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Certain Types of Vertices in *m*-Polar Fuzzy Graphs

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ABSTRACT

In this study, we explore the concept of $\boldsymbol{m}$-polar fuzzy ($\boldsymbol{m}$PF) detour $\boldsymbol{g}$-eccentric nodes within $\boldsymbol{m}$-polar fuzzy graphs ($\boldsymbol{m}$PFGs). We delve into the idea of $\boldsymbol{m}$PF detour $\boldsymbol{g}$-interior nodes and $\boldsymbol{m}$PF detour $\boldsymbol{g}$-boundary nodes, examining their significance and properties. Additionally, we establish the relationship between $\boldsymbol{m}$PF detour $\boldsymbol{g}$-boundary vertices and $\boldsymbol{m}$PF cut vertices.

Keywords: Fuzzy graphs, m-polar fuzzy graphs, mPF detour g-distance, mPF detour g-interior node, mPF detours, g-boundary node.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

In various domains such as artificial intelligence, operations research, signal processing, network routing, robotics, electrical engineering, and medical science, graph theory plays a crucial role [37]. The introduction of fuzzy sets by Zadeh in 1965 revolutionized the field, providing enhanced precision in both theory and application. Rosenfeld's pioneering work in 1975 laid the foundation for fuzzy graph theory, which finds numerous applications across different fields [32]. The concept of mPF sets, introduced by Chen et al. in 2014, led to the development of m-polar fuzzy graphs (mPFGs), explored extensively by Ghorai and Pal, Singh, and others [1, 14, 13, 21]. Singh extended this concept further by defining m-polar fuzzy graph representations using lattice theory and exploring various properties and applications [22, 23]. Bhutani, Rosenfeld, Mathew, Sunitha, and others contributed to defining different arc types, bridges, trees, cycles, cut nodes, and end nodes in fuzzy graphs [3, 29]. Rashmanlou et al. [30, 31] presented some work on bipolar and interval-valued fuzzy graphs. Samanta and Pal defined fuzzy planar graphs [35]. Ghorai and Pal investigated the isomorphic properties of m-polar fuzzy graphs [16].

Mandal et al. introduced the notion of strength of connectedness in mPFGs and explored different types of fuzzy graphs with operations and applications [28, 27, 34]. Concepts such as fuzzy detour g-distance, g-distance, g-boundary nodes, g-interior nodes, and g-eccentric nodes were introduced by Linda, Sunitha, Rosenfeld, Bhutani, Sameena, and others, expanding the understanding of fuzzy graph theory [17, 3, 18, 33]. Chartrand and his colleagues defined detour-related concepts such as detour center, detour number, detour set, and detour basis, further enriching the field [6, 9, 8, 7]. In this paper, we introduce and explore mPF detour g-distance, mPF detour g-interior nodes, and

mPF detour g-boundary nodes, along with their properties and relationships, contributing to the advancement of fuzzy graph theory [20]. For comprehensive coverage of fuzzy graph theory, readers are referred to the book [20].

2. Preliminaries

Firstly, we define mPFGs and other related terms.
In this paper, we examine the m-power of [0,1], denoted as $[0,1]^m$, as a partially ordered set (poset) with point-wise order $\leq$. The relation $\leq$ is defined as follows: for any $x', y' \in [0,1]^m$, $x' \leq y'$ if and only if $p_i(x') \leq p_i(y')$ for each $i = 1,2,\dots,m$, where $p_i: [0,1]^m \to [0,1]$ represents the i-th projection mapping.

