PG/IIIS/MATH/305A/23
(New)

M.Sc. 3rd Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Advanced Optimization*)

PAPER – MTM-305A (New)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions of the following : 2×4

   (*a*) Explain the effects of addition of a new constraint in the optimal solution of an LPP.

   (*b*) What do you mean by exact and inexact one dimensional search ?

   (*c*) What do you mean by quadratic programming problem ?

(*d*) Discuss the need of integer programming in mathematical programming.

(*e*) Write the short note on characteristics of dynamic programming.

(*f*) Show that the optimum point can be obtained for a quadratic objective function in a single step by Newton's method.

2. Answer any *four* questions of the following : 4×4

(*a*) Following is the optimal solution of an LPP

		c_j	4	6	2	0	0
c_B	X_B	b_B	y_1	y_2	y_3	y_4	y_5
4	x_1	1	1	0	–1	$\frac{4}{3}$	$-\frac{1}{3}$
6	x_2	2	0	1	2	$-\frac{1}{3}$	$\frac{1}{3}$
$z_j - c_j$		16	0	0	6	$\frac{10}{3}$	$\frac{2}{3}$

If the cost coefficient c_1 changes to 8, then find the optimal basic feasible solution of the modified problem.

(*b*) Describe the branch-and-bound method to find the optimal solution of an IPP.

(*c*) Maximize

$$f(x) = \begin{cases} \frac{2x}{3}, & x \leq 3 \\ 5 - x, & x > 3 \end{cases}$$

in the interval [1,4] by Fibonacci method for $n = 5$.

(*d*) Using steepest descent method

Minimize $f = x_1^2 + x_2^2 + 8x_1 + 10x_2 + 50$

starting from the point $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(*e*) Determine the value of x_1, x_2, x_3 so as to maximize $\{x_1.x_2.x_3\}$ subject to $x_1 + x_2 + x_3 = 10$ and $x_1, x_2, x_3, \geq 0$.

(*f*) Derive the Kuhn-Tucker necessary conditions of the following quadratic programming problem

Maximize $z = 2x_1 + 3x_2 - 2x_1^2$

subject to $x_1 + 4x_2 \leq 4$

$x_1 + x_2 \leq 2$

and $x_1, x_2 \geq 0$.

3. Answer any *two* questions of the following : 8×2

(*a*) Use dynamic programming to solve the following LPP

Maximize $z = 3x_1 + 5x_2$

subject to $x_2 \leq 6$

$x_1 \leq 4$

$3x_1 + 2x_2 \leq 18$

and $x_1, x_2 \geq 0$.

(*b*) Solve the following IPP using Gomory's method

Maximize $z = 5x_1 + 7x_2$

subject to $-2x_1 + 3x_2 \leq 6$

$$6x_1 + x_2 \leq 30$$

$x_1, x_2 \geq 0$ and integers.

(*c*) Graphically solve the following goal programming problem

Minimize $z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^-$

Subject to $2x_1 + 3x_2 \leq 30;$

$$6x_1 + 4x_2 \leq 60;$$

$$x_1 + x_2 + d_1^- - d_1^+ = 10;$$

$$x_1 + d_2^- - d_2^+ = 7;$$

$$x_2 + d_3^- - d_3^+ = 8 \text{ and}$$

$$x_1, x_2, d_i^-, d_i^+ \geq 0 (i = 1, 2, 3).$$

(*d*) Write down steps of Wolfe's modified simplex method for solving quadratic programming problem (the mathematical form of quadratic programming problem is to be assumed by you).

[Internal Assessment – 10 Marks]
