2023

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER : MTM-404A & 404B

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks.

The symbols used have their usual meanings.

Answer from *any* **one** Section.

SECTION—I

(MTM-404A)

(COMPUTATIONAL OCEANOLOGY)

1. Answer *any* **four** questions from the following : 2×4=8

(a) Derive the expression for u_w (the value of u-velocity at the west face of the control volume) for two points upwind scheme in non-uniform grids. and hence simply this expression for uniform grid.

(b) Write the advantages of use of finite volume method.

(c) Discuss about the closed boundary conditions for three unknowns u, v and h at the bottom of ocean for grid-A (cell centered) and grid-B (semi-staggered grid).

(d) Write down the pressure condition for the wave propagation at the free surface.

(e) Define the term Rossby radius and give its physical significance.

(f) Write down a short note on "Poincare wave".

2. Answer *any* **two** questions from the following : $4\times2=8$

(a) (i) Draw the (j, k)th control volume for irregular grid. Apply the finite volume method on the following equation for the aforesaid control volume

$$\frac{\partial q}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \quad \text{............(1)}$$

where the symbols have their usual meanings.

(ii) If the global grid is uniform and coincides with lines of constant x and y, then show that the above discretisation coincides with a centred difference representation for the spatial terms of the above original differential equation (1). 3+1

(b) Consider the Sommerfeld radiation condition at the outlet as

$$\frac{\partial \phi}{\partial t} + u_c \frac{\partial \phi}{\partial x} = 0$$

where ϕ is any flow variable and u_c is the local wave speed applied. Find the expression for $\phi^n_{N,j}$ using 3-point backward formula for both the derivatives at nth time step over the (N, j)th control volume. 4

(c) Using the implicit Euler scheme for time derivative and center differencing for space derivative, discretise the one-dimensional heat conduction equation $\dfrac{\partial T}{\partial t} = \alpha \dfrac{\partial^2 T}{\partial x^2}$. 4

(d) Derive depth-averaged momentum equations for shallow water theory. 4

(e) Prove that for the bell-shaped surface elevation, the horizontal velocity expression in a deep ocean is $u = \frac{gh}{2C_0}$ where $C_0 = \sqrt{gH}$. Symbols have their usual meanings. 4

(f) Derive an expression for speed of propagation of a stationary wave in the surface of a canal of finite depth. 4

3. Answer *any* **two** questions from the following : 8×2=16

(a) *(i)* Draw a general finite volume cell k, with four sides, showing cell centre data u_k^n and flux density H^n on a side with side vectors.

(ii) Consider the following shallow water equations (SWE)

$$\frac{\partial U}{\partial t} + \nabla \cdot \underline{H} = Q$$

where $\underline{H} = \begin{bmatrix} \underline{H}_1 \\ \underline{H}_2 \\ \underline{H}_3 \end{bmatrix} = \begin{bmatrix} \phi \underline{v} \\ \phi v_x \underline{v} + 0\cdot5\phi^2 \, \underline{i} \\ \phi v_y \underline{v} + 0\cdot5\phi^2 \, \underline{j} \end{bmatrix}$ and

$\underline{v}$ is the flow velocity.

Apply the finite volume method for the above equation and on the control volume drawn in part *(i)*.

(iii) On a structured mesh whose cells are indexed by (i, j) with the subscript 1/2 to denote cell interfaces in the usual way. Simplify the above expression derived in part-*(ii)*. 1+5+2

(b) *(i)* Draw the grids for grid-A (cell centered), grid-B (semi-staggered grid) and grid-C (staggered grid), and arrange the variables (u, v and h) in all these grids.

(ii) Discretize the x- and y-momentum equations of two-dimensional gravity waves with centred differencing for space derivative and backward for time derive on the grid-C. 3+5

(c) *(i)* Derive Klein-Gordon equation for long surface wave. 4

(ii) Derive the expressions for the surface elevation and the velocity distribution of a single Kelvin wave at a straight coast. 4

(d) *(i)* Prove that the total energy of stationary wave is $\frac{1}{4}\rho g a^2 \lambda$ where a, λ are the wave amplitude and wavelength respectively. 5

(ii) Define the circulation in fluid rotation and calculate the circulation within a small fluid element. 3

SECTION—II

(MTM-404B)

(NON-LINEAR OPTIMIZATION)

1. Answer *any* **four** questions from the following : $2 \times 4 = 8$

   (a) What is degree of difficulty in connection with geometric programming.

   (b) Define the terms Nash equilibrium strategy and Nash equilibrium outcome in mixed strategy.

   (c) Define Pareto optimal solution in a multi-objective non-linear programming problem.

   (d) State Kuhn-Tuker stationary point necessary optimality theorem.

   (e) State Karlin's constraint qualification.

   (f) State weak duality theorem in connection with duality in quadratic programming.

2. Answer *any* **four** questions from the following : $4 \times 4 = 16$

(a) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. θ is concave if and only if $\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$.

(b) Discuss the various solution concepts for solving multi-objective non-linear programming.

(c) State and prove Slater's theorem of alternative.

(d) Write the relationship among the solutions of local minimization problem (LMP), the minimization problem (MP), the Fritz-John stationary problem (FJP), the Fritz-John saddle point problem (FJSP), the Kuhn-Tucker stationary point problem (KTP), the Kuhn-Tucker saddle point problem (KTSP).

(e) State and prove Kuhn-Tucker saddle point necessary optimality theorem.

(f) Write a short note on constraint qualification in connection with non-linear programming.

3. Answer *any* **two** questions from the following : 8×2=16

(a) Using the chance-constrained programming technique to find an equivalent deterministic problem of the following stochastic programming problem :

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } P\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge p_i$$

$$x_j \ge 0, \; i, j = 1, 2, \ldots, n$$

when b_i is a random variable and p_i is a specified probability. 8

(b) Solve the following quadratic programming problem using Wolfe's method :

Maximize

$$Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1 x_2$$

$$\text{subject to } x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \le 9$$

$$x_1, x_2 \ge 0$$

8

(c) Describe Beale's method for solving the quadratic programming problem. 8

(d) *(i)* How do you solve the following geometric programming problem?

Find $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ that minimizes the

objective function

$$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N}\left(c_j \prod_{i=1}^{n} x_i^{a_{ij}}\right)$$

$c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall i, j$

(ii) Define the following terms in connection with duality in non-linear programming :

(I) The (primal) minimization problem (MP)

(II) The dual (maximization) problem (DP) 6+2

[Internal Assessment : 10 marks]

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