PG/IS/PHY/PH - 1102/09

2008

PHYSICS

PAPER—PH - 1102 (A+B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

[*Marks* : 20]

(*Quantum Mechanics* -I)

Answer **Q.No.1** and any *one* from the rest

1. Answer any *five* questions : 2 x 5

(*a*) Write down the postulates in Quantum Mechanics.

(*b*) Show that the eigenvalues of a Hermitian operator are real.

(*c*) Prove that, if an orthonormal set of kets $\{ | u_i >, i = 1, 2, \ldots \}$ forms a basis, then

$$\sum_i | u_i > < u_i | = 1 .$$

(*d*) Show that

$$[f(x) , \hat{p}] = i\hbar \frac{df(x)}{dx}.$$

(*e*) Using uncertainty relation find the ground state energy of a particle confined to one-dimensional motion with a potential energy $V(x) = bX^4$.

(*f*) A particle is confined to a box of length L with walls at $X = 0$ and L. The particle is described through a wave function

$$\psi = N \sin \frac{3\pi x}{2L} \cos \frac{\pi x}{2L}.$$

Find the energy of the particle.

(*g*) Write the energy and wave functions for the ground state of the harmonic oscillator. Show that the probability of finding the particle outside the classical limit is only 16%.

(*h*) Show that the transformation matrix which transforms from one basis set to the other is unitary.

2. (*a*) Discuss the equations of motion for both wave function and the operator in Interaction operator.

(*b*) Show that the product of uncertaintices associated with the measurement of variables represented through the two noncommutating hermitian operators is

$$\Delta A \cdot \Delta B \geq \frac{1}{2} | [A, B] |.$$

(*c*) The wave function of the hydrogen atom in a definite state is given as

$$\psi (r, \theta, \phi) = \text{Nr } \cos\theta \; e^{-r/2a_0}.$$

Find the most probable radius of the electron in this state. 3 + 4 + 3

3. Considering the operator

$$a = \frac{i}{(2 m\hbar\omega)^{1/2}} (\hat{p} - im\hat{x})$$

and its adjoint at prove that for a linear oscillator of Hamiltonian H,

$$H | n> = (n + \frac{1}{2}) \hbar\omega \; | n>.$$

Where the symbols have their usual meanings. Hence find the matrix form of a and a^+. 6 + 4

GROUP—B

[*Marks* : 20]

(*Crystallography and Lattice Dynamics*)

Answer **Q.No.1** & **2** and any *one* from the rest

1. Answer any *two* bits : 2x2

(*a*) Show that in a cubic crystal the [*hkl*] direction is normal to the (*hkl*) planes.

(*b*) Show that density of states becomes infinity at the zone boundary for a linear monatomic lattice.

(*c*) A plane makes intercepts of 1, 2 and 3 Å on the crystallographic axes of an orthorhombic crystal with $a : b : c = 3 : 2 : 1$. Determine the Miller indices of this plane.

2. Answer any *two* bits : 3x2

(*a*) Find the packing fraction of the HCP and diamond structures.

(*b*) Discuss the specific heat of a solid on the basis of the cut-off procedure suggested by Born.

(*c*) The Debye temperature of diamond is 2000 K. Calculate the mean velocity of sound in diamond, given the density and atomic mass of diamond as $3500\,kg\,m^{-3}$ and 12 amu respectively. If the interatomic spacing is 1·54 Å, estimate the frequency of the dominant mode of lattice vibration.

3. (*a*) Derive Bragg law from Laue equations.

(*b*) The Geometrical structure factor F_{hkl} for reflection from the (hkl) planes is

$$F_{hkl} = \sum_n f_n \exp\{2\pi i (hx_n + ky_n + lz_n)\}.$$

Hence show that the factor vanishes unless the numbers h, k, and l are all even or all odd for fcc lattice.

(*c*) Suppose that we allow two masses M_1 and M_2 in a one-dimensional diatomic lattice to become equal. What happens to the frequency gap? Compare the results with those of the monatomic lattice.

(*d*) In an assembly of 10^{23} S.H.O. each has frequency of 10^{13} Hz. Calculate the mean energy of the system (ignoring zero point energy) at 20 K. 2 + 3 + 3 + 2

4. (*a*) Show that for a lattice the rotation by $\frac{2\pi}{n}$ about an axis passing through a lattice point in a symmetry operation provided $n = 1, 2, 3, 4, 6$.

(*b*) Define a screw axis, give a diagram showing a 3_1 screw axis.

(*c*) Derive the expression for the thermal conductivity of solid.

(*d*) "Phonons are created simply by raising the temperature." Justify this statement. 3 + 2 + 4 + 1