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Finding Equilibrium Price and Quantity Using Fuzzy Linear Demand and Supply Equations

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ABSTRACT

In this research, we explore the supply and demand model and pursue the factors that influence demand and supply. We discuss the shifting of equilibrium points for the changes of supply and demand. Equilibrium is the point where both the consumers' and producer's willingness agree. Since there exists so many factors affecting the equilibrium point and the data are imprecise, we are aiming to find the equilibrium interval rather than the crisp point. We conduct our work with the help of interval arithmetic of fuzzy numbers and the classical method of solving the system of fuzzy linear equations.

Keywords: Demand, Supply, Equilibrium, Linear Demand, and Supply Model, Classical Method.

Mathematical Subject Classification (2010): 94D05

1. Introduction

A linear system of equations shows a significant function in the field where the parameters are uncertain. Linear systems of equations having unpredictable parameters are used widely owing to some inexact data on the connection of a linear system of equations. In our competitive market, the equilibrium price of a product is the only price where the desires of both consumers and producers agree. The product's quantity that consumers are willing to consume (quantity demanded) and the product's quantity that the sellers are willing to sell (quantity supplied) are equal in equilibrium price. The equilibrium quantity is the mutually intended quantity desired by both consumers and producers. At any other price, we see that there is either deficiency or abundance of the commodity, deficiency or abundance of the consumers, shortage or surplus of the producers.

But for the instability of the market, the price of a product may vary every other day. For example, we can bring to the front the previous market of onion and potato. When a market has equilibrium price and quantity, it means the interest of buyers and sellers towards a product maintains balance. But when a market isn't in equilibrium, it causes a troublous situation. To avoid the erratic situation fuzzy approach in finding the equilibrium point could be a useful technique. The purpose of this research is to find out the equilibrium price and quantity using fuzzy linear demand and supply equations in an interval rather than a crisp value, which would help to maintain the stability of the price and quantity supplied of a product. Here we use the Classical Method to solve the fuzzy system of linear equations to find the equilibrium quantity. The arithmetic operations of fuzzy numbers using α-cuts have been discussed before as we use the triangular fuzzy number in our examined demand and supply problem.

2. Related work

In [6], Horcik examined the solution of a system of linear equations having fuzzy numbers. In [7], the parametric form of fuzzy numbers was being used by Vroman et.al. (2007) to solve the general system of the linear equation having fuzzy numbers. In [8], Li. et.al. (2010) introduced a new algorithm to solve a system of the linear equation having fuzzy numbers. In [9], Sevastjanov and Dymova suggested a new technique for the interval and also for the fuzzy systems $X = AX + U$. In [10], Amirfakhrian (2012) formulated a method for solving a system of fuzzy linear equations where he had used the procedure of fuzzy distance. In [11], the Center and width-based approach for working out fuzzy systems of linear equations was developed by Chakraverty and Behera (2012). In [12], for solving fuzzy linear systems, Senthilkumara and Rajendran (2011) pertained to an algorithmic approach. In [13-14] Das and Chakraverty (2012), Senthilkumara and Rajendran (2011) also examined a fully fuzzy system of linear equations. In [20], Nasseri et al. (2011) examined a system with fuzzy right-hand side and crisp coefficients for their fuzzy procedure to supply-demand model. Applying the QR-decomposition procedure, they have unraveled a system. Rupjit Saikia, Dipjyoti Sarma examined in their work “A Case study on an Economic problem by using Fuzzy linear Equations” about solving an economic issue using interval/fuzzy outcomes of a system of linear equations.

In this research, we want to find the equilibrium prices in terms of fuzzy numbers, which would help us to know the equilibrium prices for a specific period. We aim to generate our research more specific in solving the demand and supply model and which can provide strong fuzzy solutions.

3. Overview about demand and supply [3][19]

Supply and demand play the most vital role in determining the costs and quantities of most goods and services available in a given market. Supply and demand maintain the commerce between the purchaser and seller of a product or service. The point at which the alliance between supply and demand maintains the balance is called the equilibrium price.

In this segment, we review basic knowledge about demand, supply, and their influencing factors.

3.1. Demand

In economics, demand is that the volume of a product that a consumer’s willingness and purchasing ability meet at different rates during a given time. The demand curve shows the connection between price and quantity demanded. An item's perception of need, price, perception of quality, comfort, convenient substitutes, purchasers' earnings, and savors, and other influencing factors can influence the demand for a selected item.

