M.Sc. 2nd Semester Examination, 2023

PHYSICS

PAPER – PHS-201.1 & 201.2

[Old and New]

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PHS-201.1

(*Quantum Mechanics* - II)

1. Answer any *two* of the following : 2×2

(*a*) Consider the Clebsch-Gordan coefficients $\langle j_1, j_2; j, m | j_1, j_2; m_1, m_2 \rangle$. Use the $J_\pm$ ladder operators to find $\langle 1, 2; 3, 2 | 1, 2; 0, 2 \rangle$.

(*Turn Over*)

(*b*) The Hamiltonian for studying Zeeman effect is given by

$$H = \frac{p^2}{2m_e} + V(r) + \alpha(r)\vec{L}.\vec{S} - \frac{e}{2m_e c}|\vec{B}|(L_z + 2S_z)$$

where $V(r)$ and $\alpha(r)$ are known functions. Identify a good basis that can be used for perturbation theory, when the external magnetic field $|\vec{B}|$ is (*i*) weak (*ii*) strong.

(*c*) If $\phi(\vec{p})$ is the momentum-space wave function for state $|\alpha\rangle$, that is, $\phi(\vec{p}) = \langle\vec{p}|\alpha\rangle$. Find the momentum-space wave function for the time-reversed state $\Theta|\alpha\rangle$.

(*d*) A quantum state $|\psi\rangle$ is known to be in an eigenstate of L^2 and L_z with eigenvalues $\hbar^2 l(l+1)$ and $m\hbar$ respectively. Calculate the expectation values (*i*) $\langle\psi|L_x|\psi\rangle$ (*ii*) $\langle\psi|L_x^2|\psi\rangle$

2. Answer any *two* of the following : 4×2

(*a*) Estimate the ground-state energy of a one-dimensional harmonic oscillator with $H = p^2/(2m) + mw^2x^2/2$ using the trial wave function $\psi(x) = N/(x^2 + a^2)$, where the normalization $N = (2a^3/\pi)^{1/2}$ and a is a real constant which is to be varied.

(*b*) Identify (*i*) $\pm\frac{1}{\sqrt{2}}(x+iy)$ (*ii*) z as spherical tensors and find the conditions for which the matrix elements $\left\langle l', m' \middle| \pm\frac{1}{\sqrt{2}}(x+iy) \middle| l, m \right\rangle$ and $\langle l', m' | z | l, m \rangle$ are non-zero.

(*c*) Consider the eigenstates $|l, m\rangle$ of the angular momentum operators L^2 and L_z. Show that $\pi|l,m\rangle = \lambda_{l,m}|l,m\rangle$, with $\lambda^2_{l,m} = 1$, where π is the parity operator. Use the commutator $[\pi, L_\pm]$ to show further that $\lambda_{l,m}$ is independent of m.

(*d*) A system having the Hamiltonian H_0 is perturbed by H_1 so that $H = H_0 + H_1$ where

$$H_0 = E_0 \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \text{ and } H_1 = E_0 \begin{pmatrix} 0 & \in \\ \in & 0 \end{pmatrix}$$

with $\in \ll 1$. Find the first and second order shifts in the energy levels of H_0 using perturbation theory. Compute the eigenvalues of H exactly and compare your results.

3. Answer any *one* of the following : 8×1

(*a*) Consider a one-dimensional harmonic oscillator having the Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 .$$

Suppose the particle has charge *e* and is perturbed by an electric field of strength *E* in the *x* direction.

(*i*) Explain using the parity symmetry of H_0 why the first order correction to the energy vanishes.

(*ii*) Compute the change in each energy level to second order in the perturbation.

(*iii*) Show that this problem can be solved exactly and compare the result with the perturbation approximation. 2 + 3 + 3

(*b*) Suppose that the energy wave functions for a particle in a periodic potential with periodicity a is satisfy $\psi(x+a) = -\psi(x)$.

(*i*) If $\psi(x) = e^{ikx}$ write down the allowed values k and the energy eigenvalues when the Hamiltonian is $H_0 = p^2/2m$. Show that the ground state is doubly degenerate.

(*ii*) A perturbing potential $V = V_0 \cos\left(\frac{2\pi x}{a}\right)$ is applied where $V_0 \ll \hbar^2 / ma^2$. Write down the secular equation for first order perturbation and compute the lowest two energy eigenvalues. 3 + 5

PHS-201.2

4. Answer any *two* questions : 2×2

(*a*) Find the inverse Fourier transform of

$$F(w) = \frac{-4}{\sqrt{2\pi}\, w^3}(w \cos w - \sin w)$$

(*b*) If SO(2) involves rotational symmetry about a single axis, find its generator.

(*c*) A square pulse is given by

$f(t) = A[\theta(t) - \theta(t - t_0)]$; A = amplitude.

Find its Laplace transform.

(*d*) If $\phi(x) = x + \frac{1}{2}\int_{-1}^{+1}(t - x)\,\phi(t)\,dt$

Prove that $\phi(x) \approx \frac{3}{4}x + \frac{1}{4}$.

5. Answer any *two* questions : 4×2

(*a*) $\dfrac{\partial^2 \psi}{\partial x^2} - \dfrac{\partial^2 \psi}{\partial x \partial y} = \sin x \cos 2y.$

Solve it by Lagrange's method.

(*b*) $y''(x) + w^2 y = 0$ with $y(0) = 0; y'(0) = 1.$

Show that

$$y(x) = x + w^2 \int_0^x (t - x)\, y(t)\, dt.$$

(*c*) $m\dfrac{d^2 x}{dt^2} = P\delta(t)$ where P is a constant with

$x(0) = 0;\ x'(0) = 0.$

Solve it by using Laplace transform.

(*d*) Find the eigen values and eigen functions of

$$\phi(x) = 2 \int_0^{2\pi} \cos(x - t)\, \phi(t)\, dt$$

6. Answer any *one* question : 8×1

(*a*) Solve the integral equation for $\phi(t)$.

$$f(x) = \int_{-1}^{+1} \frac{\phi(t)}{(1-2tx+x^2)^{1/2}}\, dt \qquad -1 \le x \le 1.$$

if

(*i*) $f(x) = x^{2s}$

(*ii*) $f(x) = x^{2s+1}$ $4 + 4$

(*b*) (*i*) If $\theta(t) = +1$ for $t > 0$

$= -1$ for $t < 0$

and F.T. of $\theta(t) = {}^{-2i}\!/_{w}$

Find the F.T. of $\frac{1}{2}\left[\theta\left(t+\frac{1}{2}\right)-\theta\left(t-\frac{1}{2}\right)\right]$.

(*ii*) Construct the group multiplication table of C_{4v} and find the invariant subgroup and factor group of C_{4v}. $3 + 3 + 2$
