2023

M.Sc.

4th Semester Examination

PHYSICS

PAPER : PHS-401.1 & 401.2

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Answer from **both** the Sections as directed.

SECTION—I

(PARTICLE PHYSICS)

PHS-401.1

1. Answer *any* **two** questions from the following : $2\times2=4$

(a) Using the antisymmetry of the total wave functions determine the spins of the $\ell = 0$ baryons in the octet and the decuplet.

(b) How do $|K^0\rangle$ and $|\bar{K}^0\rangle$ transform under CP? Construct linear combinations of the neutral Kaons $|K^0\rangle$ and $|\bar{K}^0\rangle$ that are eigenstates of CP.

(c) Giving reasons, determine which of the following processes is allowed by strong interactions :

(i) $K^- + p \to \pi^+ + \Sigma^+$

(ii) $K + p \to \pi + \Sigma^+$

(d) Write down the G-parity operator and explain how the G-parity of the pions $\pi^{0,+}$ can be determined.

2. Answer *any* **two** questions from the following : $4 \times 2 = 8$

(a) By writing down the $SU(3)$ group algebra, explain what is meant by a conjugate representation. Write down an explicit matrix corresponding to $V_+ = \frac{1}{2}(\lambda_4 + i\lambda_5)$ for the conjugate representation and show that $V_+\bar{u} = -\bar{s}$.

(*b*) Given that the fermion field $\psi(t, x)$ transforms under Parity (P) as $\eta_a \gamma^0 \psi(t, -x)$, determine the transformations of $\bar{\psi}\gamma^\mu\psi$ under P.

(*c*) Let G be a Lie Group and $X = x_a t_a$ be an element of the algebra where t_a's satisfy $[t_a, t_b] = if_{abc} t_c$. Define the transformation $X' = UXU^+$, where $U \in G$. Considering the infinitesimal form, write down the transformation in terms of x_a's. Rewrite the transformation as $\vec{X}' = \exp(-i\theta_a T_a)\vec{X}$, where $\vec{X}^T = (x_1, x_2, \dots)$ and obtain the representation matrices T_a in terms of the structure constants f_{abc}.

(*d*) Consider a meson X. A strong decay process $X \to \pi^+\pi^-$ has been observed but the process $X \to \pi^0\pi^0$ has never been observed. Determine the quantum numbers $J^{PC}I^G$ for the particle X.

3. Answer *any* **one** question from the following : $8 \times 1 = 8$

(a) (i) The mixed-symmetric and the mixed-antisymmetric flavor parts of the proton wavefunction are respectively

$$\frac{1}{\sqrt{6}}(2uud - udu - duu) \text{ and } \frac{1}{\sqrt{2}}(udu - duu).$$

Use the raising/lowering operators of $SU(3)$ to compute the Flavor $\otimes$ Spin wave function for the $\Xi^0(ssu)$ baryon with $s_z = \frac{1}{2}$. Further, obtain the magnetic moment of Ξ^0 in terms of those of the constituent quarks. [Given : $T_+ d = u$, $V_+ s = u$, $U_- s = d$] 5

(ii) Using isospin invariance, find the ratio of the decay rates for the following :

$$\Delta^{++} \rightarrow p + \pi^+ : \Delta^+ \rightarrow p + \pi^0 : \Delta^+ \rightarrow n + \pi^+$$ 3

(b) (i) Consider a theory of a complex scalar field φ coupled to electromagnetic field

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu\varphi|^2 - V(\varphi) \text{ with}$$

$$V(\varphi) = \mu^2 |\varphi|^2 + \lambda(|\varphi|^2)^2$$

Where $D_\mu = \partial_\mu + ieA_\mu$, μ is the mass of the φ field and λ and e are coupling constants. For $\mu^2 < 0$ show that the potential has a new minimum at $|\varphi| = v \neq 0$. Expand the potential about the new minimum taking $\phi = \frac{1}{\sqrt{2}} e^{i\chi(x)/v} (v + h(x))$ and find the masses of the $h(x)$ and $A_\mu(x)$ fields. 6

(ii) The decay of a neutral pion to two photons, $\pi^0 \rightarrow 2\gamma$ has been observed experimentally. Using this fact explain if the decay $\pi^0 \rightarrow 3\gamma$ is allowed or forbidden. 2

SECTION—II

(STAT. MECH.—II)

PHS-401.2

1. Answer *any* **two** questions from the following : $2 \times 2 = 4$

(a) Show that de-Broglie wavelength associated with an electron gas is given by

$$\lambda_F = 2\left(\frac{\pi}{3n_0}\right)^{1/3}$$

where n_0 is the number of electrons per c.c. of the gas at 0K.

(b) Plot the variation of chemical potential of ideal Fermi-gas and Bose gas with temperature.

(c) A gas of N spin-zero Bosons in d-dimensional volume V has dispersion relation $\varepsilon = \alpha |\vec{p}|^s$, where α is constant and positive. Find the condition on s and d for which BE condensation takes places.

(d) For a system of two ising spins interaction energy is given by $H = -J\vec{S_1} \cdot \vec{S_2}$

show that $\langle E \rangle = -\dfrac{J^2}{k_B T}$ at high temperature.

2. Answer *any* **two** questions from the following : $4 \times 2 = 8$

(a) How many photons are there in 1 c.c. of radiation at 10^3 K? Also find their average energy.

(b) Prove that pressure in black body radiation inside d-dimensional cavity at temperature T is

$$P = A_d I_d (k_B T)^{d+1}$$

(c) Show that in 3D solid of N atoms ensemble of harmonic oscillator zero-point energy

$$E_0 = \frac{9N}{8} k_B T_D$$

where T_D is the Debye temperature.

(d) For an ising system, spontaneous magnetization $m \sim (T_c - T)^{\beta}$

magnetic susceptibility $\chi \sim |T - T_c|^{-\gamma}$ near T_c. Find the discontinuity of specific heat C_H.

3. Answer *any* **one** question from the following : $8 \times 1 = 8$

(a) *(i)* Show that photoelectric current density is independent of temperature, if photon energy >> work function of metal.

(ii) Prove that for BE gas, find an expression of c_V^{+} (for $T > T_0$). 5+3

(b) *(i)* If $\varepsilon(l, p_z) = \left(l + \frac{1}{2}\right)\hbar\omega_c + \frac{p_z^2}{2m}$ for quantized Landau levels,

prove that $\langle N \rangle = \frac{zV}{\lambda^3} \frac{x}{\sinh x}$

where $x = \frac{\beta \hbar e H}{2mc}$ and other symbols have usual meanings.

(ii) In a lattice of $N+1$ sites has $S_i = \pm 1$ at each site and $\hat{H} = -h \sum_{i=0}^{N} S_i - J \sum_{i=1}^{N} S_i S_0$ when $h = 0$, show that

$$\langle S_i S_j \rangle = \langle S_0 S_i \rangle \langle S_0 S_j \rangle$$ 4+4

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