Chapter 8

A soft set based VIKOR approach for some decision-making problems under complex neutrosophic environment

[1]

8.1 Introduction

One of the important and challenging issues in our day-to-day life is the presence of uncertainty to be addressed. Neutrosophic set theory [149, 150] is one of the innovative generalizations of fuzzy set theory [183] where, three independent membership grades (truth membership ($T(x)$), indeterminate membership ($I(x)$) and false membership ($F(x)$) where, $T(x), I(x), F(x) \in [0, 1]$ and $0 \leq T(x) + I(x) + F(x) \leq 3$) have been considered together. In the last few decades, neutrosophic set has been improved widely and further successfully used to solve various types of real-life problems. Authors can follow the references [166, 181, 189] to study the applications of neutrosophic set in real-life

Furthermore, Romot et al. [137] extended fuzzy membership to complex membership ($\mu_C(x)$) by adding a second dimension $u(x)$ as, $\mu_C(x) = r(x)e^{iu(x)}$ and introduced the concept of *complex fuzzy set*, where, $r(x) \in [0, 1]$ is the amplitude term and $u(x)$, a real

[1] This chapter has been published in **Engineering Applications of Artificial Intelligence, 89 (2020): Article ID-103432, ELSEVIER.**

number, is the phase term. In many different fields where, adding a second decision information of an object over an attribute is needed to get a better and accurate result, complex fuzzy set can be applied to solve these problems. Latter, Alkouri and Salleh [8] proposed complex intuitionistic fuzzy set by using both the membership degree and non membership degree in complex fuzzy sense. Then, Ali and Smarandache [12] introduced complex neutrosophic set by introducing each of truth membership, indeterminate membership and false membership in complex sense.

Besides, researchers generalized soft set theory [118] to several new fields such as, Fuzzy soft set theory [22] intuitionistic fuzzy soft set theory [105], neutrosophic soft set theory [24], etc. and used them for handling real-life related problems. Meanwhile, complex fuzzy valued generalization of soft set theory have also been developed by the researchers. For instance, Thirunavukarasu et al. [158] introduced the concept of complex fuzzy soft set theory by taking all the parameters in a soft set in complex fuzzy sense. Then, Selvachandran et al. [144] initiated complex vague soft set theory by taking all the parameters in a vague soft set in complex fuzzy sense. Moreover, they also proposed some different types of distance measures on complex vague soft sets [144] to solve pattern recognition problems. Furthermore, the relations between complex vague soft sets have been thrived by Selvachandran et al. [145]. In addition, Broumi et al. [29] introduced complex neutrosophic soft set theory by taking all the parameters in a soft set in complex neutrosophic sense.

After surveying the above literature, complex neutrosophic set theory, introduced by Ali and Smarandache [12], first motivates us to use this concept in soft set theory, developed by Molodtsov [118], to solve some real-life based decision making problems. Then, we have studied complex neutrosophic soft set in decision-making, proposed by Broumi et al. [29], from which some shortcomings have been raised in our mind.

- In a complex neutrosophic soft set based real-life decision-making problem, all the parameters may not be uniform in nature i.e., conflicting parameters may exist in the parameter set. But, in their approach, they do not give any process to handle this type of the parameters.

- Moreover, all the considered parameters in a decision-problem may not have equal weight. But, their approach can only be applied to the decision-making problems where all weights of the parameters are equal. So, when parameters will have unequal weights then, we can not embed their algorithm to solve a complex neutrosophic soft set based problem.

- Besides, the main flavor of complex neutrosophic soft set is in its additional phase term. Because, through this idea, we can handle more complicated real-life related problems having a second dimension. But, in their illustrated decision-making problem, they do not give any idea about the phase part.

Therefore, to fulfil these research gaps, we are motivated to develop some decision-making methods in soft set theory under complex neutrosophic environment to take a quality of decision from several types of real-life related problems. Moreover, to avoid complexity in solving real-life data based problems, sometimes, transformation from an uncertain value into a real value becomes very necessary. But, there is no such definition in literature for a complex neutrosophic number. Besides, VIKOR method ([124]) is a popular multi-criteria decision-making methodology which provides a compromise optimal solution of a decision-making problem. So, if this familiar optimization method is used in solving complex neutrosophic soft set based decision-making problems, then compromise optimal solution would be obtained as a solution instead of a general optimal solution.

From the above aforementioned analysis, our main contributions can be highlighted as following:

- A new definition of score function of a complex neutrosophic number has been proposed to transform a complex neutrosophic number into some real value in the interval $[0, 1]$.
- Two algorithmic approaches have been developed to solve complex neutrosophic soft set based decision-making problems by using VIKOR method.
- Some real-life related problems have been discussed and solved to illustrate the working progress of our proposed approach.

The outline of this chapter is as follows. In Section 8.2, some basic terminologies have been recalled. In Section 8.3 and Section 8.5, we have introduced some set-theoretic operations for complex neutrosophic sets and complex neutrosophic soft sets respectively. In Section 8.4, a new definition of score function of a complex neutrosophic number has been proposed. After that, we have provided two algorithms for complex neutrosophic soft set based decision-making problems as given in Section 8.6. Then, some applications of our proposed approaches including company's manager selection problem, sustainable manufacturing material selection problem and medical diagnosis problem etc. have been discussed and solved in Section 8.7. Finally in Section 8.8, we have discussed the sensitivity analysis, validity and effectiveness of our proposed approach. Section 8.9 contains the conclusion of this chapter.

8.2 Some basic relevant notions

(i) **Complex neutrosophic soft set (CNSS) [29].**

A pair $(f_{\tilde{C}_N}, E)$ is said to be a complex neutrosophic soft set (CNSS) over the universal set X if and only if, $f_{\tilde{C}_N}$ a function from the parameter set E to the set of all complex neutrosophic subsets of the set X i.e., $f_{\tilde{C}_N} : E \to \tilde{\tilde{\rho}}(X)$ where, $\tilde{\tilde{\rho}}(X)$ is the set of all complex neutrosophic subsets of the set X. Mathematical it can be defined as follows,

$$(f_{\tilde{C}_N}, E) = \{(e, f_{\tilde{C}_N}(e)) | \forall e \in E\},$$

where, $f_{\tilde{C}_N}(e)$ is the set of all complex neutrosophic e-approximate elements of the set X such that,

$$f_{\tilde{C}_N}(e) = \{(x_s, (T_f(e)(x_s), I_f(e)(x_s), F_f(e)(x_s))) \mid \forall\ x_s \in X\}$$
$$= \{(x_s, (p_f(e)(x_s)e^{iu_f(e)(x_s)}, q_f(e)(x_s)e^{iv_f(e)(x_s)}, r_f(e)(x_s)e^{iw_f(e)(x_s)}))\}.$$

Here, all the amplitude terms $p_f(e)(x_s), q_f(e)(x_s), r_f(e)(x_s)$ are limited in the closed interval $[0, 1]$ with $0 \leq p_f(e)(x_s) + q_f(e)(x_s) + r_f(e)(x_s) \leq 3$ and all the phase terms $u_f(e)(x_s),\ v_f(e)(x_s),\ w_f(e)(x_s)$ are limited in the closed interval $[0, 2\pi]$.

(ii) **Complement of a complex neutrosophic set [12, 185].**

The complement of a complex neutrosophic set $\tilde{C}_N = \{(x, (p(x)e^{iu(x)}, q(x)e^{iv(x)}, r(x)e^{iw(x)}))|x \in X\}$ over the universal set X is denoted by $\tilde{C}_N^c$ and is defined as follows:
$\tilde{C}_N^c = (r(x)e^{i(2\pi - u(x))}, (1 - q(x))e^{i(2\pi - v(x))}, p(x)e^{i(2\pi - w(x))})$.

