M.Sc. 3rd Semester Examination, 2022

APPLIED MATHEMATICS

(*Discrete Mathematics*)

PAPER – C-MTM-304(CBCS)

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

A. Answer any *four* questions of the following : 2×4

1. Define bipartite graph and give an example.

2. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

3. Find a closed form for the generating function for the following sequence :

$$1, 0, -1, 0, 1, 0, -1, 0, 1, \ldots\ldots\ldots$$

4. Define chain and anti-chain with an example in connection with poset.

5. Find the dual of the Boolean expression : $wx\,(y'z + yz') + w'x'(y' + z)\,(y + z')$

6. Find the language for the regular expression given below : $(a + b)^* (a + bb)$.

B. Answer any *four* questions of the following : 4×4

7. A simple graph with n vertices and k-components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

8. Define poset and consider $P(S)$ as the power set, show that the inclusion relation $\subseteq$ is a poset on the power set $P(S)$.

9. In the Boolean algebra $(B, +, ., ')$, express the Boolean function

$$f(x, y, z) = (x + y)(x + z) + y + z'$$

in its disjunctive normal form.

10. Using mathematical induction, prove that for any integer $n > 1$.

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots\ldots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

11. Define planar graph and prove that the graph $K_{3,3}$ (Kuratowski's second graph) is non-planar.

12. Define finite-state machine (FSM). Let M be the FSM with state table appearing as

	f		g	
S \ Σ	a	b	a	b
s_0	s_1	s_0	1	0
s_1	s_3	s_0	1	1
s_2	s_1	s_2	0	1
s_3	s_2	s_1	0	0

(*i*) Find the input set $\sum$, the state set S, the output set O, and initial state of M.

(*ii*) Draw the state diagram of M.

C. Answer any *two* questions of the following : 8×2

13. (*i*) State and prove Euler's theorem for a connected planar graph.

(*ii*) If G is connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n-6$.

(*iii*) Prove that every nontrivial tree T has at least two vertices of degree 1. $3 + 3 + 2$

14. Define phrase-structure grammar. Describe the classification scheme of phrase-structure grammar introduced by Noam Chomsky. $2 + 6$

15. (*i*) Determine the generating function of the following sequences :

$$f_r = \frac{r(r+1)}{2}, \ (r > 0).$$

(*ii*) Use generating function to solve the recurrence relation :

$$a_{n+2} - 2a_{n+1} + a_n = 2^n \text{ taking } a_0 = 2, a_1 = 1.$$

3 + 5

16. State the principle of inclusion-exclusion. Use the principle of inclusion-exclusion, find the number of primes less than 100. 2 + 6

[*Internal Assessment* – 10 Marks]