2022

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

Paper : MTM - 105

(Classical Mechanics and Non-linear Dynamics)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***four*** questions : 2×4=8

   (a) Defind the Lagrangian and Hamiltonian of a dynamical system. Compare these two functions.

   (b) What do you mean by non-inertial frame? Give an example of a non-inertial frame.

   (c) Is Poisson bracket commutative? Justify your answer.

   (d) State and prove conservation laws of linear momentum and angular momentum.

   (e) Define cyclic coordinates. State when Routhian equations of motion is useful.

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(f) Define holonomic and non-holonomic constraints. Give one example of each of these constraints.

2. Answer any ***four*** questions : 4×4=16

(a) What do you mean by generalized forces? Find the expression of it in terms of generalised coordinates. 4

(b) Define constant of motion. Show that the Poisson bracket of two constants of motion is also constant of motion. 4

(c) Solve the Euler's dynamical equations

$A\dot{w}_1 - (B-C)w_2w_3 = 0$, $B\dot{w}_2 - (C-A)w_3w_1 = 0$,

$C\dot{w}_3 - (A-B)w_1w_2 = 0$, when $A = B$. 4

(d) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{d}{dt}F(q_1, q_2, ..., q_n)$ also satisfies Lagrange's equations, where F is arbitrary, but differentiable function of its arguments. 4

(e) Consider the following nonlinear dynamical system, $\dot{x} = x^2y - x^5$, $\dot{y} = -y + x^2$. Study the stability at the origin. 4

(f) What is the effect of the Coriolis force on a particle falling freely under the action of gravity? 4

3. Answer any ***two*** questions : $8 \times 2 = 16$

(a) Deduce Lagrange equations of motion for unconnected holonomic and conservative force. 8

(b) Find the curve joining two given points A and B, which is traversed by a particle moving under gravity from A to B in the shortest time (ignoring friction along the curve and the resistance of the medium). 8

(c) Derive the Lorentz transformation equations. 8

(d) Deduce Euler's dynamical equations when a rigid body is rotating about a fixed point. 8