3.2. Factor influencing demand

Demand can be dominated by several factors that are described as determinants of demand. The followings are some of the determinants of demand:

- Good's price
- Price of similar goods
- Personal earnings
- Taste of choices
- Consumer anticipations about prospective rates, earnings, and availability
- Population
- Nature of the product
- The number of buyers in a market

3.3. Supply

In its most simple form, a linear supply function is represented in general as follows: y = mx+ b. Here, x represents the independent variable and y represents the dependent variables which means that we've got the variable quantity on the x-axis and therefore the variable price on the y-axis.

In economics, supply refers to the quantity of a product available within the marketplace for sale at a specified price at a given time. The willingness of a producer or seller to produce or sell the desired amount of a product within a selected price and time is considered as supply.

3.4. Factor influencing supply

Supply can be dominated by several factors that are described as determinants of supply. The followings are some of the determinants of supply:

- Price
- Cost of production
- Natural conditions
- Technology
- Transport conditions
- Factor prices and their availability
- Government's policies
- Prices of related goods

3.4. Linear demand and supply model

Here we would like to consider a fuzzy linear system of the form $\bar{A}.\bar{X} + \bar{C} = \bar{B}$, wherein the coefficients and variables both are fuzzy numbers. We consider an economic problem where the demand and supply of a particular product are liner functions of price are given by

$$Q_d = a + bp$$
$$Q_s = c + dp$$

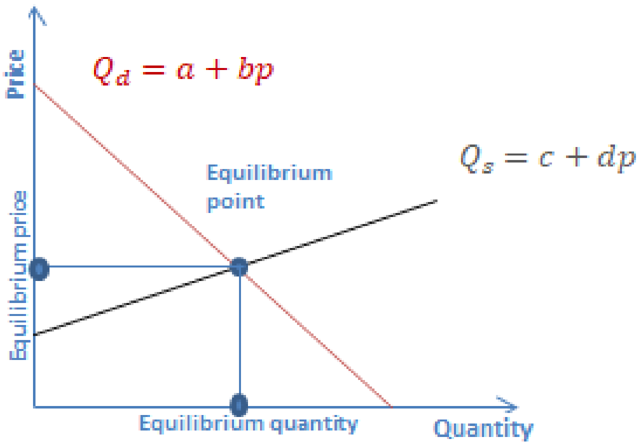
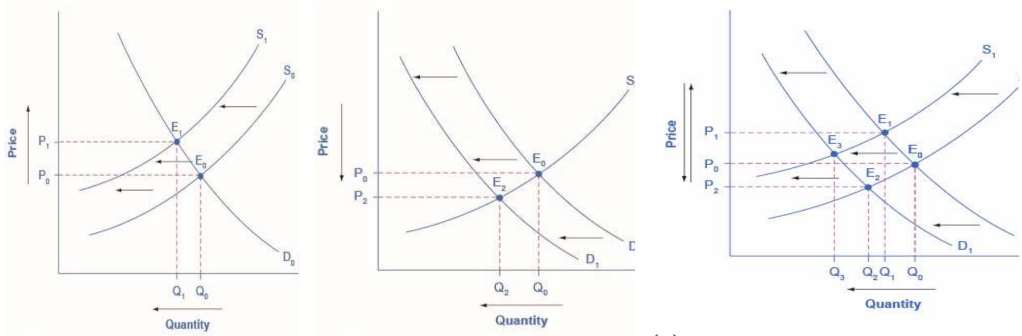


Figure 1:

In the demand equation, the quantity of demand, Q_d, is dependent on price (p), a is quantity intercept and b is inverse of the slope of demand equation, i.e., $\frac{\Delta Q}{\Delta p}$. In supply equation, c is the quantity intercept of the supply, $d = \frac{\Delta Q}{\Delta p}$ is the price coefficient of supply which is the responsiveness of the producers. We would like to solve the fuzzy linear demand and supply equations using the solution technique (Classical Method) for solving fuzzy systems of linear equations and find fuzzy equilibrium prices, at which prices the quantity supplied and the quantity demanded are in equilibrium.

3.5. Shifting of equilibrium point [15][16]



(a) Shift in supply (b) Shift in demand (c) Shift in both supply and demand

Figure 2:

Table 1: Shifting of equilibrium point for the changes of supply and demand

Change in Demand	Change in Supply	Effect on Equilibrium Quantity	Effect on Equilibrium Price
No change	Increase	Increase	Decrease
No change	Decrease	Decrease	Increase
Increase	No change	Increase	Increase
Decrease	No change	Decrease	Decrease
Decrease	Increase	Indeterminant	Decrease
Increase	Decrease	Indeterminant	Increase
Increase	Increase	Increase	Indeterminant
Decrease	Decrease	Decrease	Indeterminant

4. Arithmetic operations of fuzzy numbers using α-cuts [4]

The α-cuts of fuzzy numbers always are closed and also bounded intervals. Here we consider A and B where both A and B are fuzzy numbers and let $A(\alpha) = [a_1(\alpha), a_2(\alpha)]$ and $B(\alpha) = [b_1(\alpha),\ b_2(\alpha)]$, for $0 \leq \alpha \leq 1$, are the α-cuts A^α and B^α of the fuzzy numbers A and B respectively.

