neighbourly irregular and highly irregular interval-valued fuzzy graphs are equivalent is discussed.

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