PG/IVS/PHS-401/15

M.Sc. 4th Semester Examination, 2015

PHYSICS

PAPER – PHS-401 (A + B)

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PHS-401(A)

(*Relativity*)

[*Marks* : 20]

Answer **Q.No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 × 5

(*a*) Prove that $\left\{ \begin{matrix} p \\ p\,q \end{matrix} \right\} = \frac{\partial}{\partial x^q} \ln \sqrt{g}$.

(*Turn Over*)

(*b*) Prove that δ^p_q is a mixed tensor of second rank.

(*c*) What do you mean by Schwarzschild singularity ? Hence discuss the 'event horizon' of a black hole.

(*d*) Explain why relativistic corrections are not significant for white dwarfs, but are important in the case of neutron stars.

(*e*) Show that in Galilean co-ordinates, the energy-momentum tensor is given by

$$T^{\mu\nu} = \rho \frac{dx^{\mu}}{dS} \frac{dx^{\nu}}{dS}$$

where ρ is the co-ordinate density of matter.

(*f*) Explain the evolution of a star as a function of mass on the HR diagram.

(*g*) What are the physical processes that determine the mass of a star ? Explain for both lower and higher masses.

(*h*) How long does it take a molecular cloud (with density 10^{-17} gm/cm^3) to collapse under gravity ?

2. (*a*) If the line element

$$dS^2 = e^{\nu}\, dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$

where $\lambda = \lambda(r)$
$\nu = \nu(r)$

Then prove that curvature tensor

$$R_{11} = \frac{1}{2}\nu'' + \frac{1}{4}\nu'^2 - \frac{1}{4}\nu'\lambda' - \frac{\lambda'}{r}.$$

where $\nu' = \frac{d\nu}{dr}; \lambda' = \frac{d\lambda}{dr}$.

(*b*) A star has luminosity is 100 $L_{\odot}$ and apparent bolometric magnitude, $m_{\text{star}}^{\text{bol}} = 9 \cdot 7$, If the sun has $m_{\odot}^{\text{bol}} = +4 \cdot 7$, Calculate the distance of the star. Discuss 'Chandrasekhar Mass Limit' for a white dwarf. 10

3. (*a*) Find the differential equations for the geodesics in spherical co-ordinates.

(*b*) From a study of Newtonian Theory of Stellar equilibrium, Show that if $\gamma = \frac{4}{3}$ instability might set in the interior of the star.

$$\left(\gamma = \frac{C_P}{C_V}\right).$$

5 + 5

PHS-401 (B)

(*Statistical Mechanics*-II)

[*Marks* : 20]

Answer **Q.No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 × 5

(*a*) How many photons are there in 1c.c. of radiation at 10^3 K ? Find their average energy. Given

$$\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2 \cdot 405$$

(*b*) Draw the temperature dependence of fugacity for BE and FD distribution.

(*c*) For a non-relativistic electron gas prove that $P_e \alpha \rho^{5/3}$ and $P_{\text{gravity}} \alpha \rho^{4/3}$.

(*d*) Write down the equation of state for Bose and Fermi gas.

(*e*) Draw temperature (T) *vs* entropy (S) and specific heat (C_p) for second-order phase transition and λ-transition.

(*f*) Define spin-spin correlation function $G(i, i+1)$. What is its important in magnetic susceptibility.

(*g*) Consider a system of Bosons, each with the following properties :
Chemical potential = 0, energy $\varepsilon = \hbar w$; density of states varies as $w^{1/2}$. Show that specific heat of the system varies as $T^{3/2}$.

(*h*) Define critical exponents and Rushbrook inequality.

2. (*a*) Show that for a two-dimensional electron gas, the number of electrons per unit area is given by

$$n = \frac{4\pi m K_B T}{h^2} \ln\left(e^{E_F/K_B T} + 1\right).$$

(*b*) In an Ising model (one-dimensional) Hamiltonian

$$H = -J_1 \sum_{i=1}^{N-1} S_i S_{i+1} - J_2 \sum_{i=1}^{N-2} S_i S_{i+2}.$$

(*i*) Find the partition function in the limit $N \to \infty$.

(*ii*) Show that correlation function $< S_i S_{i+1} >$ in the limit $N \to \infty$ is given by

$$\frac{\sin h(\beta J_1)}{\sqrt{\sin h^2(\beta J_1) + e^{-4\beta J_2}}}.$$

4 + 3 + 3

3. (*a*) Show that for a two-dimensional *B* - *E* gas, number of particles per unit area.

$$N = \frac{2\pi m K_B T}{h^2} B_1(\alpha)$$

where $\alpha = -\mu\beta$; $\beta = \frac{1}{K_B T}$; other symbols have usual meanings. Can it undergo *B-E* condensation ?

(*b*) Find out the expression of the free energy of non-interacting electrons in a magnetic field. In what respect is the behaviour of this system different from a paramagnetic material and why ?

(*c*) Distinguish between He-I and He-II in the light of two fluid model. 4 + 4 + 2