The arithmetic of fuzzy numbers with the help of their α-cuts can be defined as follows:

Let $* \in (+,-,\times,\ /)$. Then the '*' operation on fuzzy numbers A and B , denoted by $A * B$, gives a fuzzy number in $\mathbb{R}$ where,

$$A * B \ = \bigcup_{\alpha} \alpha.(A * B)^\alpha$$

And $(A * B)^\alpha = A^\alpha * B^\alpha, \qquad \alpha \in (0,1]$.

WhereA^α and B^α are the intervals $A(\alpha) = \ [a_1(\alpha),\ \ a_2(\alpha)]$ and $B(\alpha) \ = [b_1(\alpha),\ \ b_2(\alpha)]$ respectively.

Let, $P = A + B$, then

$$P[\alpha] = A[\ \alpha] + B[\ \alpha],$$

or,

$$P[\alpha] = [a_1(\alpha) + b_1(\alpha), \qquad a_2(\alpha) + b_2(\alpha)], \quad for\ 0 \leq \alpha \leq 1.$$

For subtraction, if we let $Q = A - B$, then

$$Q[\alpha] = [a_1(\alpha) - b_2(\alpha), \qquad a_2(\alpha) - b_1(\alpha)], \quad for\ 0 \leq \alpha \leq 1.$$

If $R = A.B$, then

$$R[\alpha] = [\ a_1(\alpha),\ \ a_2(\alpha)]\ .[\ b_1(\alpha),\ \ b_2(\alpha)] = [r_1(\alpha),\ r_2(\alpha)], \quad for\ 0 \leq \alpha \leq 1.$$

where $\quad r_1(\alpha) = \min\{a_1(\alpha)b_1(\alpha),\quad a_1(\alpha)b_2(\alpha),\quad a_2(\alpha)b_1(\alpha),\quad a_2(\alpha)b_2(\alpha)\}$

and

$$r_2(\alpha) = \max\ \{a_1(\alpha)b_1(\alpha), \qquad a_1(\alpha)b_2(\alpha), \qquad a_2(\alpha)b_1(\alpha), \quad a_2(\alpha)b_2(\alpha)\}.$$

Lastly, if $S = A/B$, then

$$S[\alpha] = [a_1(\alpha), a_2(\alpha)]/[b_1(\alpha), b_2(\alpha)] = [s_1(\alpha), s_2(\alpha)], \quad for\ 0 \leq \alpha \leq 1.$$

Here,

$$s_1(\alpha) = \min\ \{a_1(\alpha)/b_1(\alpha),\ \ a_1(\alpha)/b_2(\alpha),\ \ a_2(\alpha)/b_1(\alpha),\ \ a_2(\alpha)/b_2(\alpha)\}$$

and

$$s_2(\alpha) = \max\ \ \{a_1(\alpha)/b_1(\alpha),\ \ a_1(\alpha)/b_2(\alpha),\ \ a_2(\alpha)/b_1(\alpha),\ \ a_2(\alpha)/b_2(\alpha)\}.$$

Assuming that zero does not belong to $B[0]$. Here $P[\alpha]$, $Q[\alpha]$, $R[\alpha]$ and $S[\alpha]$ are the α-cuts of P, Q, R and S respectively.

Example

Consider $A = (0/1/2)$ and $B = (1/2/3)$ with membership function

$$\mu_M(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 0, & otherwise \end{cases} \text{ and } \mu_N(x) = \begin{cases} x - 1, & 1 \le x \le 2 \\ 3 - x, & 2 < x \le 3. \\ 0, & otherwise \end{cases}$$

Here α-cut of A is $A[\alpha] = [\alpha,\ 2 - \alpha]$ and α-cut of B is $B[\alpha] = [1 + \alpha,\ 3 - \alpha]$.
Since the fuzzy numbers $A = (0/1/2)$ and $B = (1/2/3)$ both are positive.
Then we use simply
$R[\alpha] = A[\alpha].B[\alpha] = [a_1(\alpha)b_1(\alpha),\ a_2(\alpha)b_2(\alpha)]$ and
$S[\alpha] = A[\alpha]/B[\alpha] = [a_1(\alpha)/b_2(\alpha), a_2(\alpha)/b_1(\alpha)]$ according to interval arithmetic.
Then

1. $P[\alpha] = [\alpha + \alpha + 1, 2 - \alpha + 3 - \alpha] = [1 + 2\alpha, 5 - 2\alpha]$,
2. $Q[\alpha] = [\alpha - (3 - \alpha), 2 - \alpha - (\alpha + 1)] = [2\alpha - 3, 1 - 2\alpha]$,
3. $R[\alpha] = [\alpha(\alpha + 1),\ (2 - \alpha)(3 - \alpha)] = [\alpha^2 + \alpha, 6 - 5\alpha + \alpha^2]$,
4. $S[\alpha] = [\alpha/(3 - \alpha), (2 - \alpha)/(1 + \alpha)]$.

