M.Sc. Part-II Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – VII

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

GROUP – A

[*Marks* : 25]

Answer **Q.No. 1** and any **two** from the rest

1. Write relations between electric and magnetic fields with the electromagnetic potentials. 1

2. (*a*) Find the expression of potential energy of a system of charges in electromagnetic theory. 5

(*b*) Derive the mutual potential energy of two doublets when the doublets are non-coplanar. 7

3. (*a*) Prove the following Maxwell's equation :

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

where symbols have their usual meanings. 6

(*b*) Find potential and field due to a dipole. 6

4. (*a*) Define perfect dielectric medium and propagation constant in electromagnetic wave theory. 2 + 2

(*b*) Derive the wave equation for conducting the medium in electromagnetic wave theory. 8

GROUP – B

[*Marks* : 25]

(*Fuzzy Sets and its application in OR*]

Answer **Q.No. 5** and any **three** from **Q.No. 6 to 10**

5. Give an example of a trapezoidal fuzzy number. 1

6. Describe Werner's approach to solve a linear programming problem with fuzzy resources. 8

7. Prove that law of excluded middle and law of contradiction does not hold for fuzzy sets. Prove that for two fuzzy sets $\tilde{A}$ and $\tilde{B}$, $(\tilde{A} \cap \tilde{B})^c = \tilde{A}^c \cup \tilde{B}^c$. 8

8. (*a*) Evaluate : 4

$$3\,[2, 5] - 4\,[-1, 2, 6] + 2\,[1, 2, 3, 4] - 5[0, 1, 4, 5\,] + [-1, 4] + 2\,[1, 2, 5] - 5.$$

(*b*) Define with examples : Alpha-cut, and intersection of two fuzzy sets. 4

9. Describe the Zimmermann's method to convert fuzzy LPP into crisp LPP. 8

10. (*a*) Give the concept of uncertainty with examples. Explain the difference between random uncertainty and fuzzy uncertainty with example. 4

(*b*) Using Zadeh's extension principle, prove that $[4, 8] - [3, 5] = [-1, 5]$. 4

GROUP – C

[*Marks* : 30]

Answer any **two** questions

11. (*a*) State and prove the theorem of Blasius for fluid thrust and couple on a fixed cylinder placed in steady, two-dimensional irrotational flow. 8

(*b*) An elliptic cylinder with semi-axes a and b is rotating round its axis with angular velocity 'ω' in an infinite liquid of density 'ρ' which is at rest at infinity. Show that if the fluid is under the action of no external forces, the moment of the fluid pressure on the cylinder about the centre is

$$\frac{1}{8}\pi\rho c^2 \frac{d\omega}{dt}$$

where $c^2 = a^2 - b^2$. 7

12. (*a*) Consider the case of single row vortices each of strength k at the points $z = 0, \pm a, \pm 2a, \ldots, \pm na, \ldots\ldots$, in the complex z-plane, z being $x + iy$. Show that as $y \to \infty$, there is a uniform stream of speed $\dfrac{k}{2a}$ in the negative x-direction. 7

(*b*) An elliptic cylinder, the semi-axes of whose cross-section are a and b, is moving with velocity U parallel to the major axis of it cross-section, through an infinite liquid of density ρ which is at rest at infinity, the pressure there being II. Prove that in order that the pressure may be everywhere positive if

$$\rho U^2 < \frac{2a^2 \text{II}}{2ab + b^2}.$$ 8

13. (*a*) Find the velocity distribution in an incompressible viscous fluid of infinite expanse adjacent to an infinite flat plate which is impulsively started from rest at time $t = 0$ and then moves in its own plane with a constant velocity U. Find the thickness of the boundary layer at time t. 7

(*b*) Determine the velocity distribution in the steady flow of uniform incompressible viscous fluid between two co-axial circular pipes under the action of a uniform pressure gradient along the common axis of the pipes. Find the average velocity and the volume fluxs. 8

GROUP – D

(*Magneto Hydrodynamics*)

[*Marks* : 20]

14. Answer any *two* questions : 10 × 2

(*a*) State and prove Alfvén's Theorem.

(*Continued*)

(*b*) A viscous incompressible fluid of uniform density is confined between two horizontal non-conducting planes $z = 0$ (lower) and $z = h$ (upper). The lower plane is held at rest and the upper one is moved horizontally in its own plane with uniform velocity U_0. A uniform magnetic field H_0 acts perpendicular to the planes. Find the velocity and the magnetic field between the planes.

(*c*) Show that in an infinite mass of an in viscid, perfectly conducting incompressible fluid (of density ρ and magnetic permeability μ_e) permeated by a uniform magnetic field H_0, a small disturbance in the magnetic field is propagated in the form of transverse waves along the magnetic lines of force with velocity $H_0 (\mu e / 4\pi\rho)^{1/2}$.
