PG/1st Sem/PHYS/22

2022

1st Semester Examination

PHYSICS

Paper : PHYS 101

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Paper : 101.1

1. Answer any **two** : 2×2=4

(a) A is a non-Hermitian operator, show that $(A+A^{+})$ and $i\,(A-A^{+})$ are Hermitian.

(b) Show that a real matrix that is not symmetric can not be diagonalized by an orthogonal or unitary transformation.

(c) Prove that $\in_{ipq}\in_{jpq}=2\delta_{ij}$.

(d) State and prove Cauchy's integral theorem.

2. Answer any **two** : 4×2=8

(a) $f(z)=u(x,y)+iv(x_1y)$

where u is the velocity potential and v is the stream function $\bar{v}=\bar{\nabla}u$

Show that $\bar{\nabla}.\bar{v}=0$ and $\bar{\nabla}\times\bar{v}=0$.

P.T.O.

(b) Prove that $\left\{ {p \atop pq} \right\} = \frac{\partial}{\partial x^q} \log \sqrt{g}$

where g is the determinant of the metric tensor.

(c) $A = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$

Find A^{10} using caley Hamilton's theorem.

(d) If the two-index Levicivita symbol $\in_{ij}$ is pseudo-tensor, show that it is invariant under orthogonal similarity transformation.

3. Answer any ***one*** : 8×1=8

(a) (i) Show that a real symmetric matrix A is transformed by

$$\phi_i' = \phi_i \cos\theta - \phi_j \sin\theta$$

$$\phi_j' = \phi_i \sin\theta + \phi_j \cos\theta$$

Then $a_{ij}' = 0$ if $\tan 2\theta = \frac{2a_{ij}}{a_{jj} - a_{ij}}$

(ii) Evaluate $\int_o^\infty \frac{\ln(1+x^2)}{(1+x^2)} dx$ by using Residue-theorem. 4+4

(b) (i) If $A_k = \frac{1}{2} \in_{ijk} B^{ij}$ and $B^{ij} = -B^{ji}$

Show that $B^{mn} = \in^{mnk} A_k$

(ii) Evaluate $\int_{3+4i}^{4-3i} \left(4z^2 - 3iz\right) dz$ for an arc on the circle $|z| = 5$. 4+4=8

Paper : 101.2

(Classical Mechanics)

Answer any ***two*** of the following : 2×2=4

1. Prove that the dynamics of a particle governed by the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$ describes a free particle. 2

2. Find the value of Poisson bracket $\left[|\vec{r}|, |\vec{p}|\right]$. 2

3. The Lagrangian for a particle of mass m at a point r moving with a velocity v is given by $L = \frac{1}{2}mv^2 + c\bar{r}.\bar{v} - v(r)$, where $v(r)$ is a potential and c is a constant. Find the Hamiltonian of the system consider p_r as the canonial momentum. 2

4. Derive Hamilton's canonical equation in terms of the Poisson bracket.

P.T.O.

Answer any ***two*** of the following : 4×2=8

5. (a) Show that $p_k = \frac{\partial F_1}{\partial q_k}$ and $P_k = -\frac{\partial F_1}{\partial Q_k}$

(b) A mechanical system is described by the Hamiltonian $H(q,p) = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$. As a result of the canonical transformation generated by $F(q,Q) = -\frac{Q}{q}$, find the Hamiltonian in the new coordinate Q and momenta P. 1+3

6. Find the Poisson bracket between θ and $\dot{\theta}$, $[\theta, \dot{\theta}]$ for $L = ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$. 4

7. Prove that Poisson's bracket remains invariant under the canonical transformation. 4

8. A Particle of mass m falls a given distance z_0 in time $t_o = \sqrt{2z_0 / g}$ and the distance travelled in time t is given by $z = at + bt^2$, where a and b are such that the time t_0 is always the same. Show that the integration $\int_0^{t_0} L dt$ is an extremum for real values of the coefficient only when $a = 0$ and $b = g/2$. 4

Answer any ***one*** of the following : 8×1=8

9. (a) Prove that $\frac{du}{dt} = [u, H] + \frac{\partial u}{\partial t}$.

(b) The Lagrangian of a system moving in three dimensions is

$$L = \frac{1}{2}m\dot{x}^2 + m\left(\dot{y}^2 + \dot{z}^2\right) - \frac{1}{2}kx^2 - \frac{1}{2}k\left(y+z\right)^2$$

assuming $L_x - yp_z - zp_y$ determine $\frac{dL_x}{dt}$. 3+5

10. What is the action angle variable? Find out the frequency of a linear harmonic oscillator using the action-angle variable method. Starting from the time-dependent Schrödinger equation, obtain the Hamilton-Jacobi equation. 2+3+3