(iii) **Distance of two complex neutrosophic sets [12].**

Let $\tilde{C}_A$ and $\tilde{C}_B$ be two complex neutrosophic sets over X where, $\forall x \in X$,
$\tilde{C}_A = \big(p_A(x)e^{iu_A(x)}, q_A(x)e^{iv_A(x)}, r_A(x)e^{iw_A(x)}\big)$,
$\tilde{C}_B = \big(p_B(x)e^{iu_B(x)}, q_B(x)e^{iv_B(x)}, r_B(x)e^{iw_B(x)}\big)$.
Then, the distance between $\tilde{C}_A$ and $\tilde{C}_B$ is defined as follows:

$$\begin{aligned} \tilde{d}_{\text{CN}}(\tilde{C}_A, \tilde{C}_B) \quad &= max(max(\sup_{x\in X}|p_A(x) - p_B(x)|, \sup_{x\in X}|q_A(x) - q_B(x)|, \\ &\sup_{x\in X}|r_A(x) - r_B(x)|), max(\frac{1}{2\pi}\sup_{x\in X}|u_A(x) - u_B(x)|, \\ &\frac{1}{2\pi}\sup_{x\in X}|v_A(x) - v_B(x)|, \frac{1}{2\pi}\sup_{x\in X}|w_A(x) - w_B(x)|)) \end{aligned}$$

(iv) **Normalization process [131].**

A multi-criteria decision-making (MCDM) problem contains a set of alternatives ($\{A_1, A_2, .., A_M\}$) and a set of corresponding parameters ($\{E_1, E_2, .., E_N\}$). The rating of the alternatives are evaluated based on the associated parameters. Let, F_{sj} be the rating of an alternative $A_s; s = 1, 2, .., M$ over a parameter $E_j; j = 1, 2, .., N$. Now, in a decision-making problem, not all the parameters may be based on the same environment, therefore, they may not be commensurable to one another. Then, to handle these parameters, normalization process can be used by which, all the parameters become dimension less. Some popular normalization techniques are as follows:

(1) **Linear sum normalization.** $R_{sj} = \frac{F_{sj}}{\sum\limits_{s=1}^{M} F_{sj}};\ j = 1, 2, .., N;\ s = 1, 2, .., M.$

(2) **Linear max normalization.** $R_{sj} = \frac{F_{sj}}{F_j^+};\ where,\ F_j^+$ is the ideal evaluation over e_j.

(v) **VIKOR method [124].**

VIKOR method is a well-known optimization technique which provides a compromise optimal solution by testing '*acceptable advantage*' and '*acceptable stability*' in decision-making. This method contains the following steps:
Step 1. Define the ideal solution $A^+ = \{F_1^+, F_2^+, .., F_N^+\}$, where, $F_j^+ = max_s\{F_{sj}\}$; $s = 1, 2, .., M; j = 1, 2, .., N.$
Step 2. Obtain the group of utility (utility measure) S_s and individual regret (regret measure) R_s of each of the alternatives as,

$$S_s = \sum_{j=1}^{N} w_j d(F_{sj}, F_j^+);\ R_s = max_{j=1}^{N} w_j d(F_{sj}, F_j^+).$$

$w = \{w_1, w_2, .., w_N\}$ be the weights of the parameters such that, $w_j \geq 0$ and $\sum\limits_{j=1}^{N} w_j = 1.$
Step 3. Then, determine the VIKOR index Q_s of an alternative A_s is defined as follows,

$$Q_s = \theta\ (\frac{S_s - min_s S_s}{max_s S_s - min_s S_s}) + (1 - \theta)\ (\frac{R_s - min_s R_s}{max_s R_s - min_s R_s})$$

where, $\theta \in [0, 1]$ is the weight of strategy of the maximum group of utility.
Step 4. Rank the alternatives based on the decreasing order of S_s, R_s and Q_s; $s = 1, 2, .., M.$
Step 5. Now, suppose The alternative A^1 is the best alternative based on the VIKOR index Q. Then, it will be a compromise optimal solution if the following two conditions are satisfied:
$(C1)$ *Acceptable advantage:* $Q(A^2) - Q(A^1) \geq DQ\ (a\ threshold\ value) = \frac{1}{M-1}$, where, A^2 is the second best alternative based on Q.
$(C2)$ *Acceptable stability:* The alternative A^1 is also the best for the both S and R.
If any one of $C1$ and $C2$ is fail, then a set of compromise optimal solutions will be obtained as a optimal solution as follows:
$\Rightarrow$ If the condition $C1$ is dissatisfied, then a set of M' number of alternatives $\{A^1, A^2, .., A^{M'}\}$ will be the compromise optimal solutions, where, $A^{M'}$ is obtained from $Q(A^{M'}) - Q(A^1) < DQ$ for the maximum value of M'.
$\Rightarrow$ If the condition $C2$ is not satisfied then, A^1 and A^2 both will be the optimal solutions.

8.3 Some operations on complex neutrosophic sets

Basic set-theoretic operations are very useful to deal with real-life problems. In 2009, Zhang et al. [185] (**Appendix B**) defined some set-theoretic operations for complex fuzzy sets. Now, we have extended these ideas to complex neutrosophic sets. Here, we have used Smarandache's [149, 150] neutrosophic operations (given in **Appendix A**) as follows:

Let, $\tilde{C}_A$ and $\tilde{C}_B$ be two complex neutrosophic sets over the universal set X where,
$\tilde{C}_A = (T_{\tilde{C}_A}, I_{\tilde{C}_A}, F_{\tilde{C}_A}) = (p_A e^{iu_A}, q_A e^{iv_A}, r_A e^{iw_A})$ and
$\tilde{C}_B = (T_{\tilde{C}_B}, I_{\tilde{C}_B}, F_{\tilde{C}_B}) = (p_B e^{iu_B}, q_B e^{iv_B}, r_B e^{iw_B})$.

Definition 8.1. *Complex neutrosophic intersection.*
Then, complex neutrosophic intersection of $\tilde{C}_A$ and $\tilde{C}_B$ is denoted by, $\tilde{C}_A \tilde{\cap}_N \tilde{C}_B$ and is defined as, $\tilde{C}_A \tilde{\cap}_N \tilde{C}_B = (T_{\tilde{C}_A \tilde{\cap}_F \tilde{C}_B}, I_{\tilde{C}_A \tilde{\cap}_F \tilde{C}_B}, F_{\tilde{C}_A \tilde{\cap}_F \tilde{C}_B})$ where, $\tilde{\cap}_F$ is the complex fuzzy intersection. Now,

- **Complex neutrosophic standard intersection.**
  Complex neutrosophic standard intersection of $\tilde{C}_A$ and $\tilde{C}_B$ is denoted by, $\tilde{C}_A \tilde{\cap}_N^{min} \tilde{C}_B = \big(T_{\tilde{C}_A \tilde{\cap}_F^{min} \tilde{C}_B}, I_{\tilde{C}_A \tilde{\cap}_F^{min} \tilde{C}_B}, F_{\tilde{C}_A \tilde{\cap}_F^{min} \tilde{C}_B}\big)$ and is defined as,
  $\tilde{C}_A \tilde{\cap}_N^{min} \tilde{C}_B = \big(min(p_A, p_B) e^{imin(u_A, u_B)}, min(q_A, q_B) e^{imin(v_A, v_B)},$
  $min(r_A, r_B) e^{imin(w_A, w_B)}\big)$.

- **Complex neutrosophic product.**
  Complex neutrosophic product of of $\tilde{C}_A$ and $\tilde{C}_B$ is defined follows:
  $\tilde{C}_A \tilde{\circ}_N \tilde{C}_B = \big(p_A.p_B e^{i2\pi\left(\frac{u_A}{2\pi}.\frac{u_B}{2\pi}\right)}, q_A.q_B e^{i2\pi\left(\frac{v_A}{2\pi}.\frac{v_B}{2\pi}\right)}, r_A.r_B e^{i2\pi\left(\frac{w_A}{2\pi}.\frac{w_B}{2\pi}\right)}\big)$.

- **Complex neutrosophic bold intersection.**
  Complex neutrosophic bold intersection of $\tilde{C}_A$ and $\tilde{C}_B$ is defined as follows:
  $\tilde{C}_A \tilde{\Cap}_N \tilde{C}_B =$
  $(max(0, p_A + p_B - 1) e^{imax(0, u_A + u_B - 2\pi)}, max(0, q_A + q_B - 1) e^{imax(0, v_A + v_B - 2\pi)},$
  $max(0, r_A + r_B - 1) e^{imax(0, w_A + w_B - 2\pi)})$.

Definition 8.2. *Complex neutrosophic union.*
complex neutrosophic union of $\tilde{C}_A$ and $\tilde{C}_B$ is denoted by, $\tilde{C}_A \tilde{\cup}_N \tilde{C}_B$ and is defined as, $\tilde{C}_A \tilde{\cup}_N \tilde{C}_B = (T_{\tilde{C}_A \tilde{\cup}_F \tilde{C}_B}, I_{\tilde{C}_A \tilde{\cup}_F \tilde{C}_B}, F_{\tilde{C}_A \tilde{\cup}_F \tilde{C}_B})$, where, $\tilde{\cup}_F$ is the complex fuzzy union.

- **Complex neutrosophic standard union.**
  Complex neutrosophic standard union of $\tilde{C}_A$ and $\tilde{C}_B$ is defined as,
  $\tilde{C}_A \tilde{\cup}_N^{max} \tilde{C}_B = \big(max(p_A, p_B) e^{imax(u_A, u_B)}, max(q_A, q_B) e^{imax(v_A, v_B)},$
  $max(r_A, r_B) e^{imax(w_A, w_B)}\big)$.

