M.Sc. 2nd Semester Examination, 2014

PHYSICS

PAPER – PHS-201(A + B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

[*Marks* : 20]

Time : 1 hour

Answer **Q. Nos. 1** and **2** and any **one** from the rest

1. Answer any *three* bits : 2×3

(*a*) Simplify $\left(\vec{\sigma}.\vec{A}\right)\left(\vec{\sigma}.\vec{B}\right)$.

(*b*) A particle is in an eigenstate of J_z. Find the value of $\langle j,m|J_x|j,m\rangle$.

(*c*) An electron is in the *p* state with orbital angular momentum quantum number $l = 1$, and spin angular quantum number $s = 1/2$. Find the wave function for the state

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle \text{ and } \left|\frac{1}{2},\frac{1}{2}\right\rangle$$

using Clebsch Gordan coefficients.

(*d*) Obtain the eigenvalues of the operator

$$k = \frac{\beta(\overrightarrow{\sigma^d}.\vec{L} + \hbar)}{\hbar}$$

(*e*) Consider a system in the unperturbed state described by the Hamiltonian

$$H = \begin{bmatrix} E_0 & 0 \\ 0 & E_0 \end{bmatrix}.$$

The system is subjected to a perturbation of the form

$$H' = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$$

Where A and B are very small quantities in comparison to E_0. Find the energy eigenvalues of the perturbed system.

2. Answer any *one* bit : 4

(*a*) Write Dirac Hamiltonian for an electron in an electromagnetic field. Show that the equation in the non-relativistic limit contain terms which represent the interaction potential of magnetic moment due to both orbital and spin motion with the magnetic field.

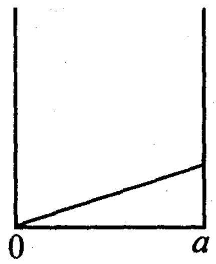
(*b*) A particle of mass m is bound by the potential $V(r) = -V_0 e^{-r/a}$. Using the variation method with a trial wave function $\psi(r) = Ne^{-br}$ find the ground state energy.

3. (*a*) Establish the stationary perturbation theory for non-degenerate states and obtain the first and second order correction to energy level and first order correction of the wave function.

(*b*) In an infinite square well potential with walls at 0 and a, a portion OAB has been sliced off as shown in the picture. The sliced portion is represented by a perturbing potential

$$H' = \frac{V_0 x}{a}.$$

Find the change in the ground state energy level to first order in the perturbing potential.



10

4. Obtain the plane-wave solution for the spin half particle in the relativistic formalism. Write the four component wave functions corresponding to the $\pm$ energy and the two spin states in matrix form. 10

GROUP – B

[*Marks* : 20]

Time : 1 hour

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* from the following : 2×5

(*a*) Find the Laplace transform of the triangular wave of period $2a$ given by

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$$

(*b*) Find the Fourier sine transform of $\dfrac{e^{-ax}}{x}$.

(*c*) Solve the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} + z = 0,$$

given that when $x = 0, z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$.

(*d*) Define Lie group.

(*e*) Write a short note on direct product of two groups.

(*f*) Write down the self adjoint differential operator in one and three dimensions.

(*g*) If $G = \{1, i, -1, -i\}$ be a group then find the class of G.

(*h*) Prove that equivalent representations have the same set of character.

2. (*a*) Show that the function $y(x) = (1 + x^2)^{-3/2}$ is a solution of the volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2}\, y(t)\, dt$$

(*b*) Use Laplace transform to solve $ty'' + 2y' + ty = \cos t$ given that $y(0) = 1$. 5 + 5

3. (*a*) Find the Green's function for

$$\frac{d^2y(x)}{dx^2}+y(x)=0,$$

subject to the conditions $y(0)=0$ and $y'(1)=0$.

(*b*) Construct the character table for the group D_3. 5 + 5