Definition 2.1. *[11] An m-polar fuzzy graph (mPFG) of a graph $G^* = (V, E)$ is a pair $G = (V, A, B)$ where $B: \widetilde{V^2} \to [0,1]^m$ and $A: V \to [0,1]^m$ are an mPF set in $\widetilde{V^2}$ and an mPF set in V respectively such that $p_i \circ B(a,b) \leq min\{p_i \circ A(a), p_i \circ A(b)\}$ for all $(a,b) \in \widetilde{V^2}$, for each $i = 1,2,\dots,m$ and $B(a,b) = 0$ for all $(a,b) \in (\widetilde{V^2} - E)$, (The smallest element in $[0,1]^m$ is $0 = (0,0,\dots,0)$).*

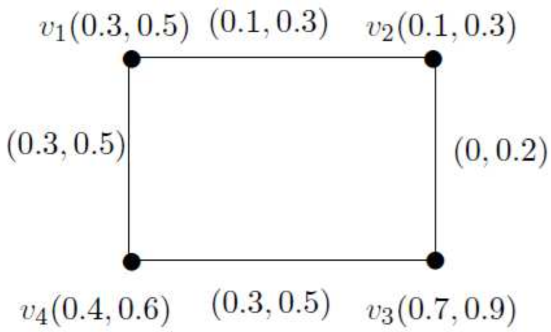

Figure 1: A 2PFG

Definition 2.2. *[10] If an mPFG $G = (V, A, B)$ satisfies the relation*

$$p_i \circ B(x,z) = min\{p_i \circ A(x), p_i \circ A(z)\}, for\ all\ x, z \in V, i = 1,2,3,\dots,m.$$

Definition 2.3. *[28] A path $u' = v_1, v_2, \dots, v_n = v'$ in mPFG G is said to be an mPF path if this path satisfies the relation $p_i \circ B(v_j, v_{j+1}) > 0$, $(j = 1,2,\dots,n-1)$ for at least one i and all the vertices are distinct except v_1 which may be the same as v_n.*

Definition 2.4. *[28] The strength of the mPF path $P: u' = v_1, v_2, \dots, v_n = v'$ in mPFG G is defined as*

$$S(P) = (B_1^n(u',v'), B_2^n(u',v'), \dots, B_m^n(u',v')),$$

where, $B_k^n(u',v') = \min_{1 \leq i < j \leq n} (p_k \circ B(v_i, v_j)), k = 1,2,\dots,m.$

$CONN_G(u',v')$ is the strength of connectedness between u' and v' and is defined as

$$CONN_G(u',v') = ((\max_{n \in N}(B_1^n(u',v')), (\max_{n \in N}(B_2^n(u',v')), \dots (\max_{n \in N}(B_m^n(u',v'))).$$

Definition 2.5. *[28] An $mPFG$ is said to be mPF connected graph if* $(p_i \circ B(a', b'))^{\infty} > 0$, *for at least one* $i = 1,2,3, \ldots, m$.

Definition 2.6. *[28] A* $u' - v'$ *path* $P: u' = v_1, v_2, \ldots, v_n = v'$ *in* $mPFG$ G *is said to be a strongest* mPF $u' - v'$ *path if* $S(P) = CONN_G(u', v')$.

Definition 2.7. *[28] An edge* (a', b') *of an* $mPFG$ G *is said to be strong* mPF *arc if* $B(a', b') \geq CONN_{G-(a',b')}(a', b')$.

Definition 2.8. *[28] A path* $P: x = x_1, x_2, \ldots, x_n = y$ *from* x *to* y *is called strong* mPF *path if* (x_i, x_{i+1}) *is strong* mPF *arc for all* $1 \leq i \leq n - 1$.

Definition 2.9. *[28] A vertex* y *is an* mPF *cut vertex of* G *if removing it from* G *reduces the connectedness strength between some other pair of nodes* G.

Definition 2.10. *[28] An* $mPFG$ G *is called an* mPF *tree if it has a spanning* mPF *subgraph* H' *which is an* m*-polar F-tree and such that for all* i, $p_i \circ B'(x, y) = 0$ *implies* $p_i \circ B(x, y) < p_i \circ CONN_{H'}(x, y)$.

Definition 2.11. *A maximum spanning* mPF *tree of a connected* $mPFG$ $G = (V, A, B)$ *is an* mPF *spanning subgraph* T *of* G, *which is a* m *polar F-tree, such that* $CONN_G(u, v)$ *is the strength of the unique strongest* uv mPF *path in* T *for all* $u, v \in G$.