- **Complex neutrosophic algebraic sum.**
  Complex neutrosophic algebraic sum $\tilde{C}_A$ and $\tilde{C}_B$ is defined as,
  $\tilde{C}_A \tilde{+}_N \tilde{C}_B =$
  $\big((p_A + p_B - p_A.p_B) e^{i2\pi\left(\frac{u_A}{2\pi} + \frac{u_B}{2\pi} - \frac{u_A}{2\pi}.\frac{u_B}{2\pi}\right)}, (q_A + q_B - q_A.q_B) e^{i2\pi\left(\frac{v_A}{2\pi} + \frac{v_B}{2\pi} - \frac{v_A}{2\pi}.\frac{v_B}{2\pi}\right)},$
  $(r_A + r_B - r_A.r_B) e^{i2\pi\left(\frac{w_A}{2\pi} + \frac{w_B}{2\pi} - \frac{w_A}{2\pi}.\frac{w_B}{2\pi}\right)}\big)$.

- **Complex neutrosophic bold sum.**
  Complex neutrosophic bold sum of $\tilde{C}_A$ and $\tilde{C}_B$ is defined as,

$\tilde{C}_A \tilde{\Cup}_N \tilde{C}_B = \big(min(1, p_A + p_B)e^{imin(2\pi, u_A+u_B)}, min(1, q_A + q_B)e^{imin(2\pi, v_A+v_B)},$
$min(1, r_A + r_B)e^{imin(2\pi, w_A+w_B)}\big).$

Definition 8.3. *Complex neutrosophic bounded difference.*
Complex neutrosophic bounded difference of $\tilde{C}_A$ and $\tilde{C}_B$ is denoted by, $\tilde{C}_A \tilde{\ominus}_N \tilde{C}_B$ and is defined as, $\tilde{C}_A \tilde{\ominus}_N \tilde{C}_B = \big(max(0, p_A - p_B)e^{imax(0, u_A - u_B)}, max(0, q_A - q_B)e^{imax(0, v_A - v_B)},$
$max(0, r_A - r_B)e^{imax(0, w_A - w_B)}\big).$

Definition 8.4. *Aggregation of m complex neutrosophic sets.*
Let, $\tilde{C}_s = (p_s e^{iu_s}, q_s e^{iv_s}, r_s e^{iw_s})$; $s = 1, 2, .., m$ be the m complex neutrosophic sets over the universe X. Then, different types of aggregations of m complex neutrosophic sets are defined as follows:

- Complex neutrosophic arithmetic mean aggregation of m complex neutrosophic sets $\tilde{C}_1, \tilde{C}_2, .., \tilde{C}_m$ is denoted by, $\tilde{C}_1 \tilde{\oplus}_N^{AM} \tilde{C}_2 \tilde{\oplus}_N^{AM} .. \tilde{\oplus}_N^{AM} \tilde{C}_m$ and is defined as follows, $\tilde{C}_1 \tilde{\oplus}_N^{AM} \tilde{C}_2 \tilde{\oplus}_N^{AM} .. \tilde{\oplus}_N^{AM} \tilde{C}_m = \big(\frac{1}{m}(p_1 + p_2 + .. + p_m)e^{i\frac{1}{m}(u_1+u_2+..+u_m)},$
$\frac{1}{m}(q_1 + q_2 + .. + q_m)e^{i\frac{1}{m}(v_1+v_2+..+v_m)}, \frac{1}{m}(r_1 + r_2 + .. + r_m)e^{i\frac{1}{m}(w_1+w_2+..+w_m)}\big).$

- Complex neutrosophic geometric mean aggregation of m complex neutrosophic sets $\tilde{C}_1, \tilde{C}_2, .., \tilde{C}_m$ is denoted by, $\tilde{C}_1 \tilde{\oplus}_N^{GM} \tilde{C}_2 \tilde{\oplus}_N^{GM} .. \tilde{\oplus}_N^{GM} \tilde{C}_m$ and is defined as follows, $\tilde{C}_1 \tilde{\oplus}_N^{GM} \tilde{C}_2 \tilde{\oplus}_N^{GM} .. \tilde{\oplus}_N^{GM} \tilde{C}_m = \big((p_1.p_2....p_m)^{1/2} e^{i2\pi(\frac{u_1}{2\pi}.\frac{u_2}{2\pi}....\frac{u_m}{2\pi})^{1/2}},$
$(q_1.q_2....q_m)^{1/2} e^{i2\pi(\frac{v_1}{2\pi}.\frac{v_2}{2\pi}....\frac{v_m}{2\pi})^{1/2}}, (r_1.r_2....r_m)^{1/2} e^{i2\pi(\frac{w_1}{2\pi}.\frac{w_2}{2\pi}....\frac{w_m}{2\pi})^{1/2}}\big)$

- Complex neutrosophic harmonic mean aggregation of m complex neutrosophic sets $\tilde{C}_1, \tilde{C}_2, .., \tilde{C}_m$ is denoted by, $\tilde{C}_1 \tilde{\oplus}_N^{HM} \tilde{C}_2 \tilde{\oplus}_N^{HM} .. \tilde{\oplus}_N^{HM} \tilde{C}_m$ and is defined as follows,
$\tilde{C}_1 \tilde{\oplus}_N^{HM} \tilde{C}_2 \tilde{\oplus}_N^{HM} .. \tilde{\oplus}_N^{HM} \tilde{C}_m = \Big(\big(\frac{m}{\frac{1}{p_1}+\frac{1}{p_2}+..+\frac{1}{p_m}}\big)e^{i2\pi\left(\frac{m}{\frac{2\pi}{u_1}+\frac{2\pi}{u_2}+..+\frac{2\pi}{u_m}}\right)},$
$\big(\frac{m}{\frac{1}{q_1}+\frac{1}{q_2}+..+\frac{1}{q_m}}\big)e^{i2\pi\left(\frac{m}{\frac{2\pi}{v_1}+\frac{2\pi}{v_2}+..+\frac{2\pi}{v_m}}\right)}, \big(\frac{m}{\frac{1}{r_1}+\frac{1}{r_2}+..+\frac{1}{r_m}}\big)e^{i2\pi\left(\frac{m}{\frac{2\pi}{w_1}+\frac{2\pi}{w_2}+..+\frac{2\pi}{w_m}}\right)}\Big).$

Example 8.1. Let, $X = \{x_1, x_2, x_3\}$ and $\tilde{C}_A = \{(x_1, (0.6e^{i2\pi}, 0.4e^{i\pi}, 0.5e^{i2\pi})),$
$(x_2, (0.8e^{i\pi}, 0.3e^{i\pi}, 0.2e^{i2\pi})), (x_3, (0.3e^{i\pi/2}, 0.5e^{i2\pi}, 0.1e^{i\pi}))\}$ and
$\tilde{C}_B = \{(x_1, (0.4e^{i\pi}, 0.6e^{i\pi}, 0.8e^{i2\pi})), (x_2, (0.7e^{i\pi}, 0.3e^{i\pi}, 0.4e^{i2\pi})),$
$(x_3, (0.1e^{i\pi}, 0.5e^{i2\pi}, 0.7e^{i\pi/2}))\}$ be two complex neutrosophic sets over X. Then,

$(i)\tilde{C}_A \tilde{\cap}_N^{min} \tilde{C}_B = \{(x_1, (0.4e^{i\pi}, 0.4e^{i\pi}, 0.5e^{i2\pi})), (x_2, (0.7e^{i\pi}, 0.3e^{i\pi}, 0.2e^{i2\pi})),$
$(x_3, (0.1e^{i\pi/2}, 0.4e^{i\pi}, 0.1e^{i\pi/2}))\}.$

(ii) $\tilde{C}_A \tilde{\circ}_N \tilde{C}_B = \{(x_1, (0.24e^{i\pi}, 0.24e^{i\pi/2}, 0.4e^{i2\pi})), (x_2, (0.5e^{i\pi/2}, 0.09e^{i\pi/2}, 0.08e^{i2\pi})),$
$(x_3, (0.03e^{i\pi/4}, 0.2e^{i\pi}, 0.07e^{i\pi/4}))\}.$

(iii)
$\tilde{C}_A \tilde{\Cap}_N \tilde{C}_B = \{(x_1, (0e^{i\pi}, 0e^{i0}, 0.3e^{i2\pi})), (x_2, (0.5e^{i0}, 0e^{i0}, 0e^{i2\pi})), (x_3, (0e^{i0}, 0e^{i\pi}, 0e^{i0}))\}$.

(iv) $\tilde{C}_A \tilde{\cup}_N^{max} \tilde{C}_B = \{x_1/(0.6e^{i2\pi}, 0.6e^{i\pi}, 0.8e^{i2\pi}), x_2/(0.8e^{i\pi}, 0.3e^{i\pi}, 0.4e^{i2\pi}),$
$x_3/(0.3e^{i\pi}, 0.5e^{i2\pi}, 0.7e^{i\pi})\}$.

