Detection and Deterrence: A Microtheoretic Approach

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Abstract

For over three decades, research on crime and corruption has been of special interest among social scientists. The present paper seeks to examine the decision-making behaviour of an illegal firm in terms of a simple micro theoretic exercise. We set up simple models of a profit maximizing illegal industrial firm and derive conditions under which illegalisation of work and production occurs, taking into consideration the fact that the probability of being detected of unlawful activity is a variable and being dependent on the scale of activity, measured in terms of output or employment. Here, we try to explore the cases where the firm is detected at different phases of its production process and also examine the impact of deterrence, if any, on the scale of illegal activity.

Key Words: *Illegal Sector, Fine, Profit Maximization, Detection, Output, Labour.*

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I

Crime and corruption are in existence from the very beginning of humanity. It can be defined as dishonest or fraudulent conduct by the people in power or those who have a prominent position in the society. It typically involves bribery, nepotism, extortion, patronage, cronyism, etc. It is like a disease to the economy of the country. Not only does it hold the economy from reaching new heights but also prevents the country's development. It is one of the biggest challenges faced by many countries all over the world. It has spread its roots so deep in the society that it has become difficult to get rid of it. It is becoming bigger day by day because of people's increasing wants in life.

Crime is any action that violates the law of country or rules of the state. It is that illegal action which is harmful to oneself or the society and is punishable by law. A crime can range from a minor level of robbery to the extent of homicide or suicide. Thus, any individual who commits a crime is known as a criminal. In the present day, we are most vulnerable to various criminals of the society. They can be kidnappers, rapists, murderers, or even terrorists. There are criminals who are indulged in crime under the influence of politicians and businessmen.

The rate of crime is increasing day by day. This can be mainly due to the rapid globalization and urbanization. In one way, the scientific advancements serve as a boon; in other way, it also serves as a bane. The same advancements can be utilized to harm the society. There have been advancements in the way the crimes are being committed. In olden days, only the uneducated and poor people were driven to be criminals but nowadays, even well-educated young people are involved in major crimes. The technology has been exploited by these educated people to commit crime or undergo terror attacks.

Reviewing literature, we can trace some of the papers that have contributed immensely for structuring our proposed work. Leys (1965) concludes that corruption is a positive developmental force by creating competition and market structures for private sector actors. Becker (1968) analyzes that relationship between punishment and an individual's willingness to engage in an illegal act. He asks what is the optimal level of punishment to deter crime. The optimal level of punishment is largely a function of the cost of enforcement, investigation, and punishment. He concludes that the optimal levels of sanction depend upon the overall cost of enforcement. Phillips and Votey (1972) state that for meting out punishment, law enforcement not only responds to crime but also tempers the process of crime generation by creating a deterrent effect. Technology and expenditures on law enforcement determines the effectiveness of law enforcement. Posner (1975) argues that competition to attain a monopoly transforms monopoly profits into social costs. Costs associated with regulation may exceed the social costs of monopoly. This is an argument that supports notions of corruption as enhancing competition. Kramer (1990) states that social harm increases due to uniformly high sanctions. There is negative demand for offenses from the society's perspective. To prevent the supply of offenses, it is willing to pay a price. But if there is no criminal activity, it is not ready to pay anything. Schleifer and Vishny (1993) focuses on the consequences of corruption for resource allocation. It argues that political structures and processes determine that extent of corruption in a given country. Unlike its sibling taxation, corruption is costly and therefore detrimental to a country's economic development. The paper draws important considerations of corruption as an activity that occurs under conditions of secrecy. Meny (1996) states that shifting values have elevated corruption as a critical problem for government. Perceptions of the costs of corruption hold that it has increased and an urgency has become apparent to control its incidence. Corruption reflects a crisis in the efficient functioning of the public sector and has become a priority for governments to control. Bliss and Tella (1997) offer an explanation of under what circumstances competition may limit the costs of corruption. Cartier-Bresson (1997) argues that most studies of corruption concentrate on occasional and unorganized behaviour. It analyzes instead networks of corrupt actors who interact in regular, organized corruption. The article concludes that the study of networks has more to offer for an understanding of corruption than the classic political economy approach. Polinsky and Shavell (2001) analyse corruption in enforcement of law: in order to extort money, threatening and framing innocent individuals, paying enforcement agents bribes, and the actual framing of innocent individuals. It is very discouraging to frame, extort, and bribe people which leads to reduced deterrence. The authors suggest that there should be imposition of optimal penalties for bribery and framing, but extortion should not be sanctioned. For reporting violations, the enforcement agents should be rewarded, which in turn, may reduce corruption. Even though the problem of bribery can be partially or completely mitigated with such rewards, but still they encourage framing. To discourage bribery, the optimal reward may be relatively high, or may be relatively low to discourage framing and extortion. Lando (2006) considers the traditional view whereby deterrence is lowered due to wrongful conviction. This is because it lowers the pay-off of an innocent person without affecting the pay-off for being guilty. However, this view cannot distinguish between mistake about the act and mistake about identity. In case of identity mistake, the view is incorrect, since a person who actually commits a criminal act will not be certain about eliminating the risk of being convicted himself of someone else's crime. There may be a positive or a negative effect on deterrence, if a person is wrongfully convicted and the magnitude of it is likely to be small because the chance of wrongful conviction is shared among many people. The paper concludes by stating that when there is a binary choice set for the offender, the traditional view is correct and if the choice set is continuous, and then there is a positive effect on deterrence if the defendant's act is wrongly assessed by the court. Mungan (2015) considers an abandoned attempt, if the criminal wishing to execute his criminal plan, forgoes the opportunity to do so. Prior to completion of an intended offence, abandonment to criminal attempts acts as an incentive to the offender to withdraw from his criminal conduct. This paper establishes a trade-off between ex-ante and marginal deterrence.

The purpose of the present exercise is to focus on the decision-making behaviour of a firm that is engaged in illegal or underground activity. We are, especially, interested to investigate the optimization conditions of the firm and the policy implications of the conditions. It needs to be asserted that the firm operating in the illegal market faces the risk of detection and, consequently, there are chances of facing penalty. Hence, while the firm in overground sector may be taken to be a profit maximiser, the firm in the underground or illegal sector should be taken as maximiser of expected profit.

Let us suppose that the firm in the illegal business is manufacturing a product that is detrimental to the interest of the society. The firm may be illegally manufacturing arms or guns or explosives without licence; or, the firm may be producing a duplicate or fake product while the authentic or branded product is being produced by some firm in the overground economy. In India, production of adulterated goods replicating the authentic goods is rather common; consumers and producers are often cheated by the presence of duplicate brands in the market – e.g. in detergents, bulbs, edible oils and even in pharmaceutical products and medicines. Only a very careful examination or serious investigation occasionally exposes such fraudulent practices.

