PG/IIS/MTM/205/23

M.Sc. 2nd Semester Examination, 2023

APPLIED MATHEMATICS

PAPER – MTM-205

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

(*General Theory of Continuum Mechanics*)

1. Answer any *four* questions : 2×4

   (*a*) Find the value of Lame's constants in terms of Young's modulus and Poisson's ratio. 2

(*Turn Over*)

(*b*) Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress. 2

(*c*) The stress tensor at a point P in a continuum body is given by

$$(E_{ij}) = \begin{pmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

Find the Cauchy's stress quadric surface at P. 2

(*d*) The state of stress at a point with respect to cartesian co-ordinate system $Ox_1x_2x_3$ is given by

$$(T_{ij}) = \begin{pmatrix} 15 & -10 & 0 \\ -10 & 5 & 0 \\ 0 & 0 & 20 \end{pmatrix}$$

Determine the stress T'_{ij} for related to the axes $Ox'_1x'_2x'_3$ for which the transformation matrix is given by 2

$$(T_{ij}) = \begin{pmatrix} \frac{3}{5} & 0 & \frac{-4}{5} \\ 0 & 1 & 0 \\ \frac{4}{5} & 0 & \frac{3}{5} \end{pmatrix}.$$

(*e*) Show that in two dimensional irrotational liquid motion, stream function $\psi(x, y)$ and velocity potential $\phi(x, y)$ satisfy the Laplace's equation and also show that the family of curves $\phi(x, y) = constant$ and $\psi(x, y) = constant$ cut orthogonally at their point of intersection. 2

(*f*) Write down the difference between stream line and path line. 2

2. Answer any *four* questions : 4 × 4

(*a*) Find the shearing stress and normal stress on the octahedral plane element through a point P at which stress matrix is given by 4

$$(T_{ij}) = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}.$$

(*b*) Prove that there can not be two different forms of irrotational motion for a given confined mass of liquid whose boundaries have prescribed velocities. 4

(*c*) Derive the extensional strain tensor. The strain tensor at a point is given by

$$(E_{ij}) = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Determine the extension of the line element in the direction of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$. 2 + 2 (*d*) Establish relation between the strain tensor and strain vector. Define principal strain. 2+2

(*e*) Derive Navier's equation of motion. 4

(*f*) What is the significance of an image. Find the image of a source with respect to a straight line. 2 + 2

3. Answer any *two* questions : 8×2

(*a*) What is the concept of stress vector ? Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 2 + 6

(*b*) (*i*) Derive the integral of Euler's Equation of motion when body forces are conservative, pressure is a function of density alone and flow is irrotational. 5

(*ii*) Show that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$$

is a possible form of boundary surface of a liquid motion. 3

(*c*) If the equations characterizing the deformation are

$$x_1 = X_1 + \in X_2,$$
$$x_2 = X_2 - \in X_1 + \in X_3$$
$$x_3 = X_3 - \in X_2$$

then determine the Lagrangian and Eulerian finite strain tensors. 8

(*d*) Define isotropic linear elastic body. Hence, derive its constitutive equation. 2 + 6

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