2023

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER : MTM-403

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.
The symbols used have their usual meanings.

UNIT—1

(*Marks* : 20)

(MAGNETO-HYDRODYNAMICS)

1. Answer *any* **two** questions from the following : 2×2=4

(a) Write down the basic difference between the flow of magneto-hydrodynamics (MHD) and magneto-fluid dynamics (MFD).

(b) Describe the working principle of MHD power generator.

(c) Write down the Maxwell's electromagnetic field equations of moving media.

(d) Define Hartmann number and explain its significance.

2. Answer *any* **two** questions from the following : 4×2=8

(a) Find the equations of motion of a conducting fluid in the context of magneto-hydrodynamics flow.

(b) State and prove Alfven's theorem.

(c) Starting from the induction equation $\frac{\delta B}{\delta t} = \nabla \times (v \times B)$ for an infinitely conducting fluid, show that the magnetic flux across any closed contour moving with the fluid remains constant. Interpret this result in terms of the motion of the lines of force.

(d) Define the terms Alfven's velocity and Alfven's waves. Hence, derive the speed of propagation is $\sqrt{c^2 + V_A^2}$ for magneto-hydrodynamic wave, where symbols have their usual meanings.

3. Answer *any* **one** question from the following : $8 \times 1 = 8$

(a) Define Couette flow and write down the mathematical formulation of the magneto-hydrodynamic Couette flow and derive its velocity and magnetic field expressions. 1+2+5

(b) *(i)* For a conducting fluid in a magnetic field, show that the magnetic body force per unit volume, i.e., $\mu(\nabla \times H) \times H$ is equivalent to a tension μH^2 per unit area along the lines of force, together with a hydrostatic pressure $\frac{1}{2}\mu H^2$, where symbols have their usual meanings.

(ii) Define magnetic energy and further, find the rate of change of magnetic energy in magneto-hydrodynamics. 5+3

UNIT—2

(*Marks* : 20)

(STOCHASTIC PROCESS AND REGRESSION)

1. Answer *any* **two** questions from the following : 2×2=4

(a) Define the terms *(i)* accessible state, *(ii)* return state, *(iii)* periodic state and *(iv)* aperiodic state.

(b) Define multiple correlation coefficient and indicate how it differs from simple correlation coefficients.

(c) Define Markov Chain with an example.

(d) What do you mean by transition matrix?

2. Answer *any* **two** questions from the following : 4×2=8

(a) State and prove the first entrance theorem.

(b) Starting from the probability-generating function of the birth and death process, find the probability of ultimate extinction in the case of the linear growth process starting with i individuals at time 0.

(c) Prove that $1-r_{1\cdot23}^2=(1-r_{12}^2)(1-r_{13\cdot2}^2)$. The symbols have their usual meanings.

(d) Prove that the state j is persistent if and only if

$$\sum_{n=0}^{\infty} p_{jj}^{(n)}=\infty$$

3. Answer *any* **one** question from the following : $8\times1=8$

(a) Deduce multiple regression equation of $x_1, x_2, x_3,\ldots, x_p$ in terms of the means, the standard deviations and the inter-correlations of the variables.

(b) Show that the generating function $P_n(s)$ for the branching process satisfies the following relations :

(i) $P_n(s) = P_{n-1}(P(s))$

(ii) $P_n(s) = P(P_{n-1}(s))$

where $P_1(s) = P(s)$.

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