We see that $P = (1/3/5)$, $Q = (-3/-1/1)$, $R = (0/2/6)$, and $S = (0/0.5/2)$.
[**Note:** For the fuzzy set P, we have $P[\alpha]$ then support
$P[0] = [0 + 1, 5 - 0] = [1,5]$, and core $P[1] = [2 + 1, 5 - 2] = [3,3] = 3$. Since the fuzzy numbers $M = (0/1/2)$ and $N = (1/2/3)$ both are triangular then P must be triangular or triangular shaped. Similarly, we can find Q, R and S**].**

4.1. Arithmetic operations of triangular fuzzy numbers

Let us consider two triangular fuzzy numbers A and B. Then

- The addition $A + B$ and subtraction $A - B$ are also triangular fuzzy numbers.
- The multiplication $A.B$ and the division A/B may or may not be triangular fuzzy numbers. In general, they are triangular-shaped fuzzy numbers (also called approximated triangular-shaped fuzzy numbers).
- 'max' and 'min' operations of triangular fuzzy numbers are not always in the form of triangular fuzzy numbers.

We can simply use the following two formulas for the addition and subtraction of two fuzzy numbers. Let two triangular fuzzy numbers be $A = (a_1/a_2/a_3)$ and
$B = (b_1/b_2/b_3)$, then

1. $A + B = (a_1 + b_1/\ a_2 + b_2/a_3 + b_3)$
2. $A - B = (a_1 - b_3/a_2 - b_2/a_3 - b_1)$ and $-B = (-b_1/-b_2/-b_3)$.

For multiplication and division, the method of α-cut is the best way. The α-cut of the triangular fuzzy number $B = (b_1/b_2/b_3)$ is

$$B^{\alpha} = [(b_2 - b_1)\alpha + b_1, -(b_3 - b_2)\alpha + b_3], \forall\alpha \in [0,1].$$

5. Solution technique for fuzzy system of linear equations [17][18]

We will consider the **classical method** for finding the solution of a fuzzy system of linear equations. Throughout work $\bar{A} \leq \bar{B}$ means $\bar{A}(x) \leq \bar{B}(x)$ for all x. Here we use the works of Buckley and Qu and Eslami.

We are to work out the fuzzy matrix equation

$$\bar{\boldsymbol{A}}.\bar{\boldsymbol{X}} + \bar{\boldsymbol{C}} = \bar{\boldsymbol{B}} \tag{1}$$

Here $\bar{\boldsymbol{A}} = [\bar{a}_{ij}]$ be a $m \times n$ matrix where each $\bar{a}_{ij}$ are real triangular fuzzy numbers for $1 \leq i \leq m$ and $1 \leq j \leq n$,

$\bar{\boldsymbol{B}}^t = (\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n)$ be a $n \times 1$ vector where each $\bar{b}_i, (1 \leq i \leq n)$, are unknown real fuzzy numbers,

$\bar{\boldsymbol{C}}^t = (\bar{c}_1, \bar{c}_2, \dots, \bar{c}_n)$ be a $n \times 1$ vector where each $\bar{c}_i, (1 \leq i \leq n)$, are unknown real fuzzy numbers,

$\bar{\boldsymbol{X}}^t = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$ be a $n \times 1$ vector of unknown real fuzzy numbers $\bar{x}_i, 1 \leq i \leq n$.

For simplicity, we investigate first the crisp theory for 2×2 systems and then fuzzify it. A linear system which is 2×2 can be written as

$$a_{11}x_1 + a_{12}x_2 = b_1 - c_1, \tag{2}$$

$$a_{21}x_1 + a_{22}x_2 = b_2 - c_2, \tag{3}$$

where $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1$ and c_2 are crisp constants and x_1, x_2 are crisp variable (unknown). We are to work out these two equations at one time for x_1, x_2.

