2022

1st Semester Examination

PHYSICS

Paper : PHYS 102

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Paper : 102.1

(Quantum Mechanics-I)

1. Answer any *two* of the following : 2×2=4

(a) Given that $|\psi_q\rangle$ is an eigenstate of operator $\hat{Q}$ such that $\hat{Q}|\psi_q\rangle = q|\psi_q\rangle$. An operator $\hat{C}$ acts on $|\psi_q\rangle$ as $\hat{C}|\psi_q\rangle = |\psi_{-q}\rangle$. Is $|\psi_q\rangle$ an eigenstate of the operator $\hat{C}\hat{Q}+\hat{Q}\hat{C}$? If so find the eigenvalue.

(b) The space translation operator is given by $\hat{T}(\Delta x) = \exp[-i\Delta x.\hat{p}/\hbar]$. By considering how $\langle\psi|\hat{p}/\psi\rangle$, changes under a small translation Δx i.e. $\langle\psi|T^{\dagger}\hat{p}T|\psi\rangle$, show that all components of the momentum operator commute with each other : $[\hat{p}_i, \hat{p}_j] = 0$.

P.T.O.

(c) Consider a particle in a one dimensional infinite square well with $0 \le x \le L$. If the wave function of the particle is

$$\psi(x,0) = \frac{1}{\sqrt{L}} \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{2\pi x}{L}\right).$$

Compute $\psi(x,t)$ and determine if it is a stationary state.

(d) For a one dimensional harmonic oscillator using the ladder operators, $\hat{a}$ and $\hat{a}^\dagger$ compute $\langle n' | \hat{x}^2 | n \rangle$ and write down the corresponding matrix. $\left[\text{Given} : \bar{x} = \sqrt{\frac{\hbar}{2m\omega}} \left(\hat{a} + \hat{a}^\dagger\right)\right]$.

2. Answer any ***two*** of the following questions : 4×2=8

(a) Consider the spin operator $\hat{S}_{\vec{n}} = \vec{\hat{S}}.\vec{n}$, where $\hat{S}_i = \frac{\hbar}{2}\sigma_i . \sigma_i$ are Pauli matrices and $\vec{n}$ is a unit vector whose polar, azimuthal angles are θ and ϕ respectively. Find the unitary matrix U such that $U\hat{S}_z U^\dagger = \hat{S}_{\vec{n}}$.

(b) Consider the following state :

$$|\psi\rangle = N \sum_{n=0}^{\infty} \left(1/\sqrt{2}\right)^n |n\rangle$$

where $|n\rangle$ is the normalized n-th excited state of a one dimensional harmonic oscillator with energy eigenvalue $E_n = \hbar\omega(n + 1/2)$. Find the normalization

N assuming that $|\psi\rangle$ is normalized and the expectation value of the Hamiltonan $\langle\psi|H|\psi\rangle$.

(c) The Hamiltonian describing the dynamics of a particle of mass m in one dimension is given by $\hat{H}=\frac{\hat{p}^2}{2m}-F\hat{x}$ where F is a constant. Find the commutator $\left[\hat{x}(t),\hat{x}(0)\right]$.

(d) Consider a three state system with orthonormal basis states $\{|1\rangle,|2\rangle$ and $|3\rangle\}$. The action of an operator $\hat{A}$ on the states is given by $\hat{A}|1\rangle=|2\rangle, \hat{A}|2\rangle=|3\rangle$ and $\hat{A}|3\rangle=|1\rangle$. Further define an operator $\hat{H}$ as $\hat{H}=-E_0\left(\hat{A}+\hat{A}^\dagger\right)$. Suppose that the system is in the state $|2\rangle$ and $\hat{H}$ is measured, what are the possible values one obtains and with what probabilities?

3. Answer any *one* of the following : $8\times1=8$

(a) (i) Consider a one-dimensional simple harmonic oscillator with the Hamiltonian $\hat{H}=\hat{p}^2/(2m)+(1/2)m\omega^2\hat{x}^2$. Using the Heisenberg equation find the time dependence of the operators $\hat{x}(t)$ and $\hat{p}(t)$. 4

P.T.O.

(ii) Now consider the following state :

$$|\psi(0)\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + |1\rangle + |2\rangle\right)$$

where $|n\rangle$ is the normalized n-th excited state of a one dimensional harmonic oscillator with energy eigenvalue $E_n = \hbar\omega(n + 1/2)$. Suppose the system is evolved in time using the Hamiltonian for a one dimensional harmonic oscillator. Find the expectation value $\langle\hat{x}\rangle(t)$ using both the Schrödinger and Heisenberg pictures. 4

(b) Let $|+\rangle$ and $|-\rangle$ be spin – 1/2 eigenstates of the operator $\hat{S}_z$ with eigenvalues $\pm\hbar/2$. A Hamilatonian operator is given by $\hat{H} = \hbar\omega\hat{S}_x$ where ω is a real constant.

(i) Suppose the system is known to be in state $|+\rangle$ at t = 0, what is the probability for finding the system in the states $|\pm\rangle$ for $t > 0$? 4

(ii) Now consider the operator $\hat{S}_y$. Find $\langle\hat{S}_y\rangle(t)$ using both the Schrödinger and Heisenberg pictures. 4

Paper : 102.2

(Condensed Matter Physics-I)

1. Answer any *two* questions : 2×2=4

   (a) Show through matrix representation the symmetry elements introduced in point group $\frac{2}{m}$.

   (b) Draw Ewald sphere and hence find Bragg's Differention condition.

   (c) Show that number of possible modes in a lattice in equal to the number of mobile atom. Assume staionary boundary conditions to prove it.

   (d) The *E*-*k* relation in a particular solid is given by $E = Ak^2 + Bk^3$ where *A* are positive constants. Find the wave vectors for which electron group velocity is zero. Find the phase velocity of electron waves as well.

2. Answer any *two* questions : 4×2=8

   (a) Describe in details the screw symmetry elements. What is symmorphic space group? 3+1

   (b) Express structure factor in terms of fractional coordinates for BCC lattice and show the condition for systematic absence of intensity. 2+2

(c) Density of states for a linear chain of atom becomes infinite at a certain condition. Find the condition at which this occur. What is Van Hove singularity? 3+1

(d) According to empty lattice approximation find the energy for first four bands for a simple cubic lattice in [100]. 4

3. Answer any ***one*** question : 8×1=8

(a) Derive Laue equation assuming scattering of X Ray from a crystal. Also express Lane equation introducing reciprocal lattice vector. 7+1

(b) What is the physical origin of energy gap? Find an expression of energy gap on the basis of free electron approximation. Find the band width of a simple cubic lattice along [III] direction following Tight Binding Approximation? 2+4+2

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