M.Sc. 3rd Semester Examination, 2023

PHYSICS

PAPER – PHS-301.1 & 301.2

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – PHS-301.1

(*Quantum Mechanics-III*)

1. Answer any *two* of the following : 2 × 2

(*a*) Suppose that the permutation of operator $\hat{p}$ commutes with the Hamiltonian in a two identical particle system. Derive the relation between the time dependent wave functions $\psi(x_1, x_2, t)$ and $\psi(x_2, x_1, t)$ for the system.

(*Turn Over*)

(*b*) Show that the solution to the evolution operator in the interaction picture $U_I(t,t_0)$ from

$$i\hbar\frac{d}{dt}U_I(t,t_0)=V_I(t)U_I(t,t_0) \quad U_I(t,t_0)|_{t=t_0}=1$$

can be written as the Dyson series.

(*c*) Given that $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$, use the Clifford algebra $\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}1_{4\times4}$ to show that $\mathrm{tr}\,\gamma^5=0$.

(*d*) Write down the Klein-Gordon equation in the presence of an electromagnetic field $A^\mu=(\phi/c,\vec{A})$. Show that in the non-relativistic limit, the equation reduces to the corresponding Schrödinger equation.

2. Answer any *two* of the following : 4×2

(*a*) Consider a charged particle in a one dimensional harmonic oscillator potential $V(x)=mw^2x^2/2$. The particle is in the ground state at time $t<0$. At time $t=0$

an electric field E is applied suddenly. Find the probability that the particle may be found in the ground state of the new potential.

(*b*) The Dirac equation is given as

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \quad \text{where } H = -i\hbar c\vec{\alpha}.\vec{\nabla} + \beta mc^2.$$

Find the conditions on α_i and β so that ψ also satisfies the Klein-Gordon equation. If β= diagonal $(I,-I)$, find a set of the α_i matrices which satisfy these conditions.

(*c*) Consider a system two identical spin half particles. One of the particles is in the ground state and the other is in the first excited state of a one dimensional harmonic oscillator potential. Write down the normalized wave function of the system $\psi(x_1,x_2)$ in terms of $\psi_0(x_1)$ and $\psi_1(x_2)$ for the singlet and triplet states of the system and find $\left\langle (x_1 - x_2)^2 \right\rangle$.

(*d*) An electron is subjected to a time varying magnetic field given by

$$\vec{B} = B_0\hat{z} + B_1(\hat{x}\cos wt + \hat{y}\sin wt)$$

If at $t = 0$, the electron is in the eigenstate $|+\rangle_z$. Find the probability of finding the electron in the state $|\pm\rangle_z$ as a function of time. You may use the Rabi's formula given in the list of formulae.

3. Answer any *one* of the following : 8×1

(*a*) (*i*) Consider S-wave ($l = 0$) scattering of a particle from a spherically symmetric potential given by, $V(r)=\alpha\delta(r-r_0)$ for α and r_0 are constants. Compute the phase shift δ_0 and the total cross section σ_{tot}. 4

(*ii*) Consider a Hamiltonian $H=H_0+V(t)$, where

$$H_0 = \frac{1}{(2m)}\hat{p}^2 + \frac{1}{2}mw^2\hat{x}^2 \text{ and}$$

$$V(t) = \frac{\hat{x}}{\left(\sqrt{\pi T}\right)} e^{-(t/T)^2}.$$

Compute the transition probability from the ground state of H_0 to an excited state as $t \to \infty$, up to first order in perturbation theory. 4

(*b*) (*i*) Use the ansatz $\psi(\vec{r}, t) = u\, e^{i(\vec{p}.\vec{r} - Et)/\hbar}$ to find the plane wave solutions of the Dirac equation in the rest frame of a particle of mass $m \neq 0$. Indicate the energy eigenvalues and the spins of the solutions. 3

(*ii*) The Lippmann-Schwinger equation in the position basis in one dimension is written as

$$\left\langle x \middle| \psi^{(\pm)} \right\rangle = \left\langle x \middle| \phi \right\rangle + \int dx\, G_{\pm}(x, x') \left\langle x \middle| V \middle| \psi^{(\pm)} \right\rangle$$

A. Find the expression for $G_{\pm}(x, x')$ for a paricle of mass m and energy E.

B. For $V(x) = V_0$ for $|x| < a/2$ and $V(x) = 0$ otherwise, find $\psi^{(+)}(x)$ up to first order in Born approximation. Take $\langle x | \phi \rangle = e^{ikx}$. 3 + 2

List of Formulae :

1. Gaussian integrals :

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi / a}; \int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx = (-1)^n \frac{\partial^n}{\partial a^n} \int_{-\infty}^{\infty} e^{-ax^2} dx;$$

$$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = e^{b^2/4a} \sqrt{\pi / a}.$$

2. Rabi's formula :

$$|c_2(t)|^2 = \frac{\gamma^2/\hbar^2}{\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4} \sin^2 \left\{ \left[\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4 \right] t \right\}$$

3. Wave functions of 1-D harmonic oscillator :

$$\psi(x)=\frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right);$$

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}\left(e^{-x^2}\right)$$

PAPER – PHS-301.2

4. Answer any *two* questions : 2×2

(*a*) Let x_i be either p_i or q_i (i=1,2,...3N) and H is the Hamiltonian then what is the value of $\left\langle x_i \frac{\partial H}{\partial x_j}\right\rangle$ and $\left\langle \sum_{i=1}^{3N} q_i\, p_i \right\rangle$.

(*b*) Prove that pure state remains pure always.

(*c*) At ordinary pressure, perfect gases are not degenerate :- Explain.

(*d*) The density matrix of a system is given by

$$\rho = \begin{pmatrix} \theta & 0 \\ 0 & 1-\theta \end{pmatrix}$$

where $0 \leq \theta \leq 1$. Find the entropy. What is the entropy in a pure state ?

5. Answer any *two* questions : 4×2

(*a*) Deduce the equation of state of ideal Bose and Fermi gas if $\varepsilon = Ap^s$ taking into account non-relativistic and relativistic gas.

(*b*) For N distinguishable particles energy $\varepsilon = pc$ where p is the momentum.

Prove that $\mu = k_B T \ln\left[\frac{h^3 c^3 p}{8\pi (k_B T)^4}\right]$

where p is the pressure.

(*c*) Calculate the number of microstates available for a particle of mass m and total energy E moving in a spherical potential $V(r) = br^2$ where b is a constant.

(*d*) Given the spin state $|\chi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$. The beam is 70% polarized along $\hat{n}$ direction. Find the spin state $|\chi'\rangle$ which is orthogonal to $|\chi\rangle$.

6. Answer any *one* question : 8×1

(*a*) (*i*) Deduce an expression of F-D distribution function from grand partition function.

(*ii*) Given N identical non-interacting magnetic ions of spin $\frac{1}{2}$, magnetic moment μ_0 in a crystal at absolute temperature T in a magnetic field H. Show that fluctuation of magnetic moment

$$\Delta M = \sqrt{N} \frac{\mu_0}{\cosh\left(\frac{\mu_0 H}{k_B T}\right)}.$$

4 + 4

(*b*) (*i*) Calculate the density matrix for a particle in a box of infinite potential at the boundary in co-ordinate representation with physical interpretation.

(*ii*) Prove that the average of dipole-magnetic moment $\langle \mu_z \rangle = jg\,\mu_B \; B_j(x)$ where $B_j(x)$ is the Brillouin function of order j and $x = \frac{g\mu_B jH}{k_B T}$. 4 + 4

[*Internal Assessment* — **10 Marks**]