II

Here we confine ourselves, for simplicity, with the case of a single illegal firm (say, producing some adulterated or duplicate product) acting as price taker in the product and the factor markets. The firm uses two inputs, capital (K) and labour (L). We postulate the following production function:

$$Q = Q\,(L, K)$$

which is considered to exhibit constant returns to scale and diminishing average and marginal productivities.

In the short run, the size of the capital stock is given, so that we may write $K = \bar{K}$. And, accordingly,

$$Q = g\,(L),\ \ g'(L) > 0, \text{and } g''(L) < 0$$

The firm's production activity is illegal and hence it has the chance of being caught and charged for the unlawful act. Now, the question that arises is at what stage of its activity the firm is found involved in an illegal act. We may visualize a set of alternative possibilities: (a) the unlawful action comes to the notice of the police or law enforcing agency after the firm has already produced and marketed its output; (b) the firm is found involved in illegal act at the time of production, but before the output has been produced and/or marketed; (c) the firm's unlawful activity is detected at a pre-mature stage, i.e. before the firm has invested some money for production and has started its production operation; and (d) the illegal activity is detected after a portion of the output has already been sold.

Our analysis of firm's behaviour and its outcome would vary from case to case. Further, in our entire analysis we refrain from dynamics. In other words, our model is a static micro theoretic exercise. It is imperative to admit that better and more realistic picture might be obtained if one works in the context of a dynamic theory of firm. However, as a first step, it is useful to work with a static analysis.

When the firm is caught, it faces a penalty. In reality, when a firm engaged in illegal activity is caught and charged, there is a legal process which usually runs for several periods before the errant firm is finally handed down the penalty. Here we assume, for simplicity, a one-time monetary fine. And since our analysis is static in nature, we, therefore, refrain from a more than one period legal process. In other words, we conceive an instantaneous fine or penalty.

Before proceeding further, let us now write down the following notations:

P = price of the product of the firm in illegal business

Q = output of the firm

w = wage rate (assumed to be fixed)

r = rental (assumed to be fixed)

f = amount of fine imposed on the firm in case of detection of unlawful activity

π^*= profit of the firm in case of not being detected of the unlawful activity

$\hat{\pi}$ = profit of the firm in case of being detected of the unlawful activity

v = probability of escape or not being caught or detected

u = probability of being caught or detected

It is reasonable to assume that the firm is a risk averter and it attempts to maximize expected profit (while escape from being caught is considered as good state, the case of being caught may be deemed as bad state from the firm's perspective).

Since there is the possibility of detection of the unlawful act of the firm at different stages, therefore, the firm's optimal decision or the outcome would depend on at what stage the firm is caught. Accordingly, we need to differentiate between alternative scenarios which we set out in the following sections.

In standard analyses, probability of being caught or not caught of unlawful activity is usually taken as constant/fixed. In an earlier and yet to be published article, we carried out our analysis on this basis. In the present exercise, we deviate from that standard practice and, instead, postulate that the probability of being detected or not being detected is variable. To be more precise, we assert that as the size or the magnitude of the firm's unlawful activities rises, the firms faces the greater risk of being caught or detected. This is, indeed, quite logical and easy to comprehend. The size of the firm's illegal activities may be measured in two alternative ways: in terms of (a) volume of production or output and (b) size of labour employment. In the following sections, we attempt to introduce this aspect in our analysis of firm behaviour and see how the results may change.

III

Case A: Firm's illegal action is detected after the output has been sold

We take up the case with variable probabilities. Assuming, 'u' (probability of being caught or detected) = $\frac{Q}{Q+1}$ and 'v' (probability of escape or not being caught or detected) = $\frac{1}{Q+1}$. The hypothesis is that the larger is the output of the firm, the higher is the chance of detection and on the other hand, lesser the output, lower is the chance of getting detected.Thus, when $Q = 0$, u = 0 and v = 1.

When the unlawful act goes undetected, firm's profit function remains unchanged. In case of detection and consequent imposition of fine by the law enforcing agency, the firm's profit calculations get altered. The revenue of the firm is (P.Q). Let us set, for simplicity and without any loss of generality, P = 1 (i.e. we normalize the price).

$$\pi^* = PQ - wL - rK$$

$$\Rightarrow \pi^* = Q - wL - rK \quad \text{................(1)}$$

In case of detection, we have:

$$\hat{\pi} = PQ - wL - rK - f$$

$$\Rightarrow \hat{\pi} = Q - wL - rK - f \quad \text{........................(2)}$$

In short run, $K = \bar{K}$, so that r$\bar{K}$ is a fixed cost, say 'F'. Thus, we can rewrite the two profit expressions as:

$$\pi^* = Q - wL - F \text{ and } \hat{\pi} = Q - wL - F - f$$

It needs to be stated that once the illegal activity is detected after the whole output has already been sold, the law enforcing agency may find it hard to estimate the actual output or find a reliable trail to the volume of sales (recall that in case of illegal activities, firm is unlikely to maintain proper accounts too). As a result, the law enforcing agency would find it sensible to impose a hefty lump sum fine.

The expected profit function of the firm is then given by:

$$\text{E}(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \quad \text{......................(3)}$$

$$\Rightarrow \tilde{\pi} = g(L) - wL - F - \frac{g(L)}{g(L)+1}f \quad \text{..............(4)}$$

The choice before the firm is to choose the value of 'L' (given $K = \bar{K}$) in such a fashion that E (π) =$\tilde{\pi}$ is maximum.