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ must be non-singular and $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1}$ exists. Then the unique solution can be written as

$$x_1 = \frac{a_{22}(b_1 - c_1) - a_{12}(b_2 - c_2)}{a_{11}a_{22} - a_{12}a_{21}}, \tag{4}$$

$$x_2 = \frac{a_{11}(b_2 - c_2) - a_{21}(b_1 - c_1)}{a_{11}a_{22} - a_{12}a_{21}}, \tag{5}$$

Or,

$$x_1 = \frac{\Delta x_1}{\Delta}, \qquad x_2 = \frac{\Delta x_2}{\Delta}$$

Where,

$$\Delta = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta x_1 = a_{22}(b_1 - c_1) - a_{12}(b_2 - c_2)$$

$$\Delta x_2 = a_{11}(b_2 - c_2) - a_{21}(b_1 - c_1)$$

Now the linear system of equations needs to be rewritten in matrix form. Consider a 2×2 matrix of coefficients as

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{6}$$

Also, consider

$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{7}$$

$$\boldsymbol{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{8}$$

And

$$\boldsymbol{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{9}$$

be two 2×1 matrices (vectors). Now we write our linear system of equations as

$$\boldsymbol{A}.\boldsymbol{X} = \boldsymbol{B} - \boldsymbol{C} \tag{10}$$

We need to fuzzify equation (10).
Consider $\overline{\boldsymbol{A}} = [\bar{a}_{ij}]$ is a 2×2 matrix and the elements $\bar{a}_{ij}$ of matrix $\overline{\boldsymbol{A}} = [\bar{a}_{ij}]$ are continuous fuzzy numbers, $\overline{\boldsymbol{B}}^t = (\bar{b}_1, \bar{b}_2)$ be a 2×1 vector whose members $\bar{b}_i$, $i = 1,2$ are given continuous fuzzy numbers and $\overline{\boldsymbol{X}}^t = (\bar{x}_1, \bar{x}_2)$ is a 2×1 vector of unknown continuous fuzzy numbers $\bar{x}_i$, $i = 1,2$.
The fuzzified equation

$$\overline{\boldsymbol{A}}.\overline{\boldsymbol{X}} = \overline{\boldsymbol{B}} - \overline{\boldsymbol{C}} \tag{11}$$

By using $\overline{\boldsymbol{A}}$, $\overline{\boldsymbol{B}}$ and $\overline{\boldsymbol{C}}$, we are to solve equation (11) for $\overline{\boldsymbol{X}}$.

5.1. Classical solution

We represent the classical solution of $\overline{\boldsymbol{A}}.\overline{\boldsymbol{X}} = \overline{\boldsymbol{B}} - \overline{\boldsymbol{C}}$ as $\overline{\boldsymbol{X}}_c$, if the solution exists.
Alter the α-cuts of $\bar{a}_{ij}$, $\bar{x}_i$, $\bar{b}_i$ and $\bar{c}_i$ for $\bar{a}_{ij}$, $\bar{x}_i$, $\bar{b}_i$ and $\bar{c}_i (1 \le i, j \le 2)$ respectively in the following system

$$\bar{a}_{11}\bar{x}_1 + \bar{a}_{12}\bar{x}_2 = \bar{b}_1 - \bar{c}_1, \tag{12}$$
$$\bar{a}_{21}\bar{x}_1 + \bar{a}_{22}\bar{x}_2 = \bar{b}_2 - \bar{c}_2, \tag{13}$$

After substituting the α-cuts of $\bar{a}_{ij}$, $\bar{x}_i$, $\bar{b}_i$ and $\bar{c}_i$ for $\bar{a}_{ij}$, $\bar{x}_i$, $\bar{b}_i$ and $\bar{c}_i (1 \le i, j \le 2)$ in the equations (12) and (13), we hereafter obtain the two-interval equations $\forall \alpha \in [0,1]$,

$$[a_{11L}(\alpha), a_{11U}(\alpha)].[x_{1L}(\alpha), x_{1U}(\alpha)] + [a_{12L}(\alpha), a_{12U}(\alpha)].[x_{2L}(\alpha), x_{2U}(\alpha)] = [b_{1L}(\alpha), b_{1U}(\alpha)] - [c_{1L}(\alpha), c_{1U}(\alpha)] \tag{14}$$

$$[a_{21L}(\alpha), a_{21U}(\alpha)].[x_{1L}(\alpha), x_{1U}(\alpha)] + [a_{22L}(\alpha), a_{22U}(\alpha)].[x_{2L}(\alpha), x_{2U}(\alpha)] = [b_{2L}(\alpha), b_{2U}(\alpha)] - [c_{2L}(\alpha), c_{2U}(\alpha)] \tag{15}$$

