PG/IIIS/MTM/302/22

M.Sc. 3rd Semester Examination, 2022

APPLIED MATHEMATICS

(*Transforms and Integral Equations*)

PAPER – MTM-302

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

A. Answer any *four* questions : 2×4

1. If $\overline{G}(k,l)$ be the two-dimensional Fourier transform of a function $G(x,y)$, then what is the Fourier inversion formula to get $G(x,y)$ from $\overline{G}(k,l)$.

2. If $F(p)$ denotes the Laplace transform of the

(*Turn Over*)

function $f(t)$, $t \geq 0$, state the conditions for which $f(t)$ must satisfy so that $F(p)$ exists.

3. Who first coined the term wavelet ? Define 'mother wavelet' and explain the utility of it.

4. Define infinite Fourier transform and state the conditions of existence of the transform.

5. Define singular integral equation with an example.

6. If the function $f(t)$ has period $T > 0$ then calculate $L\{f(t)\}$ in a simplest form.

B. Answer any *four* questions : 4×4

7. Show that if a function $f(x)$ defined on $(-\infty, \infty)$ and its Fourier transform $F(\zeta)$ are both real, then $f(x)$ is even. Also show that if $f(x)$ is real and its Fourier transform $F(\zeta)$ is purely imaginary, then $f(x)$ is odd.

8. Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform.

9. Let $F(k)$ and $G(k)$ be the Fourier transforms of $f(x)$ and $g(x)$ respectively defined in $(-\infty, \infty)$. Show that the Fourier transform of $\int_{-\infty}^{\infty} f(u)g(x-u)du$, can be expressed in terms of the product $F(k)G(k)$. Hence prove that parseval's relation

$$\int_{-\infty}^{\infty} |f(k)|^2 dk = \int_{-\infty}^{\infty} |f(x)|^2 dx.$$

10. If a and b are real constants, solve the following integral equation :

$$ax + bx^2 = \int_0^x \frac{y(t)}{(x-t)^{\frac{1}{2}}} dt.$$

11. Solve the integral equation :

$$y(x) = f(x) + \lambda \int_0^1 (x+t) y(t) dt$$

and find the eigen values.

12. Evaluate $L\left\{\int_0^t \frac{\sin u}{u} du\right\}$ by the help of initial value theorem.

C. Answer any *two* questions : 8×2

13. Find the solution of the following problem of free vibration of a stretched string of an infinite length :

$$\frac{\partial^2 u}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=0.-\infty<x<\infty, \text{ subject to}$$

$u(x,0)=f(x)$, $\frac{\partial}{\partial t}u(x,0)=g(x)$, u and $\frac{\partial u}{\partial x}$ both vanish as $|x|\to\infty$. 8

14. (*i*) Show that

$$\int_0^t\int_0^v\int_0^w f(u)\, dudvdw = L^{-1}\left\{\frac{F(p)}{p^3}\right\}$$

where $F(p)=L\{f(t)\}$.

(*ii*) State and prove final value theorem concerning on Laplace transform. 3 + 5

15. (*i*) With help of the resolvent kernel, find the solution of the integral equation

$$y(x)=1+x^2+\int_0^x\left(\frac{1+x^2}{1+t^2}\right)y(t)dt.$$

(*ii*) Discuss the solution procedure of homogeneous Fredolm integral equation of the second kind with degenerate kernel.

4 + 4

16. (*i*) Form an integral equation corresponding to the following differential equation

$$\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+e^{-x}y(x)=x^3-5x,$$

subject to conditions, $y(0)=-3$ and $y'(0)=4$.

(*ii*) If $L\{f(t)\}=F(p)$ which exists $Real(p)>\gamma$ and $H(t)$ is unit step function, then prove that for any α, $L\{H(t-\alpha)f(t-\alpha)\}=e^{-p\alpha}F(p)$, which exists for $Real(p)>\gamma$.

5 + 3

[*Internal Assessment* – 10 Marks]