2023

M. Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER : MTM-405A & 405B

Full Marks : 25

Time : 1 hour

The figures in the right-hand margin indicate marks.
The symbols used have their usual meanings.

Answer **all** questions.

(MTM-405A)

(DYNAMICAL METEOROLOGY—II)

1. Answer *any* **two** questions from the following : 2×2=4

   (a) What are frontogenesis and frontolysis?

   (b) What is global circulation in the atmosphere?

(c) What is storm surge?

(d) What is the concept of jet stream?

2. Answer *any* **two** questions from the following : 4×2=8

(a) Derive the slope of a frontal surface in the atmosphere.

(b) Using aerological diagram, briefly explain the existence of thunderstorm.

(c) Derive the meridional temperature gradient due to global circulation in the atmosphere.

(d) Write down all governing equations which are required in numerical weather prediction.

3. Answer *any* **one** question from the following : 8×1=8

(a) Mentioning the physical concept of Rossby wave, find its differential equation in the atmosphere. 8

(b) (i) Derive the pressure tendency below a frontal surface in the atmosphere. 5

(ii) Explain the significance of CAPE. 3

[Internal Assesssment marks : 05]

(MTM-405B)

(OPERATIONAL RESEARCH MODELLING—II)

1. Answer *any* **two** questions from the following : 2×2=4

(a) Show that $R(t) = \exp\left[-\int_0^\infty \lambda(t)\,dt\right]$, where $R(t)$ is the reliability function and $\lambda(t)$ represents the failure rate.

(b) Draw a general structure of a communication system and explain it.

(c) Define MTBF of a system. Suppose five components which are connected in a series with equal reliability R. Find the MTBF of such a system.

(d) What do you mean by transition matrix?

2. Answer *any* **two** questions from the following : $4 \times 2 = 8$

(a) The system connected in series consists of three independent parts A, B and C, which have MTBF of 100, 400 and 800 hours, respectively. Find MTBF of the system and reliability of the system for 30 hours. How much of MTBF of the part A has to be increased to get an improvement of the MTBF of the system by 30%?

(b) What do you mean by hazard rate $Z(t)$? Show that the reliability of a device

$$R(t) = \exp\left[-\int_0^t Z(t)\, dt\right]$$

(c) Find the stationary path $x = x(t)$ for the functional

$$J = \int_0^1 (1 + \dot{x}^2)\, dt$$

subject to the boundary conditions $x(0) = 0$, $\dot{x}(0) = 1$, $\dot{x}(1) = 1$.

(d) The following two finite probability schemes are given by $(p_1, p_2, \ldots, p_n)$ and $(q_1, q_2, \ldots, q_n)$ with $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i$. Then show that

$$-\sum_{i=1}^{n} p_i \log p_i \leq -\sum_{i=1}^{n} p_i \log q_i$$

with equality if and only if $p_i = q_i$ for all $i = 1, 2, \ldots, n$.

3. Answer *any* **one** question from the following : $8 \times 1 = 8$

(a) An electrochemical system is modelled by the differential equation $\ddot{x} = -\dot{x} + u$, where x and u are functions of time t. Minimize the cost functional

$$J = \int_0^{\infty} (x^2 + \alpha u^2)\, dt$$

when α is a disposable constant by choosing the control variable properly. Also, express the control variable u in terms of x and $\dot{x}$.

(b) (i) In a system, there is n number of components connected in parallel with reliability $R_i(t)$, $i = 1,2,...,n$. Find the reliability of the system. If $R_i(t) = e^{-\lambda t}$, $i = 1,2,...,n$, then what will be the expression of system reliability?

(ii) Find the reliability of a system with two components, of which one is a standby. The components are connected in parallel. Also, find the MTBF of this system.

[Internal Assesssment marks : 05]

★ ★ ★