2022

M.Sc.

2nd Semester Examination

PHYSICS

PAPER—PHS-201

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

PHS–201.1 QUANTUM MECHANICS–II

[Marks : 20]

1. Answer any *two* questions : 2×2

(a) Using time reversal invariance, prove that the energy eigenstates of a one dimensional harmonic oscillator are real.

(Turn Over)

(b) Show that the state $|\theta\rangle$ defined as

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle$$

is an eigenstate of the lattice translation operator. Find the corresponding eigenvalue.

(c) Suppose that $\vec{U}$ and $\vec{V}$ are two vector operators. Evaluate the commutation relation of $\vec{U} \cdot \vec{V}$ with all the components of the angular momentum operator $\vec{J}$.

(d) Why JWKB approximation is not valid near the turning points?

2. Answer any *two* questions : 2×4

(a) Consider a system whose Hamiltonian is given by

$$\hat{H} = E_0 \begin{pmatrix} -5 & 3\lambda & 0 & 0 \\ 3\lambda & 5 & 0 & 0 \\ 0 & 0 & 8 & -\lambda \\ 0 & 0 & -\lambda & -8 \end{pmatrix},$$

where $\lambda << 1$. Using first- and second-order nondegenerate perturbation theory, find the approximate eigen-energies of $\hat{H}$.

(b) A system consisting of two nonidentical spin $\frac{1}{2}$ particles is known to be in the spin-singlet state. Suppose the measurement of one of the particles gives $s_z = \frac{\hbar}{2}$. What are outcomes and the corresponding probabilities if (i) s_z for the other particle is measured (ii) s_x for the other particle is measured ?

(c) Estimate the ground-state energy of a one-dimensional simple harmonic oscillator using the trial wavefunction $\psi(x) = \sqrt{\alpha}e^{-\alpha x}$.

$$\left[\text{Use: } \int_0^\infty x^n e^{-ax} = n!/a^{n+1}\right]$$

(d) Consider a spin-one particle with the Hamiltonian $H = aS_z + bS_x^2$ where a and b are constants. Compute the energy eigenvalues of the particle in terms of a and b.

3. Answer any *one* question : 1×8

(a) (i) A 2D harmonic oscillator of mass m having hamiltonian H_0 is subjected to the following perturbation : $V(x) = \frac{1}{2} \in m\omega^2 xy$. Considering $\in$ to be a small parameter ($\in << 1$), compute the correction(s) to the energy of the two-fold degenerate first excited state of H_0 using perturbation theory.

(ii) The wave function of a particle subjected to a spherically symmetrical potential V(r) is given by $\psi = N(x + y + 2z)e^{-ar}$ where N is the normalization constant. If L_z is measured, write down the possible outcomes and the corresponding probabilities. Find the expectation value of L_z. Use the following expressions for the spherical harmonics : Y_l^m .

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \qquad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\phi}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta \qquad Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta e^{\pm i\phi}$$

(b) A particle of spin $\frac{1}{2}$ is in a d state of orbital angular momentum (i.e., l = 2). Work out the coupling of the spin and orbital angular momenta of this particle and find all the states and the corresponding Clebsch-Gordan coefficients.

PHS–201.2
METHODS OF MATHEMATICAL PHYSICS - II

[Marks : 20]

4. Answer any *two* questions : 2×2

(a) $f(t) = 1$ for $t \leq 0$
$= 0$ for $t > 0$
Find the F.T. of f(t).

(b) Solve the equation

$$\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = E \sin pt .$$

(c) Show that the transformation $x' = ax + b$, $a \neq 0$ form a lie group. Find the generators.

(d) Prove that $3 \otimes 3 = 6 \oplus 3$.

5. Answer any *two* questions : 2×4

(a) Using Parseval's identify,

prove that $\displaystyle\int_0^\infty \frac{x^2}{\left(x^2+1\right)^2} dx = \frac{\pi}{4}$.

(b) $f(t) = 0$ for $|t| > 10$
$= \cos t$ for $-10 \leq t \leq 10$.
Prove that

$$F(jw) = \frac{\sin(10(w-1))}{w-1} + \frac{\sin(10(w+1))}{w+1} .$$

(c) Show that permutation of 3 distinct (S_3 group) is isomorphic with D_3.

(d) Solve : $y(x) = x + 2 \int_0^{\pi} \cos(x - t) y(t) dt$.

6. Answer any *one* question : 1×8

(a) Transform the differential equation

$$y'' + y = x ; \quad y(0) = 0, y'(1) = 0$$

to a Fredholm integral equation, finding the corresponding Green's function.

(b) Character table

O_h	E	$6C_1$	$3C_2$	$6S_4$	$8C_3$	$8S_6$	$3\sigma_h$	i	$6\sigma_a$	$6C_2'$
T_{1g}	3	1	−1	1	0	0	−1	3	−1	−1
T_{2g}	3	−1	−1	−1	0	0	−1	3	1	1
E_g	2	0	2	0	−1	−1	2	2	0	0
T_{Ag_1}	1	1	1	1	1	1	1	1	1	1
$T_{2g} \otimes T_{2g}$	9	1	1	1	0	0	1	9	1	1

Prove that

$$T_{2g} \otimes T_{2g} = A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}$$

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PHS–202.1 SOLID STATE - II

[Marks : 20]

1. Answer any *two* questions : 2×2

(a) What is the difference between a superconductor and a perfect conductor ?

(b) Draw the variation of $\vec{M}$ and $\vec{B}$ with applied magnetic field for a superconductor.

(Turn Over)