M.Sc. 2nd Semester Examination, 2015

PHYSICS

PAPER– PHS-201(A & B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PHS-201(A)

[*Marks* : 20]

(*Quantum Mechanics*-II)

Answer **Q. Nos. 1 & 2** and any **one** from the rest

1. Answer any *three* bits : 2 × 3

(*a*) A spin $\frac{1}{2}$ particle is in the state $\frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\2\end{pmatrix}$. What is the probability of getting $\frac{\hbar}{2}$ when we measure S_x ?

(Turn Over)

(*b*) If $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$ prove that

$$\frac{p^2}{2m} = \frac{p_r^2}{2m} + \frac{L^2}{2mr^2}.$$

(*c*) A hydrogen atom in its ground state is placed under electric field $\vec{E}$. Find the change in its eigenvalue in the second order.

$$\left(\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}\right).$$

(*d*) $S_r = \frac{\vec{S} \cdot \vec{r}}{r}$ is the component of electron spin in the direction of $\vec{r}$ commutes with each component of total angular momentum $\vec{J} = \vec{L} + \vec{S}$, then prove that

$$\left[\vec{S}, S_r\right] = i\hbar \frac{\vec{r} \times \vec{S}}{r}.$$

(*e*) If $\hat{H}_D = C\alpha\hat{p}_z + \beta mc^2 + V(z)$ be the one-dimensional Dirac Hamiltonian, then prove that

$$[\hat{\sigma}, \hat{H}_D] = 0.$$

2. Answer any *one* bit : 4

(*a*) Prove that $\left[\vec{a}\cdot\vec{L}, \vec{b}\cdot\vec{L}\right] = i\hbar\left(\vec{a}\times\vec{b}\right)\cdot\vec{L}$ under the assumption that $\vec{a}$ and $\vec{b}$ commute with each other and with $\vec{L}$.

(*b*) Prove that Dirac equation in e.m. field is of the form

$$\left[(i\partial - eA)^2 - \frac{e}{2}\sigma^{\mu\nu} F_{\mu\nu} - m^2\right]\psi = 0$$

where $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right]$ and field strength $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$.

3. (*a*) A spin $\frac{1}{2}$ particle in presence of a static magnetic field along the z and x directions

$$\vec{B} = B_z\hat{e}_z + B_x\hat{e}_x \cdot$$

(*i*) Show that the Hamiltonian is

$$\hat{H} = \hbar w_0 \hat{\sigma}_z + \frac{\hbar\Omega}{2}\hat{\sigma}_x$$

where $\hbar w_0 = \mu_B B_z$ and $\hbar\Omega_0 = 2\mu_B B_x$.

(*ii*) Now put $B_x = 0$ and immediately a weak field $B_x << B_z$ is applied. Use perturbation theory to find the new eigenvectors and eigenvalues to the lowest order in $\frac{B_x}{B_z}$ for the second order shift in eigenvalues. 2 + 3

(*b*) Use a trial function $\psi(r) = re^{-\alpha r}$, α being a variable parameter and $V(r) = -\frac{1}{r}$. Calculate the energy of the hydrogen atom. To which eigenvalue does it approximate ? (Use atomic unit). 4 + 1

4. (*a*) If radial momentum p_r and radial velocity α_r for an electron in a central potential are defined by

$$p_r = \frac{\vec{r}\cdot\vec{p} - i\hbar}{r}, \quad \alpha_r = \frac{\vec{\alpha}\cdot\vec{r}}{r}$$

(*i*) Show that

$$\vec{\alpha}\cdot\vec{p}=\alpha_r p_r+\frac{i\hbar K\beta\alpha_r}{r}$$

where $K=\frac{p(\vec{\sigma}^d\cdot\vec{L}+\hbar)}{\hbar}$.

(*ii*) Obtain eigenvalue of the operator K. 3 + 2

(*b*) Show that spin magnetic moment appears as a natural consequence for Dirac electron in an e.m. field. 5

PHS-201(B)

[*Marks* : 20]

(*Methods of Mathematical Physics*-II)

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 × 5

(*a*) Find the Laplace transform of the waveform

$$f(t)=\frac{2t}{3},\ 0\le t\le 3.$$

(*b*) Find the F.T. of a rectangular pulse of duration 2 secs. and having a magnitude of 10 volts.

(*c*) Prove that

$$\delta(x)\,\delta(y) = \frac{\delta(r)}{2\pi r}.$$

(*d*) If a group is defined as

$$a * b = a + b - 1$$

Find the inverse of the group.

(*e*) If $\hat{T}(\phi)\, f(x, y) = f(x\cos\phi + y\sin\phi,$

$$-x\sin\phi + y\cos\phi)$$

Find the generator of this Lie group.

(*f*) State and prove Lagrange's theorem for group.

(*g*) Find the inverse F. T. of $\delta(w)$.

(*h*) Define structure constants of the Lie group with examples.

2. (*a*) Find the F. T. of $f(x) = e^{-|x|}$ using this prove that

$$\int_0^\infty \frac{\cos Kx}{1+K^2}\, dK = \frac{\pi}{2} e^{-|x|}.$$ 3

(*b*) Find the Green's function for the boundary value problem 3

$$\frac{d^2y}{dx^2} + k^2 x = f(x),$$

$$y(0) = 0$$

$$y(1) = 0.$$

(*c*) If $S = \{1, 2, 3\}$ be a symmetric group with three numbers and $S'_3 = \{P_0, P_1, P_2, P_3, P_4, P_5\}$ be its permutation group and $H = \{P_0, P_3\}$ be the subgroup of S'_3. Find three distinct left cosets of H in S'_3 find also the conjugate subgroup S'_3. 4

3. (*a*) Solve the integral equation

$$y(x) - \int_0^1 (2x - t)\, y(t)\, dt = \cos 2\pi x.$$

(*b*) Find particular integral of

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16\log(x+2y).$$

(*c*) For a group T_d reducible representation for character is given below :

T_4 :	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$
	4	1	0	0	2

The character table is given below for T_d :

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
T_1	3	0	−1	1	−1
T_2	3	0	−1	−1	1

Find the number of irreducible representations in T_4 and show that $T_r = A_1 \oplus T_2$.

3 + 3 + 4