M.Sc. 3rd Semester Examination, 2023

APPLIED MATHEMATICS OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-306(C)(New)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions out of *six* questions : 2×4

(*a*) How many types of variable arrangement are in the Computational Fluid Dynamics ? Discuss them by arranging the x- and y-components of velocities and pressure.

(*b*) What do you mean by grid independent study in the field of computational fluid dynamic ? Also show graphically in the plane : Error versus number of grid.

(*c*) Provide the geometric interpretation of forward, backward and centered difference formula for first derivative $\frac{\partial y}{\partial x}$ at $x = x$; graphically.

(*d*) Write the advantages of use of finite volume method.

(*e*) Define convergence of a finite difference scheme.

(*f*) Sketch the control volume for the discretization of x-momentum equation while using staggered grid arrangement with standard notation E, EE, W, WW, N, NN, S, SS.

2. Answer any *four* questions out of *six* questions : 4×4

(*a*) Discretize the heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial t^2}$ using FTCS (forward time central space), DuFort Frankel and three -level time fully implicit schemes followed by their leading term of truncation error and stability restriction.

(*b*) (*i*) Draw the control volume for u-velocity and place the variables (velocities and pressure) on the respective faces for the purpose of Quadratic Upwind Interpolation for Convective Kinematics (QUICK).

(*ii*) Then write the expressions for u-velocity at the west faces of the said control volume for both negative and positive fluxes.

(*iii*) Also with help of appropriate symbol, compare two expressions for negative and positive fluxes. 1 + 2 + 1

(*c*) Consider the following PDE :

$$\nabla^2 u = 0, 0 \le x, y \le 1$$
$$u(0, y) = u(1, y) = 0$$
$$u(x, 0) = \sin \pi x$$
$$u(x, 1) = e^{-16\left(x-\frac{1}{4}\right)^2} \sin \pi x$$

Divide the whole computational domain in to mxn number of non-overlapping control volumes with the first and second number being the number of mesh points in the x-direction and in the y-direction respectively. Here the global grid is uniform and coincides with lines of constant x and y. Applying the finite volume method (FVM) and then using the central interpolation for the first derivative, write the algebraic equation in the form of $AU = b$ for suitable A, U (unknown matrix) & b. Take the sweep direction along x-direction.

(*d*) Discretise the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using the fully implicit scheme and hence discuss the consistency of this discretise equation with that of the governing equation.

(*e*) Write the one dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

into a system of three coupled equations and then apply the Lax-Friedrich Schemes to get the discretize form of unknown u_j^{n+1}, the value of u at j^{th} node and $(n+1)^{\text{th}}$ time level.

(*f*) Write the Lax-Wendroff scheme for one-dimensional wave equation and find the stability analysis of the scheme.

3. Answer any *two* questions out of *four* questions : 8×2

(*a*) (*i*) Write the non-dimensional form of x-component Navier-Stokes equations for laminar two-dimensional incompressible fluid flow and then convert it to its conservation form.

(*ii*) Draw the control volume for u-velocity and

(*iii*) Hence apply the finite volume method to the above x-momentum equation for space derivative and three-points backward finite difference formula for time derivative to get the discretise equation in terms of fluxes only.

2 + 2 + 4

(*b*) (*i*) Apply finite volume method to the one-dimensional steady transport equation.

(*ii*) Using a linear interpolation for the advection term and second order central difference scheme for the diffusion term to calculate unknowns on each faces, derive the most simplified form of the discretised equation derived in the above part-(a). 3 + 5

(*c*) Consider the equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, 0 < x < 1, t > 0$$

where U is known for $0 \le x \le 1$ when $t = 0$, and $x = 0$ and 1 when $t > 0$.

Discuss the analytical treat of convergence of the approximate solution u, obtained using the FTCS scheme to the exact solution U when spacing tends to zero.

(*d*) Elaborate the derivation of Crank-Nicolson implicit finite difference scheme for the one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, t > 0, a < x < b$$

subject to the following initial and boundary conditions :
$u(x, 0) = f(x), a < x < b; u(a, t) = \phi_1(t)$,
$u(b, t) = \phi_2(t), t \geq 0$.
and hence determine the von-Neumann stability condition of the scheme. 6 + 2

[**Internal Assessment — 10 Marks**]