MSC/IIS/PHY/PH1201
A & B/08

2008

PHYSICS

PAPER—PH 1201 A & B

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

(*Quantum Mechanics*)

[*Marks* : 20]

Answer **Q. No. 1 & 2** and any *one* from the rest

1. Answer any *two* bits: 2×2

(*a*) Write expression for L^2 and L_Z in spherical polar co-ordinates.

(*b*) Justify that the linear momentum operator is the generator of space translations of a quantum mechanical system.

(*c*) The sum of two angular momenta $\vec{J_1}$ and $\vec{J_2}$ are given by $\vec{J} = \vec{J_1} + \vec{J_2}$. If the eigenkets of J_1^2 and J_2^2 are $|j_1 m_1\rangle$ and $|j_2 m_2\rangle$ respectively, find the number of eigenstates of J^2.

(*d*) Which of the following transitions are electric dipole allowed?

(*i*) $1s \rightarrow 2s$

(*ii*) $1s \rightarrow 2p$

(*iii*) $2p \rightarrow 3d$

(*iv*) $3s \rightarrow 5d$.

2. Answer any *two* bits: 3x2

(*a*) Obtain the matrix of Clebesch Gordan coefficients for $j_1 = 1$ and $j_2 = 1/2$.

(*b*) Evaluate the first and second order corrections to the energy of the $n = 1$ state of an oscillator of mass m and angular frequency ω subjected to the potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 + bx$$

where b is independent of x. Given the matrix elements of x on the basis set of harmonic oscillator eigenfunctions are

$$\langle k | x | n \rangle = \begin{cases} \sqrt{\dfrac{\hbar(n+1)}{2m\omega}} & \text{if } k = n+1 \\ \sqrt{\dfrac{\hbar n}{2m\omega}} & \text{if } k = n-1 \\ 0 & \text{otherwise} \end{cases}$$

(c) Let $| jm \rangle$ be a simultaneous eigenvector of J^2 and J_Z with the eigenvalues $j(j+1)\hbar$ and $m\hbar$ respectively. Show that

$$J_{\pm} |jm\rangle = [(j \mp m)(j \pm m + 1)]^{1/2} \hbar | j \cdot m \pm 1 \rangle$$

3. (a) Write the Hamiltonian for a charged particle in an electromagnetic field.

(b) Simplify the Hamiltonian keeping only up to first order in the vector potential.

(c) Use the time dependent perturbation theory and obtain an expression for transition probability.

(d) What is dipole approximation? Write the selection rules under this approximation. 10

4. (a) State and explain the variational principle to calculate the ground state energy.

(*b*) Using variation method estimate the ground state energy of the Helium atom use the following:

The ground state wave function of hydrogen like atom with atomic number Z

$$\phi_0 = \left(\frac{Z^3}{\pi a_0^3}\right)^{1/2} e^{-\frac{Z}{a_0}r_0};$$

$$\langle \phi_0 | \left(\frac{-\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{4\pi\epsilon_0 r}\right) | \phi_0 \rangle = \frac{Z^2 e^2}{8\pi\epsilon_0 a_0}$$

$$= Z^2 (-13 \cdot 6) \text{ eV.}$$ 10

GROUP—B

(*Methods of Mathematical Physics*)

[*Marks* : 20]

Answer *all* questions

1. Answer any *five* bits: 2×5

(*a*) Define Lie group and give an example of the Lie group.

(*b*) Prove that the set containing the fourth roots of unity forms a cyclic group.

(*c*) Define with example cosets of a subgroup in a group.

(*d*) Find the Laplace transform of Dirac Delta function.

(*e*) What do you mean by Integral transform and hence define Laplace transform ?

(*f*) Find the Fourier transform of

$$f(x) = e^{-x^2}.$$

(*g*) Find Inverse Laplace transform of

$$f(p) = \frac{3+4p}{9p^2-16} + \frac{6}{2p-3}.$$

(*h*) If $u = f(x+iy) + g(r-iy)$, where f and g are arbitrary functions of x and y, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

2. Answer any *two* bits : 5x2

(*a*) (*i*) Show that Laplace transform of

$$f(t) = t^2 \cos at = \frac{2p(p^2 - 3a^2)}{(p^2 + a^2)^3}, \; p > 0.$$

(*ii*) Solve the differential equation using integral transform,

$$(D^2 - 2D + 2)y = 0, \quad y = Dy = 1,$$

when $t = 0$.

(*b*) Solve the one dimensional heat conduction equation

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\nu} \frac{\partial \theta}{\partial t}$$

where θ represents the temperature and $\nu = \frac{K}{CP}$, K, C and P are thermal conductivity, specific heat and density of the material respectively. The boundary conditions are given by $\theta(x = 0, \; t) = 0$, $\theta(x = a, \; t) = 0$. The initial temperature distribution is given by $\theta(x, \; t = 0) = f(x)$.

(c) (i) Prove that the set of all symmetry transformations of an equilateral triangle forms a group.

(ii) Define with examples Homomorphisms and Isomorphisms of groups.

(d) (i) Define covolution of two functions and state the convolution theorem.

(ii) Using convolution theorem, find Laplace Inverse of

$$f(p) = \frac{p}{(p^2 + a^2)^2}.$$

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