PG 1st Semester Examination, 2023

PHYSICS

PAPER – PHS-102.1 & 102.2

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – PHS-102.1

(*Quantum Mechanics-I*)

GROUP – A

Answer any **two** questions : 2×2

1. Using the orthonormality of $|+\rangle$ and $|-\rangle$ show that $\left[S_x, S_y\right] = i\hbar S_z$ and $\{S_x, S_y\} = 0$ where

$$S_x = \hbar/2\left(|+\rangle\langle-| + |-\rangle\langle+|\right),$$

$$S_y = i\hbar/2\left(-|+\rangle\langle-| + |-\rangle\langle+|\right),$$

$$S_z = \hbar/2\left(|+\rangle\langle+| - |-\rangle\langle-|\right).$$

2. For a quantum mechanical two-state system $\{|1\rangle, |2\rangle\}$, it is given that $\left\langle 1|\hat{A}|1\right\rangle = \frac{1}{2}$ and $\left\langle 1|\hat{A}^2|1\right\rangle = \frac{1}{4}$ where $\hat{A}$ is an operator. Determine $|1\rangle$ and $|2\rangle$ which are eigenstates of $\hat{A}$.

3. For a one dimensional harmonic oscillator compute $\langle n'|\hat{x}\hat{p}|n\rangle$ and write down the corresponding matrix using the annihilation and creation operators, $\hat{a}$ and $\hat{a}^+$.

4. Given that for one dimension,

$$\langle x|p\rangle = (2\pi\hbar)^{-1/2} e^{ipx/\hbar}$$

and a state $|\alpha\rangle$, show that $\langle p|\hat{x}|\alpha\rangle = i\hbar\frac{\partial}{\partial p}\langle p|\alpha\rangle$.

GROUP – B

Answer any **two** questions : 2×4

5. For the spin state $|+, z\rangle$, evaluate $\left\langle (\Delta S_x)^2 \right\rangle$ and $\left\langle (\Delta S_y)^2 \right\rangle$ and verify that

$$\left\langle (\Delta S_x)^2 \right\rangle \left\langle (\Delta S_y)^2 \right\rangle \geq \frac{1}{4} \left| \left\langle \left[S_x, S_y \right] \right\rangle \right|^2 .$$

6. A one-dimensional harmonic oscillator has the ground state $|0\rangle$ and the first excited state $|1\rangle$. Consider the normalized state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$. Find the values of α and β so that $\langle \psi | \hat{x} | \psi \rangle$ is maximum or minimum.

7. Consider a two state system with orthonormal basis states $\{|1\rangle, |2\rangle\}$. The Hamiltonian operator for the system is given by

$$H = \in \left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1| \right)$$

where $\in$ is a constant having the dimension of energy. Suppose it is known that the system is in state $|1\rangle$ and the energy is measured. What possible values one gets and with what probabilities ?

8. A particle of mass m is confined to a one-dimensional region $0 \leq x \leq a$. At $t = 0$ the normalized wave function of the particle is $\psi(x,0) = \sqrt{8/(5a)}[1+\cos(\pi x/a)]\sin(\pi x/a)$. Find the wave function $\psi(x,t)$ after time t and hence compute the probability that the particle will be found in the region $0 \leq x \leq a/2$ as a func tion of time.

GROUP – C

Answer any **two** questions : 8×1

9. (*a*) The Hamilatonian operator for a spin-1/2 system is given by $\hat{H} = \hbar\omega\hat{S}_x$ where ω is a real constant. Writing down the Heisen-

berg equations of motion find the $\hat{S}_x(t)$, $\hat{S}_y(t)$, $\hat{S}_z(t)$. For the state at t = 0, $|+, z\rangle$ find the exprectation values of these operators as a function of time.

(*b*) A free particle of mass m moves in one dimension. At time $t = 0$ the normalized wave function of the particle is

$$\psi(x, 0) = (2\pi\sigma^2)^{-1/4} \exp(-x^2/4\sigma^2).$$

Find the wave function of the particle, $\psi(x, t)$.

4 + 4

10. (*a*) The Hamiltonian for a system is given by $H = i\omega(-|1\rangle\langle 2| + |2\rangle\langle 1|)$, where the kets $|1\rangle$ and $|2\rangle$ form a complete set and are orthonormal. Suppose that at time, $t = 0$ the state of the system is given by

$$|\psi(0)\rangle = |1\rangle.$$

Find the probability of finding the system in state $|2\rangle$ as a function time t.

(*b*) The Hamiltonian describing the dynamics of a particle of mass m in one dimension is given by $\hat{H} = \frac{\hat{p}^2}{2m} - F\hat{x}$ where F is a constant. Using the Heisenberg equation of motion evaluate the operators for position $\hat{x}(t)$ and for momentum $\hat{p}(t)$. 4 + 4

PAPER — PHS-102.2

(*Condensed Matter Physics-I*)

GROUP — A

Answer any **two** questions : 2 × 2

1. Show the stereogram of point group $\overline{2}$ and $\overline{6}$.

2. Find Structure Factor in terms of fractional coordinates and hence find the condition of systematic absences for BCC crystal.

3. Show that under periodic boundary condition the number of possible modes of vibration

for one dimensional monoatomic lattice is equal to number of mobile atoms.

4. Draw Ewald Sphere and hence find the diffraction condition.

GROUP – B

Answer any **two** questions : 4×2

5. Mention the internal symmetry elements associated with space groups of Monoclinic system and classify all space groups for this structure.

6. Find the expression of density of staes for one dimensional monoatomic linear chain of atoms and hence explain Van Hove Singularity.

7. Prove that volume of the unit cell of the reciprocal lattice is inveresely proporational to that of corresponding reciprocal lattice.

8. What is meant by extended and reduced zone

scheme ? Plot energy as a function of wave vector for a one dimensional lattice under these schemes.

GROUP – C

Answer any **one** question : 8×1

9. Derive Laue Equations considering the scattering of x-rays from a crystal. 8

10. What is the Physical origin of energy gap in a solid ? Show that magnitude of energy gap is dependent on the Fourier component of crystal potential. What is meant by effective mass of an electron. $2 + 5 + 1$

[Internal Assessment – 10 Marks]

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