(v)
$\tilde{C}_A \tilde{+}_N \tilde{C}_B = \{(x_1, (0.76e^{i2\pi}, 0.76e^{i3\pi/2}, 0.9e^{i2\pi})), (x_2, (0.94e^{i3\pi/2}, 0.51e^{i3\pi/2}, 0.52e^{i2\pi})),$
$(x_3, (0.37e^{i5\pi/4}, 0.7e^{i2\pi}, 0.73e^{i5\pi/4}))\}$.

(vi) $\tilde{C}_A \tilde{\Cup}_N \tilde{C}_B = \{(x_1, (1e^{i2\pi}, 1e^{i2\pi}, 1e^{i2\pi})), (x_2, (1e^{i2\pi}, 0.6e^{i2\pi}, 0.6e^{i2\pi})),$
$(x_3, (0.4e^{i3\pi/2}, 0.9e^{i2\pi}, 0.8e^{i3\pi/2}))\}$.

(vii) $\tilde{C}_A \tilde{\oplus}_N^{AM} \tilde{C}_B = \{(x_1, (0.5e^{i3\pi/2}, 0.5e^{i\pi}, 0.65e^{i2\pi})), (x_2, (0.75e^{i\pi}, 0.3e^{i\pi}, 0.3e^{i2\pi})),$
$(x_3, (0.2e^{i3\pi/4}, 0.45e^{i3\pi/2}, 0.4e^{i3\pi/4}))\}$.

$(viii)$
$\tilde{C}_A \tilde{\oplus}_N^{GM} \tilde{C}_B = \{(x_1, (0.49e^{i1.42\pi}, 0.49e^{i1.42\pi}, 0.63e^{i2\pi})), (x_2, (0.75e^{i\pi}, 0.3e^{i\pi}, 0.28e^{i2\pi})),$
$(x_3, (0.17e^{i0.7\pi}, 0.45e^{i1.42\pi}, 0.26e^{i0.7\pi}))\}$.

(ix) $\tilde{C}_A \tilde{\oplus}_N^{HM} \tilde{C}_B = \{(x_1, (0.48e^{i4\pi/3}, 0.48e^{i\pi}, 0.5e^{i2\pi})), (x_2, (0.75e^{i\pi}, 0.3e^{i\pi}, 0.27e^{i2\pi})),$
$(x_3, (0.15e^{i2\pi/3}, 0.44e^{i4\pi/3}, 0.175e^{i2\pi/3}))\}$.

(x) $\tilde{C}_A \tilde{\ominus}_N \tilde{C}_B = \{(x_1, (0.2e^{i\pi}, 0e^{i0}, 0e^{i0})), (x_2, (0.1e^{i0}, 0e^{i0}, 0e^{i0})),$
$(x_3, (0.2e^{i0}, 0.1e^{i\pi}, 0e^{i\pi/2}))\}$.

8.4 Score function of a complex neutrosophic number

Some times in real-life decision-making problems, comparing uncertain evaluations such as fuzzy evaluations, intuitionistic fuzzy evaluations, neutrosophic evaluations, etc. is very necessary. In such cases, if we transform these uncertain values into some real values, then we can easily compare them. Therefore, in this section, we have introduced a new definition of score function of a complex neutrosophic number to convert this uncertain number into some real number in the closed interval $[0, 1]$.

Definition 8.5. Let, $\tilde{C}_N = (T, I, F) = (p(x)e^{iu(x)}, q(x)e^{iv(x)}, r(x)e^{iw(x)})$ be a complex neutrosophic number where, $p(x), q(x), r(x)$ are the amplitude parts and $u(x), v(x), w(x)$ are the phase parts of the truth membership (T), indeterminate membership (I) and false membership (F) of $\tilde{C}_N$, $p(x), q(x), r(x) \in [0, 1]$ with $0 \leq p(x) + q(x) + r(x) \leq 3$ and

$u(x), v(x), w(x) \in [0, 2\pi]$.

The venn diagram of a complex neutrosophic number $\tilde{C}_N$ has been given in Figure 8.1.

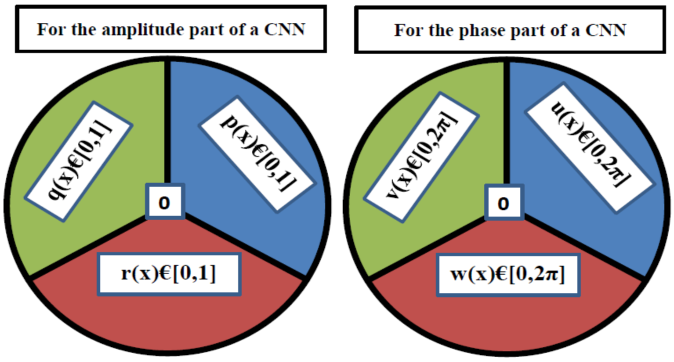

Figure 8.1: Venn diagram of a CNN

Then, the score function of $\tilde{C}_N$ is denoted by, $\hat{S}_{cr}(\tilde{C}_N)$ and is defined as follows:

$$\hat{S}_{cr}(\tilde{C}_N) = \frac{1}{6}\Big\{[p(x) + (2 - (q(x) + r(x)))] + \frac{1}{2\pi}[u(x) + (4\pi - (v(x) + w(x)))]\Big\} \quad (8.1)$$

The score function of a complex neutrosophic number measures the accuracy of the number $\tilde{C}_N$ in favor of truth degree.

Theorem 8.1. Let $\tilde{C}_A = \big(p_A e^{iu_A}, q_A e^{iv_A}, r_A e^{iw_A}\big)$ and $\tilde{C}_B = \big(p_B e^{iu_B}, q_B e^{iv_B}, r_B e^{iw_B}\big)$ be two complex neutrosophic numbers over the universal set X where,
$p_A, q_A, r_A, p_B, q_B, r_B \in [0, 1]$ with $0 \le p_A + q_A + r_A \le 3$, $0 \le p_B + q_B + r_B \le 3$ and $u_A, v_A, w_A, u_B, v_B, w_B \in [0, 2\pi]$.
Then, our proposed score function $(\hat{S}_{cr})$ satisfies the following properties.
(i) $\tilde{C}_A \le \tilde{C}_B \Leftrightarrow \hat{S}_{cr}(\tilde{C}_A) \le \hat{S}_{cr}(\tilde{C}_B)$.
(ii) If $\tilde{C}_A = (1e^{i2\pi}, 0, 0)$, then $\hat{S}_{cr}(\tilde{C}_A) = 1$.
(iii) If $\tilde{C}_A = (0, 1e^{i2\pi}, 1e^{i2\pi})$, then $\hat{S}_{cr}(\tilde{C}_A) = 0$.
(iv) $\hat{S}_{cr}(\tilde{C}_A) \in [0, 1]$.

Proof. (i) From the reference [12] we have,

$$\begin{gathered}\tilde{C}_A \le \tilde{C}_B \Leftrightarrow p_A \le p_B,\ q_A \ge q_B,\ r_A \ge r_B;\ u_A \le u_B,\ v_A \ge v_B,\ w_A \ge w_B \\ \Leftrightarrow p_A \le p_B,\ (1 - q_A) \le (1 - q_B),\ (1 - r_A) \le (1 - r_B); \\ u_A \le u_B,\ (2\pi - v_A) \le (2\pi - v_B),\ (2\pi - w_A) \le (2\pi - w_B).\end{gathered}$$

$$\begin{aligned}\Leftrightarrow\ \hat{S}_{cr}(\tilde{C}_A) &= \frac{1}{6}[(p_A+(1-q_A)+(1-r_A))+\frac{1}{2\pi}(u_A+(2\pi-v_A)+(2\pi-w_A))]\\ &\leq \frac{1}{6}[(p_B+(1-q_B)+(1-r_B))+\frac{1}{2\pi}(u_B+(2\pi-v_B)+(2\pi-w_B))]\\ &= \hat{S}_{cr}(\tilde{C}_B)\end{aligned}$$

(ii) If $\tilde{C}_A=(1e^{i2\pi},0,0)$ then, from Equation 8.1 it is obvious that, $\hat{S}_{cr}(\tilde{C}_A)=1$.

(iii) If $\tilde{C}_A=(0,1e^{i2\pi},1e^{i2\pi})$ then, from Equation 8.1 it is obvious that, $\hat{S}_{cr}(\tilde{C}_A)=0$.

(iv) Since, each of $p_A,q_A,r_A\in[0,1]$ and each of $u_A,v_A,w_A\in[0,2\pi]$, then from the Equation 8.1 it is obvious that, $\hat{S}_{cr}(\tilde{C}_A)\in[0,1]$.

