2022

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-403

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

MTM–403 (UNIT–1) MAGNETO HYDRO-DYNAMICS

1. Answer any *two* questions : 2×2

(a) Write down the basic difference between magneto-fluid-dynamics (MFD) and magneto-hydrodynamics (MHD).

(Turn Over)

(b) Define Hartmann number and explain its significance.

(c) Write down the working procedure of 'magneto-fluid-dynamics (MFD) submarines'.

(d) Write down the statement of Ferraro's law of isorotation.

2. Answer any *two* questions : 2×4

(a) Write down the basic equations of mageto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.

(b) Find the equations of motion of a conducting fluid in the context of mageto-hydrodynamics flow.

(c) State and prove Alfven's theorem.

(d) Define the terms Alfven's velocity and Alfven's waves. Hence, derive the speed of propagation is $\sqrt{c^2 + V_A^2}$ for magneto hydrodynamics wave, where symbols have their usual meaning.

3. Answer any *one* question : 1×8

(a) A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = -L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$(upper). Assume that a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field.

(b) (i) Define magnetic energy. Find the rate of change of magnetic energy in magneto-hydrodynamic.

(ii) Write the mathematical formulation of mageto-hydrodynamic Couette flow and derive its velocity expression.

[Internal assessment – 05]

MTM–403 (UNIT–2)

STOCHASTIC PROCESS AND REGRESSION

1. Answer any *two* questions : 2×2

(a) State the Chapman-Kolmogorov equation and write its significance.

(b) State Gambler's ruin problem and write the transition matrix for it.

(c) Define the order of a Markov Chain. Discuss how a Markov Chain can be represented as a graph.

(d) Show by an example, that the Markov Chain can be represented by a graph.

2. Answer any *two* questions : 2×4

(a) Starting from probability generating function for the birth and death process for the linear growth process, find the mean population size at any time and assume that the initial population size is K.

(b) Let $\{X_n, n \geq 0\}$ be a branching process. Show that if $m = E(X_1) = \sum_{k=0}^{\infty} kp_k$ and $\sigma^2 = \text{var}(X_1)$, then $E(X_n) = m^n$ and $\text{var}(X_n) =$

$$\begin{cases} n\sigma^2 & , \quad \text{if } m = 1 \\ \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & \text{if } m \neq 1. \end{cases}$$

(c) Let $\{x_n, n \geq 1\}$ be a Markov Chain having space $S = \{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Identify the states as transient, persistent, ergodic.

(d) Suppose the probability of a dry day following a rainy day is $\frac{2}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$ and t.p.m is

$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. If July 20 is a dry day then find the

probability that July 22 and July 26 are dry day.

3. Answer any *one* question : 1×8

(a) Prove that

$$r_{1.23...p} = \left(1 - \frac{|R|}{R_{11}}\right)^{1/2}$$

where the symbols have their usual meanings.

(b) Find the differential equation for the Wiener process.

[Internal assessment – 05]

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