We now require simplifying these equations.
Consider all $\bar{a}_{ij}$, $\bar{b}_i$ and $\bar{c}_i$ are triangular fuzzy numbers and set α=1 in the preceding two equations (14) and (15). Then we get the following crisp system

$$a_{11}x_1 + a_{12}x_2 = b_1 - c_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2 - c_2$$

The sign of the unknown fuzzy numbers $\bar{x}_1, \bar{x}_2$ can be determined by the sign of the solutions x_1, x_2. Let us consider for this work that all the $\bar{a}_{ij} > 0$ and all the $\bar{b}_i > 0, \bar{c}_i > 0$, so that we attempt for $\bar{x}_i > 0, i = 1,2$. We get from equations (14) and (15)

$$[a_{11L}(\alpha).x_{1L}(\alpha), a_{11U}(\alpha).x_{1U}(\alpha)] + [a_{12L}(\alpha).x_{2L}(\alpha), a_{12U}(\alpha).x_{2U}(\alpha)] = [b_{1L}(\alpha) - c_{1U}(\alpha), b_{1U}(\alpha) - c_{1L}(\alpha)]$$
$$[a_{21L}(\alpha).x_{1L}(\alpha), a_{21U}(\alpha).x_{1U}(\alpha)] + [a_{22L}(\alpha).x_{2L}(\alpha), a_{22U}(\alpha).x_{2U}(\alpha)] = [b_{2L}(\alpha) - c_{2U}(\alpha), b_{2U}(\alpha) - c_{2L}(\alpha)]$$

Which results in a 4×4 crisp system of linear equations as

$$a_{11L}(\alpha).x_{1L}(\alpha) + a_{12L}(\alpha).x_{2L}(\alpha) = b_{1L}(\alpha) - c_{1U}(\alpha) \tag{16}$$
$$a_{21L}(\alpha).x_{1L}(\alpha) + a_{22L}(\alpha).x_{2L}(\alpha) = b_{2L}(\alpha) - c_{2U}(\alpha) \tag{17}$$
$$a_{11U}(\alpha).x_{1U}(\alpha) + a_{12U}(\alpha).x_{2U}(\alpha) = b_{1U}(\alpha) - c_{1L}(\alpha) \tag{18}$$
$$a_{21U}(\alpha).x_{1U}(\alpha) + a_{22U}(\alpha).x_{2U}(\alpha) = b_{2U}(\alpha) - c_{2L}(\alpha) \tag{19}$$

We solve this system for $x_{iL}(\alpha)$ and $x_{iU}(\alpha), i = 1,2, \alpha \in [0,1]$.
This system can be written using matrix notation as

$$\begin{bmatrix} a_{11L}(\alpha) & a_{12L}(\alpha) & 0 & 0 \\ a_{21L}(\alpha) & a_{22L}(\alpha) & 0 & 0 \\ 0 & 0 & a_{11U}(\alpha) & a_{12U}(\alpha) \\ 0 & 0 & a_{21U}(\alpha) & a_{22U}(\alpha) \end{bmatrix} \begin{bmatrix} x_{1L}(\alpha) \\ x_{2L}(\alpha) \\ x_{1U}(\alpha) \\ x_{2U}(\alpha) \end{bmatrix} = \begin{bmatrix} b_{1L}(\alpha) - c_{1U}(\alpha) \\ b_{2L}(\alpha) - c_{2U}(\alpha) \\ b_{1U}(\alpha) - c_{1L}(\alpha) \\ b_{2U}(\alpha) - c_{2L}(\alpha) \end{bmatrix}$$

Using,

$$\boldsymbol{W} = \begin{bmatrix} a_{11L}(\alpha) & a_{12L}(\alpha) & 0 & 0 \\ a_{21L}(\alpha) & a_{22L}(\alpha) & 0 & 0 \\ 0 & 0 & a_{11U}(\alpha) & a_{12U}(\alpha) \\ 0 & 0 & a_{21U}(\alpha) & a_{22U}(\alpha) \end{bmatrix}, \boldsymbol{S} = \begin{bmatrix} x_{1L}(\alpha) \\ x_{2L}(\alpha) \\ x_{1U}(\alpha) \\ x_{2U}(\alpha) \end{bmatrix}$$

And

$$\boldsymbol{V} = \begin{bmatrix} b_{1L}(\alpha) - c_{1U}(\alpha) \\ b_{2L}(\alpha) - c_{2U}(\alpha) \\ b_{1U}(\alpha) - c_{1L}(\alpha) \\ b_{2U}(\alpha) - c_{2L}(\alpha) \end{bmatrix}$$