3. mPF detour g distance, mPF detour g periphery and mPF detour g eccentric subgraph

First we define m-polar fuzzy(mPF) detour g distance and then mPF geodesic g distance. Then we defined m-polar fuzzy(mPF) detour g periphery and discussed the characterization of m-polar fuzzy (mPF) detour g eccentric node.

Definition 3.1. *The length of a* $c - d$ *strong* mPF *path* P *between* c *and* d *in connected* $mPFG$ G *is called an* mPF *detour* g *distance if there does not exist other strong* mPF *path longer than* P *between* a *and* b *and we denote it by* $mPFD_g(c, d)$. *Any* $c - d$ *strong* mPF *path with length* $mPFD_g(c, d)$ *is said to be a* $c - d$ mPF g*-detour.*

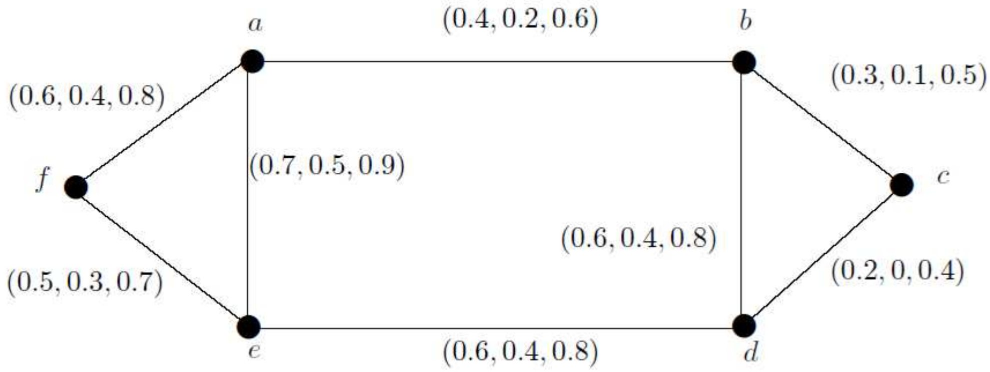

Figure 2: Connected 3PFG G.

Example 3.2. *Suppose* G *be a connected* 3PFG *of the graph* $G^* = (V, E)$ *where* $V = \{f, e, d, c, b, a\}$ *and* $E = \{(b,d), (b,c), (a,b), (d,e), (e,f), (a,f), (c,d), (a,e)\}$ *(see Fig. 1). For the* 3PF *graph of Figure 1, it is seen that all arcs except* (d,c), (a,b) *and* (f,e) *are strong* 3PF *arc and the* 3PF *detour* g*-distance of two nodes are given below:*
$3PFD_g(a,b) = 3,\ 3PFD_g(a,f) = 1,\ 3PFD_g(a,e) = 1,\ 3PFD_g(a,d) = 2,$
$3PFD_g(a,c) = 4,\ 3PFD_g(f,e) = 2,\ 3PFD_g(d,f) = 3,\ 3PFD_g(f,c) = 5,$
$3PFD_g(f,b) = 4,\ 3PFD_g(e,d) = 1,\ 3PFD_g(e,b) = 2,\ 3PFD_g(e,c) = 3,$
$3PFD_g(d,b) = 1,\ 3PFD_g(d,c) = 2$ *and* $3PFD_g(b,c) = 1$.

Definition 3.3. *The length of any smallest strong path from* a *to* b *is called the* m*PF geodesic distance, denoted by* $mPFD_g(a,b)$.