Example 8.2. Let, $\tilde{C}_A=(0.7e^{i2\pi},0.4e^{i\pi},0.3e^{i2\pi})$ be a complex neutrosophic number over X. Then, by using Equation 8.1, its score value is,
$\hat{S}_{cr}(\tilde{C}_A)=\frac{1}{6}\{[0.7+(2-(0.4+0.3))]+\frac{1}{2\pi}[2\pi+(4\pi-(\pi+2\pi))]\}=0.58$.
Let, $\tilde{C}_B=(0.2e^{i\pi},0.5e^{i2\pi},0.7e^{i2\pi})$ be another complex neutrosophic number over X. Then, by using Equation 8.1, its score value is,
$\hat{S}_{cr}(\tilde{C}_B)=\frac{1}{6}\{[0.2+(2-(0.5+0.7))]+\frac{1}{2\pi}[\pi+(4\pi-(2\pi+2\pi))]\}=0.25$.

8.5 Some operations on complex neutrosophic soft sets

Let, $X=\{x_1,x_2,..,x_m\}$ be a universal set and $E=\{e_1,e_2,..,e_n\}$ be a set of parameters. Now, consider two complex neutrosophic soft sets $(f_{\tilde{C}_N},A)$ and $(g_{\tilde{C}_N},B)$ over X where, $A,B\subseteq E$. Then, union operation and intersection operation of $(f_{\tilde{C}_N},A)$ and $(g_{\tilde{C}_N},B)$ can be defined as follows:

Definition 8.6. Complex neutrosophic soft union of two complex neutrosophic soft sets $(f_{\tilde{C}_N},A)$ and $(g_{\tilde{C}_N},B)$ is denoted by $(f_{\tilde{C}_N},A)\tilde{\cup}_N(g_{\tilde{C}_N},B)=(h_{\tilde{C}_N},D)$ where, $A\cup B=D$ and $\forall a\in D$,

$$h_{\tilde{C}_N}(a)=\begin{cases}f_{\tilde{C}_N}(a) & \text{if } a\in A-B\\ g_{\tilde{C}_N}(a) & \text{if } a\in B-A\\ f_{\tilde{C}_N}(a)\tilde{\cup}_N g_{\tilde{C}_N}(a) & \text{if } a\in A\cap B\end{cases}$$

$\tilde{\cup}_N$ is the complex neutrosophic union of $f_{\tilde{C}_N}(a)$ and $g_{\tilde{C}_N}(a)$ as defined in Definition 8.2.

- If, $\tilde{\cup}_N$ is considered as the complex neutrosophic standard union ($\tilde{\cup}_N^{max}$), then the corresponding complex neutrosophic soft union is called complex neutrosophic soft standard union, which can be denoted by, $(f_{\tilde{C}_N},A)\tilde{\cup}_N^{max}(g_{\tilde{C}_N},B)=(h_{\tilde{C}_N},D)$.

- If, $\tilde{\cup}_N$ is considered as the complex neutrosophic algebraic sum ($\tilde{\dotplus}_N$), then the corresponding complex neutrosophic soft union is called complex neutrosophic soft algebraic sum, which can be written as, $(f_{\tilde{C}_N}, A)\tilde{\dotplus}_N(g_{\tilde{C}_N}, B) = (h_{\tilde{C}_N}, D)$.

- If, $\tilde{\cup}_N$ is considered as the complex neutrosophic bold sum ($\tilde{\Cup}_N$), then the corresponding complex neutrosophic soft union is called complex neutrosophic soft bold sum which can be written as, $(f_{\tilde{C}_N}, A)\tilde{\Cup}_N(g_{\tilde{C}_N}, B) = (h_{\tilde{C}_N}, D)$.

Definition 8.7. Complex neutrosophic soft intersection of $(f_{\tilde{C}_N}, A)$ and $(g_{\tilde{C}_N}, B)$ is denoted by, $(f_{\tilde{C}_N}, A)\tilde{\cap}_N(g_{\tilde{C}_N}, B) = (H_{\tilde{C}_N}, D)$ where, $A \cap B = D$ and $\forall a \in D$, $H_{\tilde{C}_N}(a) = f_{\tilde{C}_N}(a)\tilde{\cap}_N g_{\tilde{C}_N}(a)$.

$\tilde{\cap}_N$ is the complex neutrosophic intersection of $f_{\tilde{C}_N}(a)$ and $g_{\tilde{C}_N}(a)$ as defined in Definition 8.1.

- If, $\tilde{\cap}_N$ is considered as the complex neutrosophic standard intersection ($\tilde{\cap}_N^{min}$), then the corresponding complex neutrosophic soft intersection is called complex neutrosophic soft standard intersection, which can be denoted by,
$(f_{\tilde{C}_N}, A)\tilde{\cap}_N^{min}(g_{\tilde{C}_N}, B) = (H_{\tilde{C}_N}, D)$.

- If, $\tilde{\cap}_N$ is considered a the complex neutrosophic product ($\tilde{\circ}_N$), then the corresponding complex neutrosophic soft intersection is called complex neutrosophic soft product which can be denoted by, $(f_{\tilde{C}_N}, A)\tilde{\circ}_N(g_{\tilde{C}_N}, B) = (H_{\tilde{C}_N}, D)$.

- If, $\tilde{\cap}_N$ is considered as the complex neutrosophic bold intersection ($\tilde{\Cap}_N$), then the corresponding complex neutrosophic soft intersection is called complex neutrosophic soft bold intersection which can be denoted by, $(f_{\tilde{C}_N}, A)\tilde{\Cap}_N(g_{\tilde{C}_N}, B) = (H_{\tilde{C}_N}, D)$.

Example 8.3. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. Now assume that, $(f_{\tilde{C}_N}, A)$ and $(g_{\tilde{C}_N}, B)$ be two complex neutrosophic soft sets over X where $A = \{e_1, e_2\}$ and $B = \{e_1, e_3\}$ such that,
$(f_{\tilde{C}_N}, A) = \{(e_1, ((x_1, (0.3e^{i\pi/2}, 0.1e^{i\pi}, 0.6e^{i3\pi/2})), (x_2, (0.6e^{i3\pi/2}, 0.5e^{i\pi}, 0.4e^{i\pi/2})),$
$(x_3, (0.8e^{i2\pi}, 0.3e^{i\pi}, 0.2e^{i\pi/2})))), (e_2, ((x_1, (0.7e^{i\pi/4}, 0.3e^{i\pi}, 0.2e^{i\pi})),$
$(x_2, (0.6e^{i2\pi}, 0.4e^{i\pi/2}, 0.1e^{i\pi})),$
$(x_3, (0.4e^{i\pi}, 0.7e^{i2\pi}, 0.9e^{i2\pi}))))\}$ and
$(g_{\tilde{C}_N}, A) = \{(e_1, ((x_1, (0.4e^{i\pi}, 0.3e^{i\pi/2}, 0.5e^{i2\pi})), (x_2, (0.7e^{i3\pi/2}, 0.4e^{i\pi/2}, 0.2e^{i\pi})),$
$(x_3, (0.7e^{i3\pi/2}, 0.4e^{i\pi}, 0.5e^{i\pi/4})))), (e_3, ((x_1, (0.6e^{i\pi/2}, 0.4e^{i\pi}, 0.3e^{i\pi/2})),$
$(x_2, (0.5e^{i\pi}, 0.4e^{i\pi/2}, 0.3e^{i\pi})), (x_3, (0.3e^{i\pi/2}, 0.8e^{i3\pi/2}, 0.8e^{i2\pi}))))\}$. Then,

- $(f_{\tilde{C}_N}, A)\tilde{\cup}_N^{max}(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0.4e^{i\pi}, 0.3e^{i\pi}, 0.6e^{2\pi})),$
$(x_2, (0.7e^{i3\pi/2}, 0.5e^{i\pi}, 0.4e^{2\pi})), (x_3, (0.8e^{i2\pi}, 0.4e^{i\pi}, 0.5e^{i\pi/2})))),$
$(e_2, ((x_1, (0.7e^{i\pi/4}, 0.3e^{i\pi}, 0.2e^{i\pi})), (x_2, (0.6e^{i2\pi}, 0.4e^{i\pi/2}, 0.1e^{i\pi})),$
$(x_3, (0.4e^{i\pi}, 0.7e^{i2\pi}, 0.9e^{i2\pi})))), (e_3, ((x_1, (0.6e^{i\pi/2}, 0.4e^{i\pi}, 0.3e^{i\pi/2})),$
$(x_2, (0.5e^{i\pi}, 0.4e^{i\pi/2}, 0.3e^{i\pi})),$
$(x_3, (0.3e^{i\pi/2}, 0.8e^{i3\pi/2}, 0.8e^{i2\pi}))))\}$.

