# M.Sc. 3rd Semester Examination, 2022

## APPLIED MATHEMATICS

*( Dynamical Meteorology -I/Operational Research Modelling-I )*

PAPER – MTM-306(A&B)

*Full Marks* : 50

*Time* : 2 hours

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

## PAPER–MTM-306A

*( Dynamical Meteorology- I )*

[ *Marks* : 50 ]

A. Answer any *four* questions : 2×4

1. Show that the virtual temperature is always higher than the actual temperature.

2. Show that the pressure tendency at the earth surface becomes zero when the air motion at all levels in the atmosphere is geostrophic.

3. What is the concept of Pseudo-adiabatic process ?

4. What is the concept of coriolis force ?

5. Derive the amount of heat to be required to transfer for unit mass of air parcel during the isobaric process.

6. What do you mean by barotropic and baroclinic atmosphere ?

B. Answer any *four* questions : 4×4

7. Define specific entropy. Establish the relation between the specific entropy and the potential temperature.

8. Define homogeneous atmosphere. Show that a homogeneous atmosphere has a finite height that depends only on the temperature at lower boundary.

9. Explain the concepts of divergence and convergence in the atmosphere.

10. What do you mean by isobaric cooling ? Show that the relative increase in dew-point temperature is about 5% of the sum of relative increase in mixing ratio and pressure.

11. Derive an expression for the density $\rho$ of an air parcel at pressure $p$ if it is adiabatically expands from a level where pressure and density are $p_s$ and $\rho_s$ respectively.

12. Derive the expression of the pressure gradient force in the atmosphere.

C. Answer any *two* questions : $8 \times 2$

13. Give physical concept of thermal wind in the atmosphere and then deduce its mathematical form. $3 + 5$

14. Derive the momentum equation of motion of an air parcel in the atmosphere in cartesian co-ordinate system.

15. Find the rate of change of circulation of an air parcel in the atmosphere.

16. Show that during the adiabatic process of an saturated air parcel in the atmosphere, the relation between equivalent potential temperature and potential temperature of the air parcel is

$$\theta_e = \theta e^{\frac{Lw_s}{C_p T}}$$

where all symbols have their usual meanings.

[ *Internal Assessment* — 10 Marks ]

## PAPER—MTM-306B

( *Operational Research Modelling*- I )

[ *Marks* : 50 ]

A. Answer any *four* questions : 2×4

1. What is probabilistic inventory model ?

2. What do you mean by the term 'present worth factor' ?

3. Draw the flow of goods of any real-life supply chain.

4. State Bellman's principle of optimality.

5. What are the critical paths and critical activities in network analysis ?

6. What is simulation ? Describe its advantages in solving the problems.

B. Answer any *four* questions : 4×4

7. A baking company sells cake by its weight in kg. It makes a profit of Rs. 3 on every kg sold

on the day it is baked. It disposes of all cakes not sold on the date they are baked, at a loss of Rs. 1.50 per kg. If the demand is known to be rectangular distribution between 2000 and 5000 kg, determine the optimal daily amount baked.

8. Derive the conditions of finding optimal order quantity of probabilistic inventory model of instantaneous and continuous random demand without set-up cost.

9. A machine owner finds from his past record that the costs per year of maintaining a machine whose purchase price is Rs. 6000 are given in the following table :

| Year : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Resale price: | 3000 | 1500 | 750 | 375 | 200 | 200 | 200 | 200 |
| Maintenance cost (Rs.): | 1000 | 1200 | 1400 | 1800 | 2300 | 2800 | 3400 | 4000 |

Find the most economic replacement age.

10. Explain the Monte Carlo simulation. State different mathematical steps in the Monte-Carlo method.

11. Obtain the functional equation and define state variables for solving the following problem by dynamic programming problem :

Maximize $z = g_1(x_1) + g_2(x_2) + ... + g_n(x_n)$

Subject to $x_1 + x_2 + ... + x_n = c$

$x_1, x_2, ..., x_n \geq 0$

12. Write down the rules to construct a network. Also, describe the process of numbering the events in network analysis.

C. Answer any *two* questions : $8 \times 2$

13. A team of software developers at Microsoft is planned to rise to a strength of 60 persons and then to remain at that level. Consider the following data :

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Total % who have left up to the end of the year | 5 | 30 | 50 | 60 | 70 | 75 | 80 | 85 | 90 | 100 |

On the basis of above information, determine

(*i*) What is the recruitment per year necessary to maintain the strength ?

(*ii*) There are 10 senior posts for which the length of service is the main criterion, what is the average length of service after which a new entrant can expect his promotion to one of these post.

14. A contractor of second hand motor trucks uses to maintain a stock of trucks every month. The demand of the trucks occurs at a relatively constant rate but not in a constant size. The demand follows the following probability distribution :

| Demand (r) | 0 | 1 | 2 | 3 | 4 | 5 | 6 or more |
|---|---|---|---|---|---|---|---|
| Probability(p(r)) | 0.35 | 0.30 | 0.15 | 0.12 | 0.06 | 0.02 | 0.0 |

The holding cost of an old truck in stock for one month is Rs. 120.00 and the penalty for a truck not supplied on the demand is Rs.

900.00. Determine the optimal size of the stock for the contractor.

**15.** Solve the following linear programming problem by dynamic programming method.

Maximize $z = 8x_1 + 7x_2$
Subject to $2x_1 + x_2 \leq 8$
$5x_1 + 2x_2 \leq 15$
$x_1, x_2 \geq 0.$

**16.** A project consists of eight activities with the following relevant information.

| Activity | Time estimate (Week) | | | Predecessor |
|---|---|---|---|---|
| | $t_0$ | $t_m$ | $t_p$ | |
| A | 1 | 1 | 7 | None |
| B | 2 | 4 | 7 | None |
| C | 2 | 2 | 8 | None |
| D | 1 | 1 | 1 | A |
| E | 2 | 5 | 14 | B |
| F | 2 | 5 | 8 | C |
| G | 3 | 6 | 15 | D, E |
| H | 1 | 2 | 3 | F, G |

(*i*) Draw the network and find the expected project completion time.

(*ii*) What is the probability of completing the project 4 weeks before the expected completion time ?

[ *Internal Assessment* – 10 Marks ]