For first order condition (FOC) of profit maximisation, setting $\frac{d\tilde{\pi}}{dL} = 0$, we get:

$$Q' = \frac{w\ [Q+1]^2}{[Q+1]^2 - f} = MP_L \quad \text{..................(5)}$$

where $Q' = g'(L)$ stands for the marginal productivity of labour (= $\frac{dQ}{dL}$) and by assumption $g'(L) > 0$. The above equation may, alternatively, be written as:

$$w = \frac{Q'\{[Q+1]^2 - f\}}{[Q+1]^2} \quad \text{............(6)}$$

The second order condition (SOC) for maximization of the E (π) is $\frac{d^2\tilde{\pi}}{dL^2} < 0$.

$$\Rightarrow \frac{Q''\,[Q+1]^4 - [Q+1]^2[Q''\,f] + 2f\,[Q+1][Q']^2}{[Q+1]^4} < 0$$

Under variable probabilities with u = $\frac{Q}{Q+1}$ and v = $\frac{1}{Q+1}$, there exists deterrent effect as:

$$g' = \frac{w\ [Q+1]^2}{[Q+1]^2 - f}$$

$$\Rightarrow g' = \frac{w}{[1 - \frac{f}{(g+1)^2}]} \ldots\ldots\ldots\ldots\ldots (7)$$

Since, w > 0 and g'> 0 (for meaningful solution), so the following condition should hold:

$$1 > \frac{f}{(g+1)^2}$$

$$\Rightarrow 0 < \frac{f}{(g+1)^2} < 1$$

$$\Rightarrow f < (Q+1)^2 \ldots\ldots\ldots\ldots (8)$$

Thus, the denominator of equation (7) is a fraction. This implies:

$$\frac{w}{[1 - \frac{f}{(g+1)^2}]} > w$$

Hence, $g'(L)$ is higher in equilibrium. Thus, L is lower and consequently Q is lower; thereby fine acts as a deterrent.

Also, for profit maximisation, the SOC requires:

$$Q''\ [Q+1]^4 - [Q+1]^2[Q''f] + 2f\ [Q+1][Q']^2 < 0 \text{ (since, } [Q+1]^4 > 0)$$

$$\Rightarrow f > \frac{(Q+1)^3\ Q''}{(Q+1)Q'' - 2(Q')^2} \ldots\ldots\ldots\ldots\ldots (9)$$

Since, $Q'' < 0$ and $Q' > 0$, hence f > 0. This implies that $Q'' < 0$ does not suffice to guarantee fulfilment of SOC.

Case B: Firm's illegal action is detected before the output has been sold

Let us now consider the case where the firm's unlawful activity is detected at a stage where it has completed the production process, but the output is yet to be marketed. We take up the case with variable probabilities.

Now, as before,

$$\pi^* = PQ - wL - rK$$

$$\Rightarrow \pi^* = Q - wL - rK \ldots\ldots\ldots\ldots\ldots (10)$$

However, as the firm is detected prior to sale in the market, we now have:

$$\hat{\pi} = -wL - rK - f \ \ldots\ldots\ldots\ldots\ldots (11)$$

Not only is the firm being unable to earn revenue, but also it loses the entire cost that it has already incurred plus it also faces a fine. In other words, in bad state, the firm can only incur losses.

With $K = \overline{K}$ so that r$\overline{K}$ = F (fixed cost), we can write down the profit functions under good and bad states, respectively, as:

$$\pi^* = Q - wL - F \text{ and}$$

$$\hat{\pi} = -wL - F - f = -(wL + F + f) < 0$$

The expected profit function of the firm is given as follows:

$$E(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \ldots\ldots\ldots\ldots\ldots (12)$$

$$\Rightarrow \tilde{\pi} = \frac{-g(L)\,wL - g(L)\,F - g(L)\,f + g(L) - wL - F}{g(L)+1} \ldots\ldots\ldots\ldots\ldots (13)$$

As before, the firm's problem is to choose the value of L in such a fashion that E (π) = $\tilde{\pi}$ is maximum.

Setting the FOC for first order profit maximisation, $\frac{d\tilde{\pi}}{dL} = 0$, we get:

$$g'(L) = \frac{w\,[Q+1]^2}{[1-f]} = Q' = MP_L \;\ldots\ldots\ldots\ldots\ldots(14)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

Again,

$$w = \frac{Q'\,[1-f]}{[Q+1]^2} \;\ldots\ldots\ldots\ldots\ldots(15)$$

The second order condition (SOC) for maximization of expected profit is:

$$\frac{d^2\tilde{\pi}}{dL^2} < 0$$

$$\Rightarrow \frac{[Q+1]^2[-2w\,Q'\{Q+1\} + Q''\{1-f\}] - [-w\,\{Q+1\}^2 + Q'\{1-f\}][2Q'\{Q+1\}]}{[Q+1]^4}$$

The first order condition is:

$$g'(L) = \frac{w\,[Q+1]^2}{[1-f]}$$

$$\Rightarrow g'(L) = \frac{w\,[g(L)+1]^2}{[1-f]} \;\ldots\ldots\ldots\ldots\ldots(16)$$

For $g'(L) > 0$, meaningful solution is 0 < f < 1 and this condition is same under Case A. However, in Case A, the condition is not just f < 1, but to be more specific, f < $[\,g(L)+1]^2$.

Now,

$$g'(L) = \frac{w}{\frac{1}{(g+1)^2} - \frac{f}{(g+1)^2}} \;\ldots\ldots\ldots\ldots\ldots(17), > 0 \text{ if } f < 1.$$

Let us try to compare between equations (7) and (17) – the FOCs in Case A and Case B, respectively. We may argue that:

$$1 - \frac{f}{(g+1)^2} > \frac{1}{(g+1)^2} - \frac{f}{(g+1)^2} > 0$$

This implies denominator in equation (17) is smaller than in equation (7). Hence, $g'(L)$ is higher in equation (16) than in equation (7). Consequently, value of 'L'is lower in Case B and thus, value of 'Q' is also lower in Case B. This leads to greater deterrence on unlawful activity in Case B compared to Case A.

Case C: Firm's illegal action is detected at a very nascent stage of its establishment

In this section, we consider that the firm is detected at a very nascent stage of its establishment. Here, it has just planned to operate and it is yet to hire factors (labour and capital) and, thus yet to start its unlawful production process. It needs to be noted that in many situations, the criminals are nabbed at the very early stage before they get into the act.

As before, if the unlawful act goes unnoticed, the firm earns some profit which is given by:

$$\pi^* = PQ - wL - rK$$

$$\Rightarrow \pi^* = Q - wL - rK \;..................(18)$$

However, if the illegal plan of the firm is exposed at the very outset, the firm is charged with a fine; it does not lose any sunk and variable cost. The firm's loss is just the fine or the penalty. Hence, if detected, we have:

$$\hat{\pi} = -f \;..................(19)$$

In the short run, $K = \overline{K}$, or, r$\overline{K}$ is a fixed cost, say 'F'. Thus, we can rewrite the two profit functions under good and bad states, respectively, as:

$$\pi^* = Q - wL - F \text{ and } \hat{\pi} = -f$$

The expected profit function of the firm is, therefore, given by:

$$E(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \quad \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

$$\Rightarrow \tilde{\pi} = \frac{g(L) - wL - F - f\,g(L)}{g(L) + 1} \quad \ldots\ldots\ldots\ldots\ldots(21)$$

The choice before the firm is to choose the value of 'L' in such a fashion that $E(\pi) = \tilde{\pi}$ is maximum. Let us consider short run situation, so that $K = \bar{K}$.