- $(f_{\tilde{C}_N}, A)\tilde{\dotplus}_N(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0.5e^{i1.25\pi}, 0.37e^{i1.25\pi}, 0.8e^{2\pi})), (x_2, (0.88e^{i1.88\pi}, 0.7e^{i1.25\pi}, 0.52e^{i1.25\pi})), (x_3, (0.9e^{i2\pi}, 0.58e^{i1.5\pi}, 0.6e^{i1.5\pi/2})))), (e_2, ((x_1, (0.7e^{i\pi/4}, 0.3e^{i\pi}, 0.2e^{i\pi})), (x_2, (0.6e^{i2\pi}, 0.4e^{i\pi/2}, 0.1e^{i\pi})), (x_3, (0.4e^{i\pi}, 0.7e^{i2\pi}, 0.9e^{i2\pi})))), (e_3, ((x_1, (0.6e^{i\pi/2}, 0.4e^{i\pi}, 0.3e^{i\pi/2})), (x_2, (0.5e^{i\pi}, 0.4e^{i\pi/2}, 0.3e^{i\pi})), (x_3, (0.3e^{i\pi/2}, 0.8e^{i3\pi/2}, 0.8e^{i2\pi}))))\}$.
- $(f_{\tilde{C}_N}, A)\tilde{\Cup}_N(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0.7e^{i3\pi/2}, 0.4e^{i3\pi/2}, 1e^{2\pi})), (x_2, (1e^{i2\pi}, 0.9e^{i3\pi/2}, 0.6e^{i3\pi/2})), (x_3, (1e^{i2\pi}, 0.7e^{i2\pi}, 0.7e^{i3\pi/4})))), (e_2, ((x_1, (0.7e^{i\pi/4}, 0.3e^{i\pi}, 0.2e^{i\pi})), (x_2, (0.6e^{i2\pi}, 0.4e^{i\pi/2}, 0.1e^{i\pi})), (x_3, (0.4e^{i\pi}, 0.7e^{i2\pi}, 0.9e^{i2\pi})))), (e_3, ((x_1, (0.6e^{i\pi/2}, 0.4e^{i\pi}, 0.3e^{i\pi/2})), (x_2, (0.5e^{i\pi}, 0.4e^{i\pi/2}, 0.3e^{i\pi})), (x_3, (0.3e^{i\pi/2}, 0.8e^{i3\pi/2}, 0.8e^{i2\pi}))))\}$.
- $(f_{\tilde{C}_N}, A)\tilde{\cap}_N^{min}(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.5e^{i3\pi/2})), (x_2, (0.6e^{i3\pi/2}, 0.4e^{i\pi/2}, 0.2e^{\pi/2})), (x_3, (0.7e^{i3\pi/2}, 0.3e^{i\pi}, 0.2e^{i\pi/2}))))\}$.
- $(f_{\tilde{C}_N}, A)\tilde{\circ}_N(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0.12e^{i\pi/4}, 0.03e^{i\pi/4}, 0.3e^{3\pi/2})), (x_2, (0.42e^{i9\pi/8}, 0.2e^{i\pi/4}, 0.08e^{i\pi/4})), (x_3, (0.56e^{i3\pi/4}, 0.12e^{i\pi/2}, 0.1e^{i\pi/16}))))\}$.
- $(f_{\tilde{C}_N}, A)\tilde{\Cap}_N(g_{\tilde{C}_N}, B) = \{(e_1, ((x_1, (0e^{i0}, 0e^{i0}, 0.1e^{3\pi/2})), (x_2, (0.3e^{i\pi}, 0e^{i0}, 0e^{i0})), (x_3, (0.5e^{i3\pi/2}, 0e^{i0}, 0e^{i0}))))\}$.

Complex neutrosophic soft ideal solution.
Let, $X = \{x_1, x_2, .., x_m\}$ be a universal set and $E = \{e_1, e_2, .., e_n\}$ be a corresponding parameter set. Now, consider $(f_{\tilde{C}_N}, E)$ be a complex neutrosophic soft set over X such that,

$$\begin{aligned}(f_{\tilde{C}_N}, E) &= \Big\{(e_1, f_{\tilde{C}_N}(e_1)), (e_2, f_{\tilde{C}_N}(e_2)), .., (e_n, f_{\tilde{C}_N}(e_n)\Big\}\\ &= \Big\{(e_1, \{(x_1, x_{11}), (x_2, x_{21}), .., (x_m, x_{m1})\}), (e_2, \{(x_1, x_{12}), (x_2, x_{22}), ..,\\ &\quad (x_m, x_{m2})\}), .., (e_n, \{(x_1, x_{1n}), (x_2, x_{2n}), .., (x_m, x_{mn})\})\Big\}\end{aligned}$$

where, x_{sj} is the complex neutrosophic valued evaluation of an alternative x_s over a parameter e_j.

Definition 8.8. Then, complex neutrosophic soft ideal solution of a CNSS $(f_{\tilde{C}_N}, E)$ is the best reference solution (as given in Figure 8.2) which can be defined by taking complex neutrosophic standard union among the evaluations of all the alternatives with respect to each of the parameters. Mathematically, it can be denoted by x^+ and is defined as,

$$x^+ = \{f^+_{\tilde{C}_N}(e_1), f^+_{\tilde{C}_N}(e_2), .., f^+_{\tilde{C}_N}(e_n)\}$$

where,

$$f^+_{\tilde{C}_N}(e_j) = x_{1j}\tilde{\cup}_N^{max} x_{2j}\tilde{\cup}_N^{max} ..\tilde{\cup}_N^{max} x_{mj};\ j = 1, 2, .., n.$$

$f^+_{\tilde{C}_N}(e_j)$ is called the complex neutrosophic soft ideal evaluation of a parameter e_j and $\tilde{\cup}_N^{max}$ is the complex neutrosophic standard union as defined in Definition 8.2.

8.6 Decision-making problems based on soft set theory under complex neutrosophic environment.

Decision-making problems have been studied by the researchers under different environments like, fuzzy soft set [22], intuitionistic fuzzy soft set [105], neutrosophic soft set [24], etc. Now, in this section, we have proposed two methodologies to solve decision-making problems under complex neutrosophic soft set theory.

8.6.1 Complex neutrosophic soft decision-making for single decision maker based problems

Problem description

Let, $X = \{x_1, x_2, .., x_m\}$ be the universal set and $E = \{e_1, e_2, .., e_n\}$ be the set of corresponding parameters which are in complex neutrosophic sense. Now, we have assumed that, $(f_{\tilde{C}_N}, E)$ be a complex neutrosophic soft set over X such that,

$$(f_{\tilde{C}_N}, E) = \{(e_1, f_{\tilde{C}_N}(e_1)), (e_2, f_{\tilde{C}_N}(e_2)), .., (e_n, f_{\tilde{C}_N}(e_n))\}$$

where, $f_{\tilde{C}_N}(e_j) = \{(x_1, x_{1j}), (x_2, x_{2j}), .., (x_m, x_{mj})\}$ and $x_{sj} = (T_{sj}, I_{sj}, F_{sj})$; $s = 1, 2, .., m, j = 1, 2, .., n$ is the complex neutrosophic valued evaluation of an alternative x_s over a parameter e_j. T_{sj}, I_{sj} and F_{sj} are the complex-valued truth membership degree, complex-valued indeterminate membership degree and complex-valued false membership degree such that, $T_{sj} = p_{sj}e^{iu_{sj}}$, $I_{sj} = q_{sj}e^{iv_{sj}}$, $F_{sj} = r_{sj}e^{iw_{sj}}$; $p_{sj}, q_{sj}, r_{sj} \in [0, 1]$ with $0 \leq p_{sj} + q_{sj} + r_{sj} \leq 3$ and $u_{sj}, v_{sj}, w_{sj} \in [0, 2\pi]$.
Tabular representation of the complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ has been given in Table 8.1.

Table 8.1: Tabular form of a CNSS $(f_{\tilde{C}_N}, E)$ (in general case)

	e_1	e_2	$\dots$	e_n
x_1	x_{11}	x_{12}	$\dots$	x_{1n}
x_2	x_{21}	x_{22}	$\dots$	x_{2n}
.	.	.	$\dots$	.
.	.	.	$\dots$	.
x_m	x_{m1}	x_{m2}	$\dots$	x_{mn}

In Figure 8.3, graphical representation of a complex neutrosophic number (x_{sj}) has been given.

