PG 1st Semester Examination, 2023

PHYSICS

PAPER – PHS-101.1 & 101.2

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – PHS-101.1

GROUP – A

Answer any **two** questions : 2×2

1. Show that $\frac{\hat{d}}{dx}$ is anti-Hermitian and the trace of an operator is base independent.

2. If $\hat{A}=\begin{pmatrix}3 & 2\\ 4 & 1\end{pmatrix}$ Find $S^{-1}\hat{A}S$.

3. $f(z)=\dfrac{1}{z(z-1)}$ expand in Laurent series about $z=0$ and $z=1$.

4. Prove that $\left(\varepsilon^{i}.\varepsilon^{j}\right)\left(\varepsilon_{j}.\varepsilon_{k}\right)=\delta^{i}_{k}$.

GROUP – B

Answer any **two** questions : 4×2

5. Show that $\int_{0}^{1+i} z^{*}\,dz$ depends on path (i.e. Cauchy's integral theorem does not apply)

6. Find an orthonormal set by using gram-schmidt process from $\{1,x,x^{2}\}$ in the internal -1 to $+1$.

7. Evaluate $\int_0^{\infty} \frac{\ln(x)dx}{x^3+1}$ by using Residue theorem.

8. The covariant vector A_i is the gradient of a scalar. Show that the difference of covariant derivatives $A_{i;j} - A_{j;i}$ vanishes.

GROUP – C

Answer any **one** question : 8 × 1

9. (*a*)

$$A = \begin{pmatrix} 0 & 0 & 0 & \frac{\pi}{4} \\ 0 & 0 & \frac{-\pi}{4} & 0 \\ 0 & \frac{\pi}{4} & 0 & 0 \\ \frac{-\pi}{4} & 0 & 0 & 0 \end{pmatrix}$$

Find $T_r\left(e^{\hat{A}}\right)$.

(*b*) Represent ε_{ij} by a 2×2 matrix and using the 2×2 rotation matrix, show that ε_{ij} is invariant under orthogonal similarity transformations. 4 + 4

10. (*a*) Using residue theorem prove that

$$\int_0^\infty \frac{\cos x}{\sqrt{x}}\, dx = \sqrt{\frac{\pi}{2}}.$$

(*b*) Given $A_k = \frac{1}{2}\varepsilon_{ijk} B^{ij}$ with $B^{ij} = -B^{ji}$ show that $B^{mn} = \varepsilon^{mnk} A_k$. 5 + 3

PAPER – PHS-101.2

(*Classical Mechanics*)

GROUP – A

Answer any **two** questions : 2×2

1. A particle of mass *m* moves in a potential

$V(x) = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\mu v^2$, where x is the position coordinate, v is the velocity, ω and μ are constants. Find the canonically conjugate momentum of the particle.

2. The Hamiltonian of a classical one-dimensional harmonic oscillator is $H = \frac{1}{2}(p^2 + x^2)$ in suitable unit. Find the total time derivative of the dynamical variable $\left(p + \sqrt{2}x\right)$.

3. Obtain the relation between Hamilton's principal function and Hamilton's characteristic function.

4. Show that, if the Lagrangian function does not contain the coordinate q_k explicitly, then the generalized momentum p_k is a constant of motion.

GROUP – B

Answer any **two** questions : 4×2

5. The Hamiltonian of a classical particle is given by $H(q,p)=\frac{p^2}{2m}+\frac{1}{2}kq^2$. Given

$$F(p,q,t)=\ln(p+im\omega q)-i\alpha\omega t$$

is a constant of motion $\left(\text{where } \omega=\sqrt{\frac{k}{m}}\right)$. Find the value of α.

6. The Hamiltonian of mass m is given by $H=\frac{1}{2m}(p-\alpha q)^2$, where α is a non-zero constant. Find the value of $\ddot{q}$.

7. Prove that if $F(q,p,t)$ and $G(q,p,t)$ are two integral of motion, then $[F, G]$ is also integral of motion.

8. Find the Lagrangian of a particle of charge q, mass m, and linear momentum p, enters an electromagnetic field of vector potential A and scalar potential φ.

GROUP – C

Answer any **one** question : 8×1

9. Derive Euler's-Lagrange equation of motion and hence prove that the shortest distance between two points in a plane is a straight line. 5 + 3

10. Outline the Hamilton-Jacobi equation. Explain the physical significance of Hamilton's principle function. Solve the harmonic oscillator problem by Hamilton-Jacobi method. 2 + 2 + 4

[Internal Assessment – 10 Marks]