The mPF detour g eccentricity $e_{mPFD_g}(y)$ for a node y is an mPF detour g distance from y to a vertex maximum from y which implies $e_{mPFD_g}(y) = \max(mPFD_g(y,a)),\ \forall a \in G$. Suppose y be a node and each node whose mPF detour g distance is equal to $e_{mPFD_g}(y)$ then these vertex is called an mPF detour g eccentric node. The set of all mPF detour g eccentric nodes of x is denoted by $mPFD_g(x)$. The mPF detour g radius of G, symbolized as $rad_{mPFD_g}(G)$ and which is defined as $\min e_{mPFD_g}(x), \forall x \in G$. If $e_{mPFD_g}(x) = rad_{mPFD_g}(G)$, then the vertex $x \in G$ is said to be the mPF detour g central node of G. The mPF detour g diameter of G is symbolized by $diam_{mPFD_g}(G)$, is defined as $\max e_{mPFD_g}(x), \forall x \in G$. A node d in a G is called an mPF detour g peripheral node of G if $e_{mPFD_g}(d) = diam_{mPFD_g}(G)$.

Example 3.4. *For the connected* $mPFG$ G *in Fig. 1,* $e_{3PFD_g}(c) = 5$, $e_{3PFD_g}(b) = 4$, $e_{3PFD_g}(a) = 4$, $e_{3PFD_g}(d) = 3$, $e_{3PFD_g}(e) = 3$, $e_{3PFD_g}(f) = 5$ *and* $rad_{3PFD_g}(G) = 3$, $diam_{3PFD_g}(G) = 5$.

Definition 3.5. *An* m *PFG* G *is an* m *PF* g *-detour graph if* $mPFD_g(b,a) =$

$mPFD_g(b,a), \forall\ (b,a) \in E$.

Definition 3.6. *The* mPF *subgraph of an* $mPFG$ G *is induced by the only* mPF *detour* g *peripheral node of* G, *now the subgraph is called* mPF *detour* g *periphery of* G *and it is symbolized by* $(Per_{mPFD_g}(G))$.

Definition 3.7. *If each node of a connected* $mPFG$ G *is* mPF *detour* g *eccentric node, then* G *is said to be an* mPF *detour* g *eccentric graph. An* mPF *detour* g *eccentric subgraph of* G *is an* mPF *subgraph of* G, *generated by the set of all* mPF g*-eccentric nodes of* G *is called, it is symbolized as* $Ecc_{mPFD_g}(G)$.

Example 3.8. *For the* $3PF$ *graph of Figure 2, nodes* a, b, d *are* mPF *detour* g*-periphery nodes since* $e_{3PFD_g}(a) = 4$, $e_{3PFD_g}(b) = 4$, $e_{3PFD_g}(c) = 3$, $e_{3PFD_g}(d) = 4$, $e_{3PFD_g}(e) = 3$ *and* $diam_{3PFD_g}(G) = 4$. *Here* $Per_{3PFD_g}(G)$ *of mPFG shown in Figure 2.*

