M.Sc. 3rd Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Dynamical Oceanology : Advanced Wave Hydrodynamics)

PAPER – MTM-306B (New)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *four* of the following questions : 2×4

(*a*) Describe shallow water, intermediate and deep water waves.

(*b*) Write down the kinematical condition of the free surface flow.

(*c*) Write down a short note on "Poincare waves".

(*d*) Explain, group velocity of a wave.

(*e*) Define the term "Rossby radius" and write down its value for Shallow Ocean whose depth is 100m.

(*f*) Write down the basic differences between stationary wave and progressive wave.

2. Answer any *four* of the following questions : 4×4

(*a*) Prove that, the path of the particle describes an ellipse of a progressive wave in the surface of a channel of finite depth.

(*b*) Define capillary waves and prove that the surface tension (T) of capillary waves satisfies

$$\frac{\partial^2 \phi}{\partial t^2} - g\frac{\partial \psi}{\partial x} + \frac{T}{\rho}\frac{\partial^3 \psi}{\partial x^3} = 0,$$

where symbols have usual meaning.

(*c*) Derive depth-average continuity equations for shallow water theory.

(*d*) Prove that

(*i*) The bottom stream-lines are constant

(*ii*) upper surface is stream-lines free, for steady motion of progressive wave.

(*e*) Determine the horizontal velocity expression for linear waves in the absence of rotation of the earth in the form of

$$u = -\frac{g}{2c_0}\{F(x + c_0 t) - F(x - c_0 t)\}$$

where symbols have their usual meaning.

(*f*) For a stationary wave, show that the fluid particles below the nodes move horizontally and those below the antinodes move vertically.

4. Answer any *two* of the following questions : 8×2

(*a*) (*i*) Write down the pressure condition for the wave propagation at the free surface. 3

(*ii*) Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a and λ are the wave amplitude and wave length respectively. 5

(*b*) (*i*) Show that a submarine whose depth is half a wave length, would hardly notice the motion of a submarine due to surface waves. 4

(*ii*) Prove that the reduced form for deep water wave propagation speed is

$$C^2 = \left(\frac{g}{m} + \frac{Tm}{\rho} \right)$$

due to effect of capillary on surface waves on a channel, where symbols have their usual meaning. 4

(*c*) Derive Klein-Gordon equation for long surface wave and hence, prove that geostrophic velocity in y-direction is given by

$$\bar{v} = \frac{gh}{2C_0} \exp(-|x|/a),$$

where symbols have their usual meaning. 4 + 4

(*d*) Determine the velocity of propagation for steady motion of progressive waves of an interface. Hence, show that, if velocity of the wind is just great enough to prevent the propagation of wave of length against it, the velocity of propagation of waves

of wind is $2C\left(\frac{\sigma}{1+\sigma}\right)^{1/2}$ where σ is the specific gravity of the air and C is the wave velocity when no wind is present. 5 + 3

[*Internal Assessment* — 10 Marks]