For first order profit maximisation, setting $\frac{d\tilde{\pi}}{dL} = 0$, we get:

$$Q' = \frac{w\,[Q + 1]}{[1 - f + wL + F]} = MP_L \quad \ldots\ldots\ldots\ldots\ldots(22)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

Again,

$$w = \frac{Q'\,[1 - f + F]}{[Q - L\,Q' + 1]} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

The second order condition for (expected) profit maximisation is given by:

$$[Q+1]^2\{Q'\,w + [1 - f + wL + F]Q''\} - \{Q'[1 - f + wL + F]\}\{2\,[Q+1]Q'\} - [Q+1]^2\{w\,Q'\} + \{w\,[Q+1]\}\{2\,[Q+1]Q'\} \,/\, [Q+1]^4 < 0$$

The first order condition is:

$$g'(L) = Q' = \frac{w\,[g(L) + 1]}{[1 - f + wL + F]}$$

For meaningful solution, $g'(L) > 0$. Thus, $(1 + wL + F) > f$.

$$\Rightarrow 0 < f < (1 + wL + F) \quad \ldots\ldots\ldots\ldots\ldots(24)$$

Also, from FOC:

$$w = \frac{g'(L)\,[1 - f + F]}{L\,[AP_L - MP_L] + 1} \quad \ldots\ldots\ldots\ldots\ldots(25)$$

If $g''(L) < 0$, then MP_L is diminishing, which implies $AP_L > MP_L$. This further signifies that denominator is positive.

Therefore, $w > 0$ and $g'(L) > 0$.

$$1 + F > f \;\Rightarrow f < 1 + F$$

Thus, upper bound on feasible value of 'f' has now risen.

From FOC, $[1 - f + wL + F] > 0$ and $Q'' = g''(L) < 0$.

Let us now consider SOC. The first term on the numerator is negative, the second term is also negative (as there exists a minus sign) and finally the third term on the numerator is negative too (as there exists a minus sign). Hence, $g''(L) < 0 \Rightarrow Q''(L) < 0$, satisfies SOC or FOC gives profit maximising solution.

Case D: The illegal activity is detected after a portion of the output has already been sold

This case is, in a sense, more realistic one. The illegal action comes to the notice after some portion of the fake product finds its way into the market. Let us now introduce the following additional notations:

θ = fraction of output already marketed $(0 < \theta < 1)$

$(1 - \theta)$ = fraction of output unsold at the time of detection; this is also the fraction of output confiscated or seized by the law enforcing agency

f = per unit fine imposed on the confiscated output $(f > 0)$

Now, profit in the bad state (i.e. when caught) is given by:

$$\hat{\pi} = P\,(\theta Q) - wL - r\bar{K} - f\,(1 - \theta)\,Q$$

$$\Rightarrow \hat{\pi} = \theta g(L) - fg(L) + fg(L)\theta - wL - F \text{..............(26)}$$

Profit in the good state (i.e. when not detected at all) is given by:

$$\pi^* = PQ - wL - F$$

$$\Rightarrow \pi^* = Q - wL - F = g(L) - wL - F \text{.................(27)}$$

Hence, the expected profit is given by:

$$E\,(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \text{ ....(28)}$$

$$\Rightarrow \tilde{\pi} = \frac{\theta[g(L)]^2 - f[g(L)]^2 + f\theta[g(L)]^2 - wLg(L) - Fg(L) + g(L) - wL - F}{[g(L) + 1]}$$

The objective of the firm is to choose the level of employment of labour (L) in such a way that expected profit is maximized. The FOC for expected profit maximization is:

$$\frac{d\tilde{\pi}}{dL} = 0$$

$$\Rightarrow g'(L) = Q' = MP_L = \frac{w\,[Q + 1]^2}{[[Q^2 + 2Q][\theta - f(1 - \theta)] + 1]} \text{ ....(29)}$$

where, marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

Again,

$$w = \frac{Q'[[Q^2 + 2Q][\theta + f\theta - f] + 1]}{[Q + 1]^2} \text{ ....(30)}$$

The second order condition for profit maximisation is given by:

$$[Q + 1]^2\,[Q''\{2\theta Q - fQ^2 - 2fQ + 2f\theta Q + 1 + \theta Q^2 + f\theta Q^2\} + 2Q'\{\theta Q' - fQQ' - fQ' + f\theta Q' + \theta QQ' + f\theta QQ' - wQ - w\}] - [2Q'\{Q + 1\}][Q'\{2\theta\,Q - fQ^2 - 2fQ + 2f\theta Q + 1 + \theta Q^2 + f\theta Q^2\} - w\{Q + 1\}^2]\;/\;[Q + 1]^4 < 0$$

Let $x = \theta + f\theta - f = \theta - f\,(1 - \theta)$

We have already obtained:

$$Q' = \frac{w\,[Q + 1]^2}{[xQ\,(Q + 2) + 1]}$$

When $\theta = 0 \Rightarrow x = -f$ and $\theta = 1 \Rightarrow x = 1$.

For $Q' > 0 \Rightarrow xQ\,(Q + 2) + 1 > 0 \Rightarrow xQ\,(Q + 2) > -1$.

When $x = \theta = 1$, we have:

$$Q' = \frac{w}{\frac{Q}{Q + 1} + \frac{(Q + 1)}{(Q + 1)^2}} = w \text{ ....(31)}$$

The FOC indicates that the illegal firm's behaviour and decision with respect to employment of variable input (labour) is same as that of its counterpart operating in the legal sector. The fine in this case (i.e. $\theta = 1$) has, surprisingly, no impact on employment and output of the illegal firm and thus a fixed fine, however hefty, has no deterrent impact on the illegal activity, at least in the short run.