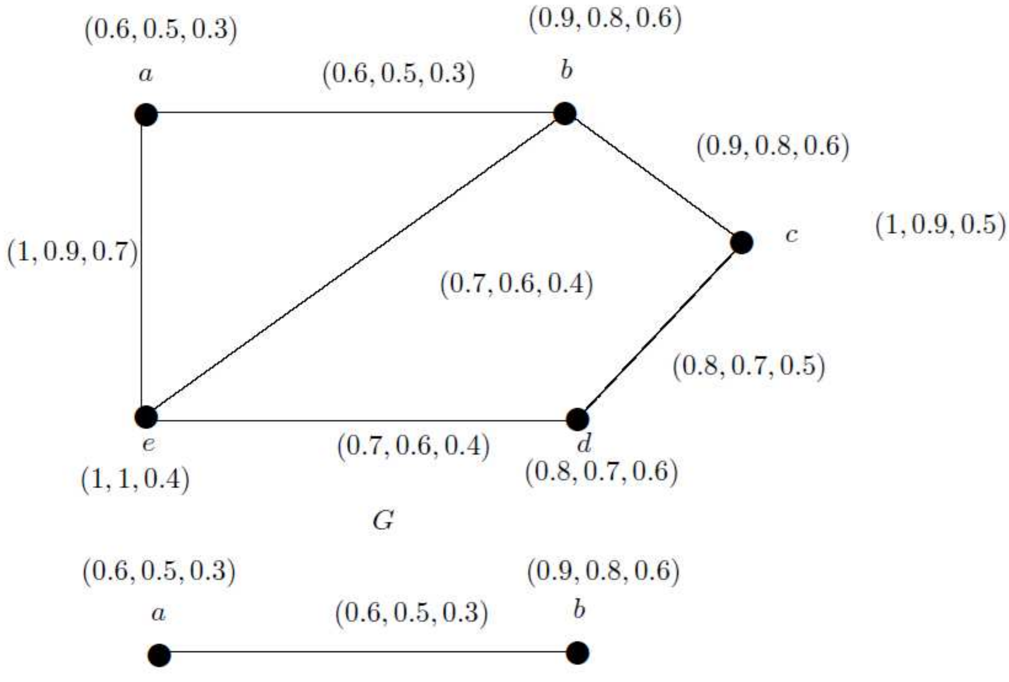

Figure 3: Connected 3PF graph G and its $Per_{3PFD_g}(G)$.

Example 3.9. *From Figure 1, we get* $3PFD_g(a) = \{d, b\}$, $3PFD_g(b) = \{a\}$, $3PFD_g(c) = \{a, d, b\}$, $3PFD_g(d) = \{a\}$, $3PFD_g(e) = \{d, b\}$. *Its* $Ecc_{3PFD_g}(G)$ *is shown in Figure 2.*

Definition 3.10. *The* mPF *subgraph of an* $mPFG$ G *is induced by the only* mPF *detour*

g central nodes is called mPF detour g centre subgraph, symbolized by $C_{mPFD_g}(G)$. A graph G is called mPF detour g self centered graph if each vertices of G are mPF detour g central nodes. In every mPF detour g self centered graph, $rad_{mPFD_g}(G) = diam_{mPFD_g}(G)$.

Theorem 3.11. *Each node of an $mPFG$ G is an mPF detour g eccentric iff G is an mPF detour g self centrad.*

Proof: Let, every vertex be an mPF detour g eccentric node in G. Here we assume that G is not an mPF detour g self-centrad graph. So $rad_{mPFD_g}(G) \neq diam_{mPFD_g}(G)$ and then $\exists$ a vertex $l \in G$ such that $e_{mPFD_g}(l) = diam_{mPFD_g}(G)$. Also, let $r \in mPFD_g(l)$. Let B be a $l - r$ mPF detour in G. Then a vertex k on B must exist for which the vertex k is not an mPF detour g eccentric node of B. Also, k cannot be an mPF detour g eccentric node for the other node. Again if k is an mPF detour g eccentric node of a node a (say), means $k \in mPFD_g(a)$. Then $\exists$ an extension of $a - k$ mPF g −detour up to l or up to r. But, there is a contradiction between the facts that $k \in mPFD_g(a)$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G)$. Hence G is an mPF detour g self centrad graph.

Conversely, let us consider G to be an mPF detour g self-centred graph and $x \in V$. Let $a \in mPFD_g(x)$. So this implies $e_{mPFD_g}(x) = mPFD_g(a, x)$. Again we know each node of G is mPF detour g central node i.e. $e_{mPFD_g}(y) = rad_{mPFD_g}(G)\ \forall\ y \in G$ because G is an mPF detour g self centrad graph, which means. So we have, $e_{mPFD_g}(a) = e_{mPFD_g}(x) = mPFD_g(a, x)$ and which implies that $x \in mPFD_g(a)$. Hence x is an mPF detour g eccentric node.

Theorem 3.12. *If G is an mPF detour g self-centred graph with n number of nodes, then $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$.*

Proof: Suppose G be an mPF detour g self-centred graph. If possible, let $diam_{mPFD_g}(G) = l < n - 1$.