A crisp system is obtained of the form

$$\boldsymbol{W}.\boldsymbol{S} = \boldsymbol{V} \tag{20}$$

For attaining the fuzzy solution for the fully fuzzy linear system of equations, in the converted crisp system the coefficient matrix must be non-singular $\forall \alpha \in [0,1]$.
We partition the system (20) into two systems as

$$\begin{bmatrix} a_{11L}(\alpha) & a_{12L}(\alpha) \\ a_{21L}(\alpha) & a_{22L}(\alpha) \end{bmatrix} . \begin{bmatrix} x_{1L}(\alpha) \\ x_{2L}(\alpha) \end{bmatrix} = \begin{bmatrix} b_{1L}(\alpha) - c_{1U}(\alpha) \\ b_{2L}(\alpha) - c_{2U}(\alpha) \end{bmatrix}$$

and

$$\begin{bmatrix} a_{11U}(\alpha) & a_{12U}(\alpha) \\ a_{21U}(\alpha) & a_{22U}(\alpha) \end{bmatrix} . \begin{bmatrix} x_{1U}(\alpha) \\ x_{2U}(\alpha) \end{bmatrix} = \begin{bmatrix} b_{1U}(\alpha) - c_{1L}(\alpha) \\ b_{2U}(\alpha) - c_{2L}(\alpha) \end{bmatrix}$$

That is, $\boldsymbol{W_L}.\boldsymbol{S_L} = \boldsymbol{V_L}$ and $\boldsymbol{W_U}.\boldsymbol{S_U} = \boldsymbol{V_U}$**.** The solution of these crisp systems specifies $x_{iL}(\alpha)$ and $x_{iU}(\alpha), i = 1,2, \alpha \in [0,1]$, which are used for reconstruction of the components of 2×1 fuzzy vector $\overline{\boldsymbol{X}}$.

After working out for the $x_{iL}(\alpha)$ and $x_{iU}(\alpha), i = 1,2\ \alpha \in [0,1]$ we test to see if the intervals $\bar{x}_i = [x_{iL}(\alpha),\ x_{iU}(\alpha)],\ \ i = 1,2, \alpha \in [0,1]$ define continuous fuzzy numbers for $i = 1,2$.
What is required is:

1. $\frac{\partial}{\partial \alpha}(x_{iL}(\alpha)) > 0$,
2. $\frac{\partial}{\partial \alpha}(x_{iU}(\alpha)) < 0$, and
3. $x_{iL}(1) \leq x_{iU}(1)$ for $i = 1,2$.

6. Numerical illustration

Price	Quantity Supplied	Quantity Demanded
1	10	50
2	20	40
3	30	30
4	40	20
5	50	10

From this table, we can easily find out the linear demand and linear supply equations.

Demand equation:

The general linear demand equation is $Q_d = a + bp$

In the linear demand equation, the term b is used as the price coefficient of demand or, the amount by which quantity will fall for every unit increase in price.

Here, $b = \frac{\Delta Q}{\Delta P} = \frac{10-20}{5-4} = -10$

At price $p = 3$, $Q_d = 30$, so, $a = 60$

Now we have the linear demand equation

$$Q_d = 60 - 10p \tag{21}$$

Supply equation:

The general linear supply equation is $Q_s = c + dp$

In this equation, the price coefficient of supply,$d = \frac{\Delta Q}{\Delta P} = \frac{50-40}{5-4} = 10$

In the linear supply equation, the term c reflects the quantity intercept of the supply. c will always be 0. In the case of subsidy, c may be positive.

At price $p = 3$, $Q_s = 30$, so, $c = 0$.

So, the linear supply equation

$$Q_s = 10p \tag{22}$$

Consider the fuzzy form of the linear demand and supply equation

$$Q_d = 60 - 10p$$
$$Q_s = 10p$$

as

$$\left.\begin{array}{c}[q_1, q_2] + (8,10,12)[p_1, p_2] = (50,60,70) \\ [q_1, q_2] - (8,10,12)[p_1, p_2] = (-1,0,1)\end{array}\right\} \tag{23}$$

We find the solution of these fuzzy matrix equations using the **classical method.**

Here $\bar{a}_{11} = 1$, $\bar{a}_{12} = (8,10,12), \bar{a}_{21} = 1, \bar{a}_{22} = -(8,10,12)$. Also, we have, $\bar{b}_1 = (50,60,70)$, $\bar{b}_2 = (-1,0,1)$.