When $\theta = 0 \Rightarrow x = -f$, we have:

$$Q' = \frac{w}{\frac{-fQ}{Q + 1} + \frac{(1 - fQ)}{(Q + 1)^2}}$$

$$\Rightarrow\ Q' = \frac{w\,(Q+1)^2}{1 - fQ^2 - 2fQ} \quad \ldots\ldots\ldots\ldots\ldots..(32)$$

For $Q' > 0$, we require:

$$1 - f(Q^2 + 2Q) > 0$$

$$\Rightarrow f < \frac{1}{Q\,(Q+2)} < 1 \text{ [depending on the value of Q]}$$

In general:

$$Q' = \frac{w}{\frac{xQ}{Q+1} + \frac{xQ+1}{(Q+1)^2}}$$

$$\Rightarrow f < \frac{\theta}{(1-\theta)} + \frac{1}{(Q^2+2Q)\,(1-\theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

If $\theta = 1 \Rightarrow f < \infty$. Thus, any positive rate of fine is possible.
If $\theta = 0 \Rightarrow f < \frac{1}{Q^2+2Q} < 1$ [depending on the value of Q].
When $f = \frac{\theta}{(1-\theta)} + \frac{1}{(Q^2+2Q)\,(1-\theta)}$, we have:

$$\frac{df}{d\theta} = \frac{(1-\theta)\,1 - \theta\,(-1)}{(1-\theta)^2} + \frac{0 - (Q^2+2Q)(-1)}{[(Q^2+2Q)(1-\theta)]^2}$$

$$\Rightarrow\ \frac{df}{d\theta} = \frac{1}{(1-\theta)^2} + \frac{(Q^2+2Q)}{[(Q^2+2Q)(1-\theta)]^2} > 0 \quad \ldots\ldots\ldots\ldots\ldots..(34)$$

This implies as θ rises, f also rises – the upper limit of f rises.
Now, let us again consider SOC.
Let $x = \theta + f\theta - f = \theta - f\,(1-\theta)$
Let $a = 2\theta Q - fQ^2 - 2fQ + 2f\theta Q + 1 + \theta Q^2 + f\theta Q^2$
$= 2Qx + Q^2x + 1$
Let $b = \theta Q' - fQQ' - fQ' + f\theta Q' + \theta QQ' + f\theta QQ' - wQ - w$
$= (Q+1)(Q'x - w)$
Let $c = 2\theta\, Q - fQ^2 - 2fQ + 2f\theta Q + 1 + \theta Q^2 + f\theta Q^2$
$= 2Qx + Q^2x + 1$
Thus, it can be verified that:

$$a = 2Qx + Q^2x + 1, b = (Q+1)(Q'x - w), c = 2Qx + Q^2x + 1$$

Further, FOC yields:

$$Q' = \frac{w\,[Q+1]^2}{[(Q^2+2Q)\,x + 1]}$$

$$\Rightarrow\ w\,[Q+1]^2 = Q'\,[Qx\,(Q+2) + 1]$$

Substituting in SOC:

$$[Q+1]^2[Q''\{2Qx + Q^2x + 1\} + 2Q'\{(Q+1)(Q'x - w)\}]$$
$$-\ [2Q'\{Q+1\}][Q'\{2Qx + Q^2x + 1\} - Q'\,\{Qx\,(Q+2) + 1\}]$$
$$=\ [Q+1]^2[Q''\{2Qx + Q^2x + 1\} + 2Q'\{(Q+1)(Q'x - w)\}]$$

Thus, $Q'' < 0$ does not suffice to guarantee fulfilment of SOC, given $[Q+1]^4 > 0$.

IV

The preceding analysis has been made on the assumption that the size of the unlawful activity, and consequently, the chances of detection are determined by the size of production or output level. Instead of relying on output as a determinant, we may explore the issue by considering employment of labour as the indicator of size of the activity. The larger is the size of the labour force associated with the production of illegal activity, the greater is the chance of being detected. In this variable probability case, we, therefore, consider labour size as the determinant of the probability (of detection and/or escape). We repeat the above four cases taking u = $\frac{L}{L+1}$ and v = $\frac{1}{L+1}$.

Case E: Firm's illegal action is detected after the output has been sold

When the unlawful act goes undetected, firm's profit function remains unchanged. In case of detection and consequent imposition of fine by the law enforcing agency, the firm's profit calculations get altered. The revenue of the firm is (P.Q). Let us set, for simplicity and without any loss of generality, P = 1 (i.e. we normalize the price).

$$\pi^* = PQ - wL - rK$$

$$\Rightarrow \pi^* = Q - wL - rK \text{ ....(35)}$$

In case of detection, we have:

$$\hat{\pi} = PQ - wL - rK - f$$

$$\Rightarrow \hat{\pi} = Q - wL - rK - f \text{ ....(36)}$$

In short run, $K = \overline{K}$, so that r$\overline{K}$ is a fixed cost, say 'F'. Thus, we can rewrite the two profit expressions as:

$$\pi^* = Q - wL - F \text{ and } \hat{\pi} = Q - wL - F - f$$

The expected profit function of the firm is then given by:

$$E(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \text{ ....(37)}$$

$$\Rightarrow \tilde{\pi} = \frac{L\,g(L) - wL^2 - LF - Lf + g(L) - wL - F}{L+1} \text{ ……………….(38)}$$

The choice before the firm is to choose the value of 'L' (given $K = \overline{K}$) in such a fashion that E (π) = $\tilde{\pi}$ is maximum.

For first order condition (FOC) of profit maximisation, setting $\frac{d\tilde{\pi}}{dL}$ = 0, we get:

$$g'(L) = Q' = MP_L = \frac{f + w\,[L+1]^2}{[L+1]^2} \text{ ……………….(39)}$$

where marginal productivity of labour (Q') = $\frac{dQ}{dL}$ > 0.

The FOC may be rewritten as:

$$w = \frac{Q'[L+1]^2 - f}{[L+1]^2} \text{ ………………(40)}$$

The second order condition (SOC) for maximization of the E (π) is $\frac{d^2\tilde{\pi}}{dL^2} < 0$.

$$\Rightarrow [L+1]^2\left[Q''\{L+1\}^2 + 2Q'\{L+1\} - 2w\{L+1\}\right] - [Q'\{L+1\}^2 - w\{L+1\}^2 - f][2(L+1)] \,/\, [L+1]^4 < 0$$

The first order condition is given by:

$$g'(L) = \frac{f + w\,[L+1]^2}{[L+1]^2}$$

$$\Rightarrow g'(L) = w + \frac{f}{[L+1]^2} > w \text{ ……………(41)}$$

We may try to make a comparison between equation (7) and equation (41), by considering some hypothetical values.

Let $L = 4, f = 50, w = 100$ and $Q = A\sqrt{LK} = \sqrt{L}$, say ($A = 1, \overline{K}$ = 1).

Then, $Q = g(L) = \sqrt{4} = 2$.

Under equation (7),

$$\frac{f}{[g+1]^2} < 1 \Rightarrow f < [g+1]^2 \Rightarrow f < [2+1]^2 = 9$$

So, for meaningful solution, $f < 9$. Suppose, $f = 3$.

