M.Sc. Part-I Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VA

Full Marks : 50

Time : 2 hours

Answer **Q. No. 6** and any **three** from the rest

The figures in the right-hand margin indicate marks

1. (*a*) What is infinitesimal strain tensors ? Give geometrical interpretation of such tensors. 8

(*b*) What are principal stress and principal direction of stress ? The stress tensor (T_{ij}) at a point is given by

$$\left(T_{ij}\right)=\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Determine the principal stresses and corresponding principal directions. Also verify the stress invariants. 8

2. (*a*) State and prove the Cauchy's first equation of motion. Deduce the equation of equilibrium. When the continuum is in static equilibrium. 8

(*b*) What is strain deformation ? Given the displacement field :

$$x_1 = x_1 + 2x_3, x_2 = x_2 - 2x_3, x_3 = x_3 - 2x_1 + 2x_2$$

determine the Lagrangian and Eulerian finite strain tensor. 8

3. (*a*) State and prove Milne-Thomson circle theorem. Deduce the image of a source relative to a circular cylinder. 8

(*b*) The velocity components for a two dimensional fluid system can be given in the Eulerian system by

$$v_1 = 2x_1 + 2x_2 + 3t$$
$$v_2 = x_1 + x_2 + \frac{t}{2}$$

Find the displacement of a fluid particle in the Lagrangian system. 8

4. (*a*) State and prove the Kelvin's minimum energy theorem. 8

(*b*) Define source and sink. If the Fluid fills the region of space on the positive side of x-axis, which is a rigid boundary and if there be as source $+m$ at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side of the boundary be the same as the pressure of the field be infinity, show that the resultant pressure on the boundary is

$$\frac{\pi\rho m^2(a-b)^2}{ab(a+b)}$$

where ρ is the density of the fluid. 8

5. (*a*) Derive the Cauchy's integrals of Lagrange's equation of motion of perfect fluid in terms of vorticity. 8

(*b*) What is strain energy ? Show that the strain energy function of a elastic body is a homogeneous function of second degree. 8

6. In two-dimensional incompressible fluid motion, show that both the stream function and velocity potential satisfy Laplace's equation. 2