2022

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

Paper : MTM - 102

(Complex Analysis)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***four*** questions : 2×4=8

(a) With necessary conditions, write the Homotopy form of the Cauchy's Theorem.

(b) Find the fixed points of the transformation $w = \frac{z-1}{z+1}$.

(c) Define the isolated singular point z_0 and then write the general form of Laurent series expansion about the z_0 with the region where it is valid.

(d) State the theorem for transformations of harmonic functions from a domain *Dz* in the *z*-plane onto the domain *Dw* in the *w*-plane.

P.T.O.

(e) Define branch and branch cut for a multi-valued function $f(z)$.

(f) Find the points at which $w = \cos(z)$ is not conformal.

2. Answer any ***four*** questions : 4×4=16

(a) Find Taylor or Laurent series expansion of the function $f(z) = \dfrac{z}{(z^2 - 4z + 3)}$ with center at $c = 1$, where region of convergence is $0 < |z - 1| < 2$.

(b) When $\log(z) = \ln(r) + i\theta,\ r = |z| > 0$, show that $\log(i^2) \neq 2\log(i)$ for $\dfrac{3\pi}{4} < \theta < \dfrac{11\pi}{4}$, while $\log(i^2) = 2\log(i)$ for $\dfrac{\pi}{4} < \theta < \dfrac{9\pi}{4}$.

(c) Does Cauchy-Goursat theorem hold separately for the real or imaginary part of an analytic function $f(z)$? Justify your answer.

(d) Using the calculus of residue evaluate $\int_0^\infty \dfrac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$.

(e) With the help of residue, find the inverse Laplace transformation $f(t)$ of $F(s) = \dfrac{s}{(s^2 + a^2)^2} (a > 0)$.

(f) Show that series $\sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuation of each other.

3. Answer any *two* questions out of *four* questions : 8×2=16

(a) (i) State and prove the Casorati-Weierstrass's theorem.

(ii) Find the value of $\int \mathrm{Log}(z+3i)\,dz$ where $c : |z| = 1$. 6+2

(b) (i) Find a conformal map, which maps the unit disk $|z| < 1$ in the z-plane onto the right half-plane $\mathrm{Re}(w) > 0$ in the w-plane.

(ii) Find the Mobious transformation that maps $1, 0, -1$ to the respective points $i, \infty, 1$. 6+2

(c) (i) With the concept of integration along a branch cut, show that $\int_0^{\infty} \frac{x^{-a}}{x+1} dx = \frac{\pi}{\sin a\pi}$ where $0 < a < 1$.

(ii) Find the singular points of the function $z|z|$, if any. Justify your answer. 6+2

(d) (i) State the Rouches theorem.

P.T.O.

(ii) Let $f(z)=z^4+z^2+8z-4$. Prove that exactly three roots of the polynomial $f(z)$ lie in $1<|z|<3$ and there is exactly one root of the polynomial $f(z)$ in the disc $|z+2|<\frac{1}{2}$.

2+6