- **Nature of the parameters in the considered decision-making problem.**

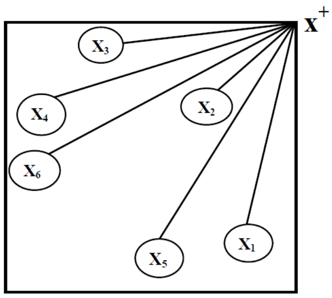

Figure 8.2: Complex neutrosophic soft ideal solution

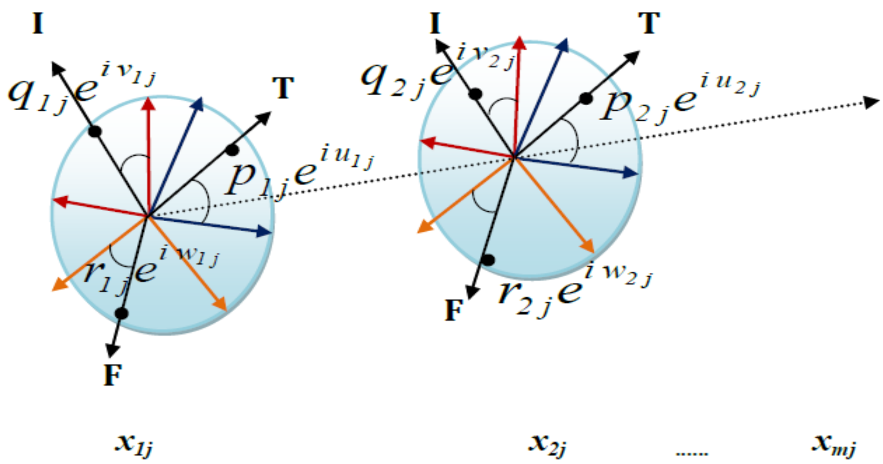

Figure 8.3: Graphical representation of CNNs

Molodtsov's soft set is a parameterized family of subsets of the universal set X. In this set, any parameterization can be considered to handle real-life problems. For instance, we can take the parameters with the help of words, sentences, functions, real numbers, etc. However, not all the parameters come from same environment, therefore, conflicting criteria may exist in the parameter set. i.e., some of the parameters may be benefit criteria and some of the parameters may be cost criteria.

So, in a complex neutrosophic soft set based decision-making problem, there may exist *conflicting criteria* in the considered parameter set. Then, to deal with the conflicting criteria, firstly, we have equalized the sense of all the parameters.

Here, we have converted the evaluations of all the alternatives with respect to each of the cost parameters into benefit sense by taking their complex neutrosophic complement.

- **Weights of the considered parameters.**

Weights of the parameters play an important role in a decision-making problem. However, weights of the parameters may or may not be given at the initial stage. Moreover, an incomplete information about the weights of the parameters may be provided in the decision-making problem. In such cases, to solve the problems, now, we have to derive the exact weights of the parameters.

If, $W = \{W_1, W_2, .., W_n\}$; $W_j \in [0, 1]$ be the associated weights of parameters, then they should satisfy the condition, $\sum\limits_{j=1}^{n} W_j = 1$.

So, based on these above discussions, our targets are as follows:
(i) *To equalize the sense of all the considered parameters associated with the CNSS* $(f_{\tilde{C}_N}, E)$ *to deal with the conflicting criteria.*
(ii) *To use a normalization process for handling non commensurable parameters.*
(iii) *To derive the exact weights of the parameters, if the weights of the parameters are not given or an incomplete information about the weights of the parameters is given in the decision-making problem.*
(iv) *Finally, to construct a ranking order the alternatives based on the complex neutrosophic soft set* $(f_{\tilde{C}_N}, E)$.

Complex neutrosophic soft VIKOR approach for single decision maker based problems

To fulfil the above targets, now we have proposed an algorithm in complex neutrosophic soft set framework. In this algorithm we have used VIKOR approach ([124]) to obtain a *compromise optimal solution* by testing *'acceptable advantage'* and *'acceptable stability'* in this decision-making problem. This compromise optimal solution would be closest to the ideal solution. Our proposed approach contains the following steps.

Algorithm I: Complex neutrosophic soft VIKOR approach for single decision maker (CNSVISDM-approach).

Step 1. *Input the necessary substance.*
Input the selected m alternatives ($X = \{x_1, x_2, .., x_m\}$) and n corresponding parameters ($E = \{e_1, e_2, .., e_n\}$). Input the evaluations of all the alternatives with respect to n parameters through a complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ as given in Table 8.1. If the weights of the parameters are given in a problem then, input them ($W = \{W_1, W_2, .., W_n\}$) also.

Step 2. *Equalization of the sense of all the parameters for dealing with conflicting criteria.*
To deal with the conflicting parameters in a complex neutrosophic soft set based decision-making problem, we have transformed the evaluations of all the alternatives over each of the cost parameters into benefit sense to uniform the sense of all the parameters. In this regard,

(i) First, identify the benefit parameters (with respect to these parameters, large evaluation of an alternative is better) and cost parameters (with respect to these parameters, small evaluation of an alternative is better). Let A be the set of benefit parameters and B be the set of cost parameters such that, $A \cup B = E$ and $A \cap B = \phi$.

(ii) Transform the evaluations of all the alternatives with respect to each of the parameters of the set B into benefit sense by taking their complement. Mathematically, it can be defined as follows, $\forall\ e_j \in B$,

$$(f_{\tilde{C}_N}(e_j))^c = \{(x_1, x^c_{1j}), (x_2, x^c_{2j}), .., (x_m, x^c_{mj})\}$$

c-denotes the complement of a complex neutrosophic evaluation x_{sj} as given in the Section 8.2.

Step 3. *Derivation of the complex neutrosophic soft ideal evaluation of each of the parameters.*
Since our goal is, select the alternative which satisfies all the parameters with maximum evaluation level therefore, by using Definition 8.8, we have derived the complex neutrosophic soft ideal evaluation of each of the parameters, denoted by $f^+_{\tilde{C}_N}(e_j);\ j = 1, 2, .., n$, over the complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ as follows:

$$f^+_{\tilde{C}_N}(e_j) = x_{1j} \tilde{\cup}^{max}_N x_{2j} \tilde{\cup}^{max}_N .. \tilde{\cup}^{max}_N x_{mj}$$

Then, from Definition 8.2 it is concluded that,

$$\begin{aligned} f^+_{\tilde{C}_N}(e_j) &= x_{1j} \tilde{\cup}^{max}_N x_{2j} \tilde{\cup}^{max}_N .. \tilde{\cup}^{max}_N x_{mj} \\ &= \big(max\{p_{1j}, p_{2j}, .., p_{mj}\} e^{i max\{u_{1j}, u_{2j}, .., u_{mj}\}}, \\ &\quad max\{q_{1j}, q_{2j}, .., q_{mj}\} e^{i max\{v_{1j}, v_{2j}, .., v_{mj}\}}, max\{r_{1j}, r_{2j}, .., r_{mj}\} e^{i max\{w_{1j}, w_{2j}, .., w_{mj}\}}\big) \\ &= (p^+_j e^{iu^+_j}, q^+_j e^{iv^+_j}, r^+_j e^{iw^+_j}); j = 1, 2, .., n \end{aligned}$$

Step 4. *Determination of the bounded difference between $f^{+}_{\tilde{C}_N}(e_j)$ and x_{sj}.*
By using Definition 8.3, we have obtained the complex neutrosophic bounded difference of the evaluation x_{sj} of an alternative x_s with respect to a parameter e_j from the corresponding complex neutrosophic soft ideal evaluation $f^{+}_{\tilde{C}_N}(e_j)$.

Mathematically, the resultant value is denoted by, $d^{\ominus}_{sj}$ and is defined as, $d^{\ominus}_{sj} = (f^{+}_{\tilde{C}_N}(e_j)\tilde{\ominus}_N x_{sj})$ such that,

$$\begin{aligned} d^{\ominus}_{sj} &= (f^{+}_{\tilde{C}_N}(e_j)\tilde{\ominus}_N x_{sj}) \\ &= (max(0, p^{+}_j - p_{sj})e^{i max(0, u^{+}_j - u_{sj})}, max(0, q^{+}_j - q_{sj})e^{i max(0, v^{+}_j - v_{sj})}, \\ &\quad max(0, r^{+}_j - r_{sj})e^{i max(0, w^{+}_j - w_{sj})}) \\ &= (\check{p}_{sj}e^{i\check{u}_{sj}}, \check{q}_{sj}e^{i\check{v}_{sj}}, \check{r}_{sj}e^{i\check{w}_{sj}}) \end{aligned}$$

The resultant soft set is called complex neutrosophic bounded difference soft set. It is denoted by, $(f^{\ominus}_{\tilde{C}_N}, E)$ and its tabular form has been given in Table 8.2.

Table 8.2: Complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$

	e_1	e_2	$\ldots$	e_n
x_1	$d^{\ominus}_{11}$	$d^{\ominus}_{12}$	$\ldots$	$d^{\ominus}_{1n}$
x_2	$d^{\ominus}_{21}$	$d^{\ominus}_{22}$	$\ldots$	$d^{\ominus}_{2n}$
.	.	.	$\ldots$	.
.	.	.	$\ldots$	.
x_m	$d^{\ominus}_{m1}$	$d^{\ominus}_{m2}$	$\ldots$	$d^{\ominus}_{mn}$

Step 5. *Derivation of score value of each of the entries of the soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (given in Table 8.2).*
Now, by using our proposed score function of a complex neutrosophic number (Definition 8.5), we have deduced the score value ($\hat{S}_{cr}$) of each of the entries
$d^{\ominus}_{sj} = (\check{p}_{sj}e^{i\check{u}_{sj}}, \check{q}_{sj}e^{i\check{v}_{sj}}, \check{r}_{sj}e^{i\check{w}_{sj}})$ in the complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ as follows:

$$\hat{S}_{cr}(d^{\ominus}_{sj}) = \frac{1}{6}\Big\{[\check{p}_{sj} + (2 - (\check{q}_{sj} + \check{r}_{sj}))] + \frac{1}{2\pi}[\check{u}_{sj} + (4\pi - (\check{v}_{sj} + \check{w}_{sj}))]\Big\}$$

These score values have been presented in Table 8.3 which is named as score valued matrix.