Then, $g'(L) = \frac{100}{1-\frac{3}{9}} = \frac{(100*3)}{2} = 150.$

Under equation (41),

$$g'(L) = 100 + \frac{3}{(4+1)^2} = 100 + \frac{3}{25} < 150.$$

In equation (7), $g'(L)$is raised by a proportional factor; in equation (41), it is raised by a fixed value. This indicates impact on lowering illegal labour is less in case of variable probabilities with labour compared to the analogous case of variable probabilities with output.Thus, the negative impact on output produced is less in the case of variable probabilities with labour. Hence, imposition of fine has a more deterring effect on the illegal activity in case of variable probabilities with output compared to the analogous case of variable probabilities with labour employed.

In equation (41),

$$g'(L)[L+1]^2 - w\,[L+1]^2 = f$$
$$\Rightarrow [L+1]^2[g'(L) - w] = f$$

As long as $g'(L) > w, f > 0$ and as the difference between $g'(L)$ and w rises, $f$rises (i.e., upper limit of 'f' is raised).

Case F: Firm's illegal action is detected before the output has been sold

Let us now consider the case where the firm's unlawful activity is detected at a stage where it has completed the production process, but the output is yet to be marketed. We take up the case with variable probabilities.

Now, as before,

$$\pi^* = PQ - wL - rK$$
$$\Rightarrow \pi^* = Q - wL - rK \;................(42)$$

However, as the firm is detected prior to sale in the market, we now have:

$$\hat{\pi} = -\,wL - rK - f \;.....................(43)$$

Not only is the firm being unable to earn revenue, but also it loses the entire cost that it has already incurred plus it also faces a fine. In other words, in bad state, the firm can only incur losses.

With $K = \overline{K}$ so that r$\overline{K}$ = F (fixed cost), we can write down the profit functions under good and bad states, respectively, as:

$$\pi^* = Q - wL - F \text{ and}$$
$$\hat{\pi} = -\,wL - F - f = -(wL + F + f) < 0$$

The expected profit function of the firm is given as follows:

$$\text{E}(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \;.................(44)$$

$$\Rightarrow \tilde{\pi} = \frac{-wL^2 - LF - Lf + g(L) - wL - F}{L+1} \;\ldots\ldots\ldots\ldots\ldots.(45)$$

As before, the firm's problem is to choose the value of 'L' in such a fashion that E $(\pi) = \tilde{\pi}$ is maximum.

Setting the FOC for first order profit maximisation, $\frac{d\tilde{\pi}}{dL} = 0$, we get:

$$g'(L) = Q' = MP_L = \frac{w\,[L+1]^2 + f + Q}{L+1} \;\ldots\ldots\ldots\ldots\ldots.(46)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

Again,

$$w = \frac{Q'[L+1] - f - Q}{[L+1]^2} \text{ ……………….}(47)$$

The second order condition (SOC) for maximization of expected profit is:

$$\frac{d^2\tilde{\pi}}{dL^2} < 0$$

$$\Rightarrow \frac{[L+1]^3[Q'' - 2w] - [Q'\{L+1\} - w\{L+1\}^2 - f - Q]\,[2(L+1)]}{[L+1]^4} < 0$$

From the first order condition, we obtain:

$$g'(L) = \frac{w\,[L+1]^2 + f + g(L)}{L+1}$$

$$\Rightarrow g'(L) = w\,(L+1) + \frac{f + g(L)}{(L+1)}$$

Now, there does not exists any need to constrain $f < 1$ for meaningful solution in order to ensure $g'(L) > 0$.

Since, denominator is always positive, the SOC requires:

$$[L+1]^3[Q'' - 2w] - [Q'\{L+1\} - w\{L+1\}^2 - f - Q][2(L+1)] < 0$$

$$\Rightarrow \frac{[L+1]^3}{[L+1]^2}[Q'' - 2w] - \left[\frac{Q'\{L+1\} - f - Q}{[L+1]^2} - \frac{w\{L+1\}^2}{[L+1]^2}\right][2(L+1)] < 0$$

Substituting from FOC, we get:

$$(L+1)[Q'' - 2w] < 0$$

Since, $Q'' < 0$ and $w > 0$, this automatically implies that SOC holds.

Case G: Firm's illegal action is detected at a very nascent stage of its establishment

In this section, we consider that the firm is detected at a very nascent stage of its establishment. Here, it has just planned to operate and it is yet to hire factors (labour and capital) and, thus yet to start its unlawful production process. As before, if the unlawful act goes unnoticed, the firm earns some profit which is given by:

$$\pi^* = PQ - wL - rK$$

$$\Rightarrow \pi^* = Q - wL - rK \text{ ....}(48)$$

However, if the illegal plan of the firm is exposed at the very outset, the firm is charged with a fine; it does not lose any sunk and variable cost.The firm's loss is just the fine or the penalty. Hence, if detected, we have:

$$\hat{\pi} = -f \text{ ....}(49)$$

In the short run, $K = \bar{K}$, or, r$\bar{K}$ is a fixed cost, say 'F'. Thus, we can rewrite the two profit functions under good and bad states, respectively, as:

$$\pi^* = Q - wL - F \text{ and } \hat{\pi} = -f$$

The expected profit function of the firm is, therefore, given by:

$$E(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \text{....................}(50)$$

$$\Rightarrow \tilde{\pi} = \frac{-Lf + g(L) - wL - F}{[L+1]} \text{ ……………….}(51)$$

The choice before the firm is to choose the value of 'L' in such a fashion that E (π) =$\tilde{\pi}$ is maximum. Let us consider short run situation, so that $K = \bar{K}$.

For first order profit maximisation, setting $\frac{d\tilde{\pi}}{dL}$ = 0, we get:

$$g'(L) = Q' = MP_L = \frac{f + w + Q - F}{[L+1]} \text{ ……………….}(52)$$

where marginal productivity of labour (Q') = $\frac{dQ}{dL}$ > 0.

From the FOC we find:

$$w = Q^{'}\{L+1\} - f - Q + F \quad \dots\dots\dots\dots\dots\dots (53)$$

The second order condition for (expected) profit maximisation is obtained as follows:

$$\frac{[L+1]^3 Q^{"} - [Q^{'}\{L+1\} - f - w - Q + F]\,[2(L+1)]}{[L+1]^4} < 0$$

The impact on comparison of L (& Q) is not clear, however. The first order condition obtained:

$$g^{'}(L) = \frac{f + w + g(L) - F}{[L+1]}$$

$g^{'}(L) > 0$ requires:

$$f + w + g(L) - F > 0$$
$$\Rightarrow f > F - w - g(L) = F - [w + g(L)]$$

Alternatively,

$$g^{'}(L)[L+1] - f - w - g(L) + F > 0$$
$$\Rightarrow f < g^{'}(L)[L+1] - [w + g(L)] + F \quad \dots\dots\dots\dots\dots..(54)$$

It is not possible to directly or unambiguously compare FOC of this case with FOC of Case C where u $= \frac{Q}{Q+1}$ and v $= \frac{1}{Q+1}$. Also, it is not clear whether $\frac{f+w+g(L)-F}{[L+1]} > w$ or not. We therefore cannot assert if probability of detection determined by the size of the labour force associated/employed with the unlawful activity acts as greater deterrence on the scale of production.