Then the α-cuts are:

$$\bar{a}_{12}[\alpha] = [a_{12L}(\alpha), a_{21U}(\alpha)] = [8 + 2\alpha, 12 - 2\alpha],$$
$$\bar{a}_{22}[\alpha] = [a_{22L}(\alpha), a_{22U}(\alpha)] = -[8 + 2\alpha, 12 - 2\alpha],$$
$$\bar{b}_1[\alpha] = [b_{1L}(\alpha), b_{1U}(\alpha)] = [50 + 10\alpha, 70 - 10\alpha],$$
$$\bar{b}_2[\alpha] = [b_{2L}(\alpha), b_{2U}(\alpha)] = [-1 + \alpha, 1 - \alpha],\ \forall \alpha \in [0,1].$$

Now substituting the α-cuts of $\bar{a}_{12}$, $\bar{a}_{22}$,$\bar{b}_1$, $\bar{b}_2$ for $\bar{a}_{12}$, $\bar{a}_{22}$,$\bar{b}_1$, $\bar{b}_2$ in the system (23), we get

$$\left.\begin{array}{c}[q_1, q_2] + [8 + 2\alpha, 12 - 2\alpha][p_1, p_2] = [50 + 10\alpha, 70 - 10\alpha] \\ [q_1, q_2] - [8 + 2\alpha, 12 - 2\alpha][p_1, p_2] = [-1 + \alpha, 1 - \alpha]\end{array}\right\} \tag{24}$$

The system (24) can be partitioned into two systems as

$$\left.\begin{aligned} q_1 + (8+2\alpha)p_1 &= 50 + 10\alpha \\ q_1 - (8+2\alpha)p_1 &= -1 + \alpha \end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned} q_2 + (12-2\alpha)p_2 &= 70 - 10\alpha \\ q_2 - (12-2\alpha)p_2 &= 1 - \alpha \end{aligned}\right\} \quad (26)$$

In matrix form, system (25) can be written as

$$\begin{pmatrix} 1 & 8+2\alpha \\ 1 & -8-2\alpha \end{pmatrix} \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} = \begin{bmatrix} 50+10\alpha \\ -1+\alpha \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} = \begin{pmatrix} 1 & 8+2\alpha \\ 1 & -8-2\alpha \end{pmatrix}^{-1} \begin{bmatrix} 50+10\alpha \\ -1+\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} q_1 \\ p_1 \end{bmatrix} = \begin{bmatrix} \dfrac{(-8-2\alpha)(-1+\alpha)}{-16-4\alpha} + \dfrac{(-8-2\alpha)(50+10\alpha)}{-16-4\alpha} \\ \dfrac{-1+\alpha}{-16-4\alpha} - \dfrac{50+10\alpha}{-16-4\alpha} \end{bmatrix}$$

In matrix form, system (26) can be written as

$$\begin{pmatrix} 1 & 12-2\alpha \\ 1 & -12+2\alpha \end{pmatrix} \begin{bmatrix} q_2 \\ p_2 \end{bmatrix} = \begin{bmatrix} 70-10\alpha \\ 1-\alpha \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} q_2 \\ p_2 \end{bmatrix} = \begin{pmatrix} 1 & 12-2\alpha \\ 1 & -12+2\alpha \end{pmatrix}^{-1} \begin{bmatrix} 70-10\alpha \\ 1-\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} q_2 \\ p_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(70-10\alpha)(-12+2\alpha)}{-24+4\alpha} + \dfrac{(1-\alpha)(-12+2\alpha)}{-24+4\alpha} \\ -\dfrac{70-10\alpha}{-24+4\alpha} + \dfrac{1-\alpha}{-24+4\alpha} \end{bmatrix}$$

Now we have,

$$[p_1, p_2](\alpha) = [\frac{-1+\alpha}{-16-4\alpha} - \frac{50+10\alpha}{-16-4\alpha}, -\frac{70-10\alpha}{-24+4\alpha} + \frac{1-\alpha}{-24+4\alpha}]$$

$$[q_1, q_2](\alpha) = [\frac{(-8-2\alpha)(-1+\alpha)}{-16-4\alpha} + \frac{(-8-2\alpha)(50+10\alpha)}{-16-4\alpha}, \frac{(70-10\alpha)(-12+2\alpha)}{-24+4\alpha} + \frac{(1-\alpha)(-12+2\alpha)}{-24+4\alpha}]$$

We here analyze as follows,

$\alpha \in [0,1]$	$p(\alpha)$		$q(\alpha)$	
	$p_1(\alpha)$	$p_2(\alpha)$	$q_1(\alpha)$	$q_2(\alpha)$
1	3	3	30	30
.8	3.03	2.97	28.9	31.1
.5	3.08	2.93	27.25	32.75
.3	3.12	2.90	26.15	33.85
0	3.18	2.87	24.5	35.5

7. Conclusion

Equilibrium is the state in which the supply and demand of a product or service make harmony with each other, and consequently, prices become stable. Any case in economics and finance related to linear equations can be worked out using the fuzzy form of the linear system if the data are inexact.

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