M.Sc. 3rd Semester Examination, 2022

APPLIED MATHEMATICS

(*Dynamical Oceanology/Advanced Optimization and Operations Research*)

PAPER – MTM-305(A & B)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER—MTM-305 A

(*Dynamical Oceanology*)

A. Answer any *four* questions : 2×4

1. Write the z-component of Reynolds Averaged *Navier-Stokes* (RANS) equations.

2. Write reasons for the use of scaling or ordering terms in Oceanology.

3. For the inertial motion, show that the inertia forces are of same size as Coriolis forces.

4. For the horizontal equations of motion when friction is included, schematically show how Coriolis, Friction and Pressure forces are related.

5. For water speed 0.15 m/s, calculate the radius of the circle of the inertia motion at latitude 45°.

6. Write the similarity/dissimilarity between circulation and vorticity.

B. Answer any *four* questions : 4×4

7. Derive the pair of equations describing the mass transports **Mx** and **My** under the assumptions of Sverdrup for the wind-driven circulation.

8. Derive the vorticity-conservation equation for the geostrophic flow.

9. For typical horizontal length scale (L) of 700 km, horizontal speeds (U) are of the order of $0.1\ ms^{-1}$ and a vertical scale length (H) of 1100 m, estimate a typical vertical speed (W).

10. (*a*) With the help of the x- and y-momentum equations of two dimensional motion for incompressible viscous and laminar flow, derive the equation of vorticity for this flow.

(*b*) Also write the physical interpretation of each terms of this equation.

11. Define the vortex doublet and hence find the complex potential at any point in the ocean.

12. Let, vortices each of strenght $-k$ at $z = 0$, $\pm a, \pm 2a, \pm 3a, \ldots\ldots$ and vortices each of strength k at $z = \pm a - ib, \pm 2a - ib,$

$\pm 3a - ib$, ... Show that the vortex system moves parallel to itself with velocity $u_0 = k \cot(\pi b/a)$, $v_0 = 0$.

C. Answer any *two* questions : 8 × 2

13. (*a*) Define the Karman Vortices by showing their positions.

(*b*) Find the complex potential of the above Karman vortices arrangement. 3 + 5

14. (*a*) What is the inertial motion ? State the necessary assumptions for the inertial motion and then derive the horizontal equation of motion in oceanography.

(*b*) For the above case- (*a*), show that the motion is described by a circle. 4 + 4

15. (*a*) Write all the equations of motion in terms of eddy viscosities.

(*b*) For the ocean with horizontal and vertical length scales 10^3 KM and 2 KM, respectively and horizontal speed of order 0.15 m/s, scale all the above equations written in part 15(a) and reduces to approximated equations with order of accuracy 1%. 2 + 6

16. Derive the Reynolds equation for the y-component of velocity. 8

[*Internal Assessment* – 10 Marks]

PAPER—MTM-305 B

(*Advanced Optimization and Operations Research*)

A. Answer any *four* questions of the following : 4×2

1. Write the advantages of revised simplex method.

2. Write the limitations of golden section searching method.

3. State the integer and mixed integer programming problems.

4. Explain different types of achievements in goal programming problem.

5. Write the general scheme of numerical optimization.

6. Explain the deletion of an existing variable from the optimal table of an LPP.

B. Answer any *four* questions : 4 × 4

7. Following is the optimal table of an LPP

		C_j	7	9	0	0
C_B	B	x_B	y_1	y_2	y_3	y_4
9	x_2	$\frac{7}{2}$	0	1	$\frac{7}{22}$	$\frac{1}{22}$
7	x_1	$\frac{9}{2}$	1	1	$-\frac{1}{22}$	$\frac{3}{22}$
$z_j - c_j$			0	0	$\frac{28}{11}$	$\frac{15}{11}$

Find range of discrete changes of c_1 and c_2 such that the optimal solution does not alter.

8. Write the procedure of cutting plane method for constraint optimization problem.

9. Derive the expression of Gomory's constant for the constraint corresponding to non-integer value.

10. Using Fibonacci method, minimize

$$f(x)=\begin{cases}6-x; & x\le 3\\ x; & x>3\end{cases}$$

in the interval [2, 5] using $n=5$.

11. A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day, and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2.

Management has established the following goals it would like to achieve in determining the daily product mix :

p_1 : The joint total production at least 10 units

p_2 : Producing at least 7 units of product B.

p_3 : Producing at least 8 units of product A.

Formulate above goal programming problem.

12. Using Newton's method minimize

$$f(x_1, x_2) = 8 + x_2 - x_1^2 + x_1 x_2 - 4x_2^2$$

with $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ as a starting point.

C. Answer any *two* questions of the following : 8×2

13. Solve the following LPP by revised simplex method

maximize $z = 7x_1 + 9x_2$

subject to

$$-x_1 + 3x_2 \le 6$$

$$7x_1 + x_2 \le 35$$

and $x_1, x_2 \ge 0$

14. Solve the following IPP using branch-and-bound method :

$$\text{maximize} \quad z = 2x_1 + 2x_2$$

subject to

$$5x_1 + 3x_2 \le 8$$

$$x_1 + 2x_2 \le 4$$

and $x_1, x_2 \ge 0$, and are integers.

15. Solve the following goal programming problem :

$$\text{minimize} \quad z = P_1(2d_2^- + 3d_3^-) + P_2 d_1^-$$

subject to

$$20x_1 + 10x_2 \le 60$$

$$10x_1 + 10x_2 \le 40$$

$$30x_1 + 60x_2 + d_1^- - d_1^+ = 600$$

$$x_1 + d_2^- - d_2^+ = 4$$

$$x_2 + d_3^- - d_3^+ = 4$$

$$x_1, x_2, d_i^- \, d_i^+ \ge 0, i = 1, 2, 3.$$

16. Use the artificial constraint method to find the initial basic solution of the following problem and then apply the dual simplex method to solve it.

maximize $z = 2x_1 - 3x_2 - 2x_3$

subject to

$$2x_2 + x_3 \leq 10$$
$$x_2 - 2x_3 \geq 4$$
$$x_1 - 2x_2 - 3x_3 = 8$$

and $x_1, x_2, x_3 \geq 0$.

[*Internal Assessment* – 10 Marks]