Since, the denominator $[L+1]^4 > 0$, let us now consider SOC which requires:

$$[L+1]^3 Q^{"} - [Q^{'}\{L+1\} - f - w - Q + F][2(L+1)] < 0$$

Substituting from FOC, we get:

$$[L+1]^3 Q^{"} = [L+1]^3 g^{"}(L) < 0 \text{ [as } g^{"}(L) < 0]$$

Case H: The illegal activity is detected after a portion of the output has already been sold

This case is, in a sense, more realistic one. The illegal action comes to the notice after some portion of the fake product finds its way into the market. Let us now introduce the following additional notations:

θ = fraction of output already marketed ($0 < \theta < 1$)

(1 – θ) = fraction of output unsold at the time of detection; this is also the fraction of output confiscated or seized by the law enforcing agency

f = per unit fine imposed on the confiscated output ($f > 0$)

Now, profit in the bad state (i.e. when caught) is given by:

$$\hat{\pi} = P\,(\theta Q) - wL - r\bar{K} - f\,(1 - \theta)\,Q$$
$$\Rightarrow \hat{\pi} = \theta g(L) - fg(L) + fg(L)\theta - wL - F \quad \dots\dots\dots\dots\dots..(55)$$

Profit in the good state (i.e. when not detected at all) is given by:

$$\pi^* = PQ - wL - F$$
$$\Rightarrow \pi^* = g(L) - wL - F \quad \dots\dots\dots\dots\dots\dots (56)$$

Hence, the expected profit is given by:

$$E\,(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^* \quad \dots\dots\dots\dots\dots\dots (57)$$

$$\Rightarrow \tilde{\pi} = \frac{L\theta g(L) - Lfg(L) + Lf\theta g(L) - wL^2 - LF + g(L) - wL - F}{[L+1]} \quad \dots\dots..(58)$$

The FOC for expected profit maximization is:

$$\frac{d\tilde{\pi}}{dL} = 0$$

$$\Rightarrow g'(L) = Q' = MP_L = \frac{w[L+1]^2 - Q[(1+f)(\theta-1)]}{[\theta L\{(L+1)(f+1)\} + L\{1-fL-f\}+1]} \;\ldots\ldots(59)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

From the FOC, we get:

$$w = \frac{Q'[\theta L\{(L+1)(f+1)\} + L\{1-fL-f\}+1] + Q[(1+f)(\theta-1)]}{[L+1]^2} \;\ldots\ldots.(60)$$

The second order condition for (expected) profit maximisation is given by:

$$(L+1)^2[Q''\{\theta L(L+1+fL+f) + L(1-fL-f)+1\} + 2Q'\{\theta(L+1+fL+f) - f((L+1)\} - 2w\{L+1\}] - [2(L+1)][Q'\{\theta L(L+1+fL+f) + L(1-fL-f)+1\} + Q\{(1+f)(\theta-1)\} - w\{L+1\}^2] \,/\, [L+1]^4 < 0$$

Recall that the FOC is:

$$g'(L) = \frac{w[L+1]^2 - Q[(1+f)(\theta-1)]}{[\theta L\{(L+1)(f+1)\} + L\{1-fL-f\}+1]}$$

When $\theta = 1$,

$$g'(L) = \frac{w[L+1]^2}{[L\{(L+1)(f+1)\} + L\{1-fL-f\}+1]}$$

$$\Rightarrow g'(L) = Q' = w \;\ldots\ldots\ldots\ldots\ldots.(61)$$

The FOC indicates that the illegal firm's behaviour and decision with respect to employment of variable input (labour) is same as that of its counterpart operating in the legal sector. The fine in this case has, surprisingly, no impact on employment and output of the illegal firm and thus a fixed fine, however hefty, has no deterrent impact on the illegal activity, at least in the short run.

When $\theta = 0$,

$$g'(L) = \frac{w[L+1]^2 - Q\,[-1-f]}{L\{1-fL-f\}+1}$$

$$\Rightarrow g'(L) = Q' = \frac{w[L+1]^2 + Q + fQ}{L - fL^2 - fL + 1} \;\ldots\ldots\ldots\ldots\ldots\ldots.(62)$$

For $Q' > 0$, we require:

$$L - fL^2 - fL + 1 > 0$$

$$\Rightarrow f < 1 \text{ [if } L \geq 1] \;\ldots\ldots\ldots\ldots\ldots(63)$$

V

Case I: Lump-sum fine is imposed when the illegal activity is detected after a portion of the output has already been sold (with probability of detection dependent on output)

Let f^* be the lump-sum fine imposed. In the earlier case (i.e. Case D), we considered a per unit fine, 'f', on unsold output $(1-\theta)\,Q$, so that total penalty amounted to $f\,(1-\theta)\,Q$.

Now, profit in the bad state (i.e. when caught) is given by:

$$\hat{\pi} = P\,(\theta Q) - wL - r\bar{K} - f^*$$

$$\Rightarrow \hat{\pi} = \theta Q - wL - F - f^* \;\ldots\ldots\ldots\ldots\ldots(64)$$

Profit in the good state (i.e. when not detected at all) is given by:

$$\pi^* = PQ - wL - r\bar{K}$$

$$\Rightarrow \pi^* = Q - wL - F \;\ldots\ldots\ldots\ldots\ldots.(65)$$

Hence, the expected profit is given by:

$$E\,(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^*$$

$$\Rightarrow \tilde{\pi} = \frac{\theta[g(L)]^2 - wLg(L) - Fg(L) - g(L)f^* + g(L) - wL - F}{[g(L) + 1]} \dots\dots(66)$$

The objective of the firm is to choose the level of employment of labour (L) in such a way that expected profit is maximized. The FOC for expected profit maximization is:

$$\frac{d\tilde{\pi}}{dL} = 0$$

$$\Rightarrow Q' = \frac{w\,[Q+1]^2}{\theta Q\,(Q+2) + (1-f^*)} \dots\dots\dots\dots\dots(67)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

When $\theta = 1$, we have:

$$Q' = \frac{w\,[Q+1]^2}{Q\,(Q+2) + (1-f^*)}$$

When $\theta = 0$, we have:

$$Q' = \frac{w\,[Q+1]^2}{(1-f^*)}$$

We observe that for meaningful solution, $f^* < 1$ when $\theta = 0$; but when $\theta = 1$, the condition becomes less stringent. Now, the upper bound of f^* is higher $[f^* < Q\,(Q+2)+1]$.

