2008

PHYSICS

PAPER—PH 1202 A & B

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

(*Classical Mechanics*)

[*Marks* : 20]

Answer *all* questions

1. Answer any *two* of the following: 2×2

(*a*) Prove that if the transformation equations are given by $r_v = r_v(q_1, q_2 \ldots q_n)$, i.e. do not involve the time t explicitly, then the kinetic energy can be written as

$$T = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} a_{\alpha\beta}\, \dot{q}_\alpha\, \dot{q}_\beta$$

where $a_{\alpha\beta}$ are functions of q_α.

(*b*) Prove that if a generalised coordinate is cyclic in the Lagrangian it should also be cyclic in the Hamiltonian.

(*c*) A particle of mass m moves in a force field whose potential in spherical co-ordinates is $V = -(K\cos\theta)/r^2$. Write the Hamilton - Jacobi equation describing its motion.

2. Answer any *two* of the following: 3×2

(*a*) Let U be a generating function dependent only on Q_α, p_α, t. Prove that

$$P_\alpha = -\frac{\partial U}{\partial Q_\alpha},\ q_\alpha = -\frac{\partial U}{\partial p_\alpha},\ K = \frac{\partial U}{\partial t} + H.$$

(*b*) Is the Lagrangian of a dynamical system unique ? Explain.

(*c*) Assuming the canonical invariance of Poisson bracket prove that

$$[F, G]_{q, p} = [F, G]_{Q, P}.$$

3. Answer any *one* of the following: 10 x 1

(*a*) Use Hamilton - Jacobi method for solving the motion of a mechanical system with the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}Kq^2.$$

What is the physical significance of Hamilton's principal function ? Starting from the time dependent Schrödinger equation obtain the Hamilton - Jacobi equation. 5 + 2 + 3

(*b*) Prove that Poisson bracket

$$[p_\alpha, q_\beta] = \delta_{\alpha\beta} = 1 \quad \text{if } \alpha = \beta$$

$$= 0 \quad \text{if } \alpha \neq \beta$$

l_x, l_y, l_z are the three components of the angular momentum of a particle, find the following Poisson brackets:

$$[p_x, p_y], [l_x, p_y] \text{ and } [l_z, l_x].$$

Find the normal modes of vibration of freely vibrating linear triatomic molecules. (Neglect the interaction between the end atoms). 2 + 4 + 4

GROUP—B

(*Solid State Physics*)

[*Marks* : 20]

Answer **Q. No. 1** and any *one* from the rest

1. Answer any *four* bits: $2\frac{1}{2} \times 4$

(*i*) Prove that effective no. of free electrons is maximum in a solid when the outermost band is half filled.

(*ii*) State Matthiessen's rule. From this, comment about the temperature variation of resistivity at low and high temperatures.

(*iii*) Calculate the magnetic susceptibility of a perfect diamagnetic substance.

(*iv*) Give the physical significance of the negative effective mass of electron near the top of the band.

(*v*) Find the expression of Fermi energy in an intrinsic semiconductor at $T > 0°$ K. What is law of mass action ?

(*vi*) Explain with neat diagrams how Pauli's paramagnetism is developed in a metal at $T = 0°$K with application of external magnetic field.

2. Describe the essential features of Kronig penny model in details. What is reduced zone scheme ? 8 + 2

3. (*a*) Deduce an expression of the Fermi energy and the density of states for the 3D electron gas. Plot the density of states as a function of energy.

(*b*) What is Hall effect ? Find an expression of Hall coefficient in a metal. (3 + 1 + 1) + 5