Suppose B_1 and B_2 are two distinct mPF detour g peripheral paths. Let $a \in B_1, b \in B_2$. So a strong mPF path exists in between a and b, because of the connectedness of G. Then there exist nodes on B_1 and B_2, whose eccentricity $> l$, but which is impossible because $diam_{mPFD_g}(G) = l$. Hence B_1 and B_2 are not distinct. Since B_1 and B_2 are arbitrary, so then there exists a vertex x in G which x present in all mPF detour g peripheral paths. So, $e_{B.F.D_g}(x) < l$, which is also impossible, because G is an mPF detour g self centrad. Hence, $diam_{mPFD_g}(G) = n - 1 = rad_{mPFD_g}(G)$.

Corollary 1. *Let G be a connected $mPFG$ with n number of vertices. Then $Per_{mPFD_g}(G) = G$ iff the mPF detour g eccentricity of every node of G is $n - 1$.*

Proof: Let $Per_{mPFD_g}(G) = G$. Then $e_{mPFD_g}(a) = diam_{mPFD_g}(G), \forall\ a \in G$. So every node of G is an mPF detour g periphery node. Therefore, G is an mPF detour g self-centred graph and $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$. So, the mPF detour g eccentricity of each node of G is $n - 1$.

Conversely, let the mPF detour g eccentricity of each node of G is $n - 1$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$. All nodes of G are mPF detour g peripheral nodes and hence $Per_{mPFD_g}(G) = G$.

Corollary 2. *For a connected $mPFG$ G, $Ecc_{mPFD_g}(G) = G$ if and only if the mPF detour g eccentricity of each vertex of G is $n - 1$.*

Proof: Suppose $Ecc_{mPFD_g}(G) = G$. So every node of G is mPF detour g eccentric node. Therefore G is m PF detour g self centrad graph and $red_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$. Hence the mPF detour g eccentricity of each node of G is $n - 1$.

Conversely, let the mPF detour g eccentricity of each node of G is $n - 1$. So $rad_{mPFD_g}(G) = diam_{mPFD_g}(G) = n - 1$. All nodes of G are mPF detour g peripheral nodes as well as mPF detour g eccentric node. Hence, $Ecc_{mPFD_g}(G) = G$.

Theorem 3.13. *In a connected $mPFG$ G, a node a is an mPF detour g peripheral node if and only if a is an mPF detour g eccentric node.*

Proof: Let us assume that $b \in Per_{mPFD_g}(G)$. So there exists an mPF detour g peripheral node, say b (distinct from a). Therefore, a is an mPF detour g eccentric node of a.

Conversely, let us that a be an mPF detour g eccentric node of G and let $a \in mPFD_g(b)$. Let x and y be two mPF detour g peripheral nodes, then $mPFD_g(x, y) = diam_{mPFD_g}(G) = k(say)$. Let B_1 and B_2 be any $x - y$ and $b - a$ mPF g detour in G respectively. Then two cases will arise.

Case 1: When a is not an internal node in G i.e., there is only one node, say c which is adjacent to a. So $c \in B_2$. Since G is connected, c is connected to a node of B_1, say c'. So either $c' \in B_2$ or $c' \in (B_1 \cap B_2)$. Thus in any case the path from b to m or b to n through c and c' is longer than B_2. But it is impossible since a is an mPF detour g eccentric node of b. Hence $e_{mPFD_g}(b) = diam_{mPFD_g}(G)$ i.e, $a \in Per_{mPFD_g}(G)$.

Case 2: When a is an internal node in G, then there exists a connection between a to m and a to n, because of the connectedness of G. Then $b - a$ mPF g detour can be extended to m or n. This is impossible because a is an mPF detour g eccentric node of b. Hence $e_{mPFD_g}(b) = diam_{mPFD_g}(G)$ i.e., a is an mPF detour g peripheral node of G.

4. Conclusion

In this article, we introduced concepts such as mPF detour g-distance, and mPF detour g-interior nodes within the context of m-polar fuzzy graphs (mPFGs), along with exploring their properties. Theorems pertaining to mPF detour g-interior nodes, mPF detour g-boundary nodes, and mPF cut nodes in mPFGs were established, utilizing the framework of maximum mPF spanning trees. Additionally, we are extending our research to define the connectivity index on m-polar fuzzy graphs and investigate its properties.

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