2008

PHYSICS

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER—PH 2101 (A)

[*Marks* : 20]

(*Advance Quantum Mechanics*)

Answer any *two* questions

1. (*a*) Starting from given linear equation for relativistic free particle

$$(E - c\,\vec{\alpha}\,.\,\vec{p} - \beta m_0 c^2)\psi = 0,$$

(*Turn Over*)

show that the solution ψ is also a solution of Klein - Gordon equation provided that α and β are matrices. Find properties of α_i and β matrices. Find a representation for α and β matrices.

(*b*) Find the velocity operator for the Dirac Hamiltonian.

(*c*) If radial momentum p_r and radial velocity α_r for an electron in a central potential are defined by

$$p_r = \frac{\vec{r}\cdot\vec{p} - i\hbar}{r}, \quad \vec{\alpha}_r = \frac{\vec{\alpha}\cdot\vec{r}}{r}$$

show that $\vec{\alpha}\cdot\vec{p} = \alpha_r p_r + \dfrac{i\hbar k\beta\alpha_r}{r}$

where $k = \dfrac{\beta(\sigma^d\cdot\vec{L} + \hbar)}{\hbar}$. 5+2+3

2. (*a*) $P(\vec{r}, t)$ is defined as

$$P = \frac{i\hbar}{2mc^2}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$

where ψ is a solution for the free particle Klein – Gordon equation.

(*i*) If P satisfies the following equation :

$$\frac{\partial p}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

then find $\vec{J}$.

(*ii*) Using phase transformation

$$\tilde{\psi}(\vec{r}, t) = \psi(\vec{r}, t)\, e^{imc^2 t/\hbar}$$

(m = rest mass)

show that $P(\vec{r}, t)$ reduces to just $|\psi(\vec{r}, t)|^2$ in the non-relativistic limit.

(*b*) Define helicity operator. Using the eigenfunction of the Hamiltonian operator

$$\psi = \frac{1}{h^{3/2}} \sqrt{\frac{mc^2 + \lambda E_p}{2\lambda E_p}} \begin{pmatrix} u \\ \dfrac{c\vec{\sigma}.\vec{p}}{mc^2 + \lambda E_p} u \end{pmatrix} e^{\frac{i}{\hbar}\vec{p}.\vec{r}}$$

Show that the probable solutions of u are

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. (3 + 2) + 5

3. (*a*) Show that the spin orbit interaction term can be obtained automatically in the nonrelativistic limit of the Dirac equation.

(*b*) For a Dirac particle moving in a central potential show that the orbital angular momentum is not a constant of motion, rather the total angular momentum is conserved? 5 + 5

PAPER-PH 2101 (B)

[*Marks* : 20]

(*Statistical Mechanics*)

Attempt **Q.No. 1** and any *one* from the rest

1. Write explanatory notes on (any *four*): $2\frac{1}{2} \times 4$

(*a*) Degree of freedom, μ–phase space and *F* phase space.

(*b*) Density of states and probability density.

(*c*) Quantum mechanical average and ensemble average.

(*d*) Microstate and concept of ensemble.

(*e*) Thermodynamical equilibrium and fluctuation.

(*f*) Liouville's theorem and stationary ensemble.

2. Write down the single-dipole partition function for a magnetic system in terms of Lands of factor (g), Bohr magneton (μ_B), total angular momentum quantum number (j) as well as corresponding magnetic quantum number (m_j) and the magnetic field (H) in which it is placed. Use this partition function to obtain the average of dipole magnetic moment μ_z such that

$$\bar{\mu}_z = g\mu_B\, j \;\; B_j(x)$$

where $B_j(x)$ is the Brillouin function of order j and $x = (g\mu_B\, jH/\text{kT})$ so that for high temperature and weak fields the comic constant is given by

$$C_j = \frac{N_0\, g^2\, \mu_B^2\, j(j+1)}{3k}.$$

when N_0 is the Avogadro number and k the Boltzmann constant. 2 + 6 + 2

3. (*a*) Considering photons as an ideal Bose gas, show that the radiation pressure is (1/3) times its energy density.

(*b*) Spontaneous magnetisation exists only at zero temperature in a one-dimensional Ising model (without magnetic field). Why ?

(*c*) At zero temperature all spins align in a one dimentional Ising model. What is the nature, energy and entropy of the first excited state ? 4 + 3 + 3