The second order condition for profit maximisation is given by:

$$[Q+1]^2[Q''\{\theta Q\,(Q+2) + (1-f^*)\} + 2Q'\{(Q+1)(\theta Q' - w)\}] - [2Q'(Q+1)]\,[Q'\{\theta Q\,(Q+2) + (1-f^*)\} - w\{Q+1\}^2]\,/\,[Q+1]^4 < 0$$

Assuming $[Q+1]^4 > 0$ and substituting from FOC, we get SOC as:

$$[Q+1]^2[Q''\{\theta Q\,(Q+2) + (1-f^*)\} + 2Q'\{(Q+1)(\theta Q' - w)\}] < 0.$$

Case J: Lump-sum fine is imposed when the illegal activity is detected after a portion of the output has already been sold (with probability of detection dependent on employment)

Let f^* be the lump-sum fine imposed. In the earlier case (i.e. Case H), we considered a per unit fine, 'f', on unsold output $(1-\theta)\,Q$, so that total penalty amounted to $f\,(1-\theta)\,Q$.

Now, profit in the bad state (i.e. when caught) is given by:

$$\hat{\pi} = P\,(\theta Q) - wL - r\bar{K} - f^*$$

$$\Rightarrow \hat{\pi} = \theta Q - wL - F - f^* \dots\dots\dots\dots\dots(68)$$

Profit in the good state (i.e. when not detected at all) is given by:

$$\pi^* = PQ - wL - r\bar{K}$$

$$\Rightarrow \pi^* = Q - wL - F \dots\dots\dots\dots\dots(69)$$

Hence, the expected profit is given by:

$$E\,(\pi) = \tilde{\pi} = u\hat{\pi} + v\pi^*$$

$$\Rightarrow \tilde{\pi} = \frac{L\,\theta g(L) - wL^2 - LF - Lf^* + g(L) - wL - F}{[L+1]} \dots\dots\dots\dots\dots(70)$$

The objective of the firm is to choose the level of employment of labour (L) in such a way that expected profit is maximized. The FOC for expected profit maximization is:

$$\frac{d\tilde{\pi}}{dL} = 0$$

$$\Rightarrow g'(L) = Q' = MP_L = \frac{Q\,(1-\theta) + f^* + w\,[L+1]^2}{(L+1)(\theta L + 1)} \dots\dots\dots\dots\dots(71)$$

where marginal productivity of labour $(Q') = \frac{dQ}{dL} > 0$.

When $\theta = 1$,

$$Q' = \frac{f^* + w\,[L+1]^2}{[L+1]^2} = \frac{f^*}{[L+1]^2} + w$$

When $\theta = 0$,

$$Q' = \frac{Q + f^* + w\,[L+1]^2}{(L+1)} = \frac{Q + f^*}{(L+1)} + w\,(L+1)$$

The second order condition for profit maximisation is given by:

$$(L+1)^3[Q''(\theta L+1) + 2Q'\theta - 2w] - [2(L+1)][Q'\{(L+1)(\theta L+1)\} + Q\,\{\theta - 1\} - f^* - w\,(L+1)^2] \;/\; [L+1]^4 < 0$$

Since $[L+1]^4 > 0$, substituting from FOC, we get SOC as:

$$(L+1)^3[Q''(\theta L+1) + 2Q'\theta - 2w] < 0$$

Clearly, mere fulfilment of $Q'' < 0$, does not necessarily guarantee that the SOC holds. So, for meaningful or feasible economic solution we require that the expression within bracket be negative. This is true in Case I also.

Our analysis shows that in almost all cases, fine acts as a deterrent in restricting level of employment/recruitment of labour and, consequently, in production of illegal output: as f rises, employment and output decrease. However, in Case D and Case H, when $\theta = 1$, fine imposed on per unit basis does not affect employment and production. On the other hand, when, instead, fine is lump-sum in nature, it affects profit maximizing employment and output solution, irrespective of whether probability of detection of the unlawful activity is dependent on the size of the output or the size of the labour force engaged. The standard microeconomic analysis of the theory of firm establishes that imposition of lump-sum tax or increase in the fixed component of the cost does not alter profit maximizing output or behaviour of the firm. In our analysis, we observe that this assertion no longer holds for an illegal firm, once the probability of detection of unlawful activity becomes a variable and is dependent on Q or L.

VI

Conclusion

We can now summarize our results. In Case A, labour employed is lower and consequently output produced is also lower. Thus, fine acts as a deterrent. In Case B, value of labour employed is lower as compared to Case A, and thus volume of output produced is also lower. This leads to greater deterrence on unlawful activity in Case B compared to Case A. In Case C, the upper bound on feasible value of fine has risen as compared to Case A.

Case E indicates impact on lowering illegal labour is less in case of variable probabilities with labour compared to the analogous case of variable probabilities with output. Hence, imposition of fine has a more deterring effect on the illegal activity in case of variable probabilities with output compared to the analogous case of variable probabilities with labour employed. Here, also upper limit of fine is raised. In Case F, there does not exists any need to constrain $f < 1$ for meaningful solution in order to ensure $g'(L) > 0$. In Case G, it is not possible to directly or unambiguously compare FOC of this case with FOC of Case C where $u = \frac{Q}{Q+1}$ and $v = \frac{1}{Q+1}$. Also, it is not clear whether $\frac{f + w + g(L) - F}{[L+1]} > w$ or not. We, therefore, cannot assert if probability of detection determined by the size of the labour force associated/employed with the unlawful activity acts as greater deterrence on the scale of production.

Finally, in Case D and Case H, when $\theta = 1$, the FOC indicates that the illegal firm's behaviour and decision with respect to employment of variable input (labour) is same as that of its counterpart operating in the legal sector. The fine in this case has, surprisingly, no impact on employment and output of the illegal firm and thus a fixed fine, however hefty, has no deterrent impact on the illegal activity, at least in the short run. In such case, rather than relying on per unit fine, it is, however, possible to constrain the scale of the illegal activity by imposing, instead, a lump-sum fine (Case I and Case J) – a conclusion which sharply differs from the standard microeconomic theory of firm's behaviour.

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