Step 6. *Normalization process to deal with non commensurable parameters.*
In a complex neutrosophic soft set based decision-making problem, there may exist non commensurable criterion in the parameter set. Then, to deal with the non commensurable data, we have used the linear max normalization technique (given in Section 8.2) to make all

Table 8.3: Tabular form of the score valued matrix $(\hat{S}_{cr}(d^{\ominus}_{sj}))_{m\times n}$

	e_1	e_2	$\ldots$	e_n
x_1	$\hat{S}_{cr}(d^{\ominus}_{11})$	$\hat{S}_{cr}(d^{\ominus}_{12})$	$\ldots$	$\hat{S}_{cr}(d^{\ominus}_{1n})$
x_2	$\hat{S}_{cr}(d^{\ominus}_{21})$	$\hat{S}_{cr}(d^{\ominus}_{22})$	$\ldots$	$\hat{S}_{cr}(d^{\ominus}_{2n})$
.	.	.	$\ldots$	.
.	.	.	$\ldots$	.
x_m	$\hat{S}_{cr}(d^{\ominus}_{m1})$	$\hat{S}_{cr}(d^{\ominus}_{m2})$	$\ldots$	$\hat{S}_{cr}(d^{\ominus}_{mn})$

the parameters unit less.

6.1 Firstly, we have derived the maximum score value over all the alternatives with respect to a parameter e_j. Mathematically it is denoted by, $\hat{S}^{+}_{cr}(d^{\ominus}_{j})$ and is defined as follows:

$$\hat{S}^{+}_{cr}(d^{\ominus}_{j}) = max^{m}_{s=1}\hat{S}_{cr}(d^{\ominus}_{sj})$$

6.2 Then, we have applied the linear max normalization technique to normalize an entry $\hat{S}_{cr}(d^{\ominus}_{sj})$ associated with the Table 8.3 as follows:

$$\phi_{sj} = \frac{\hat{S}_{cr}(d^{\ominus}_{sj})}{\hat{S}^{+}_{cr}(d^{\ominus}_{j})} \tag{8.2}$$

So, by using the above equation, each of the evaluations $\phi_{sj}; s = 1, 2, .., m;\ j = 1, 2, .., n$ over each of the parameters now becomes unit less.

Step 7. *Determination of the group utility value of an alternative x_s (Utility measure).*
The group utility value of an alternative x_s is denoted by, $\hat{S}^{W}_{cr}(x_s)$ and is derived by adding the weighted values $\phi_{s1}, \phi_{s2}, .., \phi_{sn}$. Mathematically, it is defined as follows,

$$\hat{S}^{W}_{cr}(x_s) = \sum_{j=1}^{n} W_j\phi_{sj};\ s = 1, 2, .., m;\ j = 1, 2, .., n \tag{8.3}$$

where, W_j is the weight of the parameter e_j and ϕ_{sj} the normalized bounded difference of an evaluation x_{sj} from the corresponding ideal evaluation $f^{+}_{\tilde{C}_N}(e_j)$ in terms of score value. If the weights of the parameters are not given or an incomplete information about the weights of the parameters is given then, we have to determine the exact weights of the parameters by using Subsection 8.6.2.

So, from Equation 8.3 it is concluded that, the alternative x_s which has smallest value of $\hat{S}^{W}_{cr}$ is best.

Step 8. *Determination of the individual regret value of an alternative x_s (Regret measure).*
The individual regret of an alternative x_s is denoted by, $\hat{S}^{max}_{cr}$ and is derived by taking the

maximum weighted value over all the weighted values of $\phi_{s1}, \phi_{s2}, .., \phi_{sn}$. Mathematically, it is defined as follows,

$$\hat{S}_{cr}^{max}(x_s) = \max_j\{W_j\phi_{sj}\};\ s = 1, 2, .., m;\ j = 1, 2, .., n \tag{8.4}$$

Step 9. *Derivation of the compromising index $\hat{C}(x_s)$ of an alternative x_s.*
The compromising index of an alternative x_s over all the parameters can be defined by the following equation:

$$\hat{C}(x_s) = \theta\frac{\hat{S}_{cr}^{W}(x_s) - \overline{\hat{S}_{cr}^{W}}(x_s)}{\underline{\hat{S}_{cr}^{W}}(x_s) - \overline{\hat{S}_{cr}^{W}}(x_s)} + (1-\theta)\frac{\hat{S}_{cr}^{max}(x_s) - \overline{\hat{S}_{cr}^{max}}(x_s)}{\underline{\hat{S}_{cr}^{max}}(x_s) - \overline{\hat{S}_{cr}^{max}}(x_s)} \tag{8.5}$$

where, $\overline{\hat{S}_{cr}^{W}}(x_s) = \min_s \hat{S}_{cr}^{W}(x_s)$, $\underline{\hat{S}_{cr}^{W}}(x_s) = \max_s \hat{S}_{cr}^{W}(x_s)$, $\overline{\hat{S}_{cr}^{max}}(x_s) = \min_s \hat{S}_{cr}^{max}(x_s)$, $\underline{\hat{S}_{cr}^{max}}(x_s) = \max_s \hat{S}_{cr}^{max}(x_s)$.
Here, the parameter $\theta \in [0, 1]$ indicates the weight of the strategy of maximum group of utility and $(1-\theta)$ indicates the weight of the strategy of minimum individual regret.

Step 10. *Ranking of the alternatives based on the compromising index*
Construct the ranking order of m alternatives based on the descending order of their utility measure ($\hat{S}_{cr}^{W}$), regret measure ($\hat{S}_{cr}^{max}$) and compromising index ($\hat{C}$). Therefore, we will get three ranking results.

Assume that, the alternative x_L has minimum compromising index value among m alternatives. Then, the alternative x_L will be a compromise optimal solution if the following two conditions are satisfied:

- *Condition 1.* ***Acceptable advantage:***
  $\hat{C}(x_L^{'}) - \hat{C}(x_L) \geq DQ\ (a\ threshold\ value) = \frac{1}{m-1}$,
  where, $x_{L'}$ is the alternative placed at the second position in ranking order of the alternatives based on compromising index and m is the number of alternatives.

- *Condition 2.* ***Acceptable stability:***
  The alternative x_L is the best solution based on the ranking order of utility measure ($\hat{S}_{cr}^{W}$) and regret measure ($\hat{S}_{cr}^{max}$).

Now, if any one of the above two conditions is not satisfied then, we will get a set of compromise optimal solutions as follows:

- If, *Condition 1* is not satisfied then, we will take the first $m^{'}$ alternatives as a compromise optimal solutions for this decision problem for maximum number of $m^{'}$, where the number $m^{'}$ will be derived by the following equation:
  $$\hat{C}(x_{m'}) - \hat{C}(x_L) < \frac{1}{m-1}$$

- If, *Condition 2* is not satisfied then, x_L and $x_{L'}$ both will be the compromise optimal solutions for this decision-making problem.

8.6.2 Determination of the weights of the parameters

Weights of the parameters play an important role in a decision-making problem. However in a problem, weights of the parameters may or may not be given. Moreover, an incomplete information about the weights of the parameters may be given in the problem. So, when exact weights of the parameters are not given in a problem initially, then we have to determine the exact weights of the parameters.

Let, $W = \{W_1, W_2, .., W_n\}$ be the weighs parameters where, each $W_j \in [0,1]$; $j = 1, 2, .., n$ and $\sum_{j=1}^{n} W_j = 1$.

Definition 8.9. The mean potentiality of a parameter e_j associated with the complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ (after equalization) is derived by taking arithmetic mean aggregation (as given in Definition 8.4) of the evaluations of all the alternatives with respect to the parameter e_j. Mathematically, it is denoted by m_{e_j} and is defined as follows:

$$m_{e_j} = (x_{1j} \tilde{\oplus}_N^{AM} x_{2j} \tilde{\oplus}_N^{AM} .. \tilde{\oplus}_N^{AM} x_{mj}) \tag{8.6}$$

Then, from Definition 8.4 it is obtained that,

$$\begin{aligned} m_{e_j} &= \big(x_{1j} \tilde{\oplus}_N^{AM} x_{2j} \tilde{\oplus}_N^{AM} .. \tilde{\oplus}_N^{AM} x_{mj}\big) \\ &= \big(\frac{1}{m}(p_{1j} + p_{2j} + .. + p_{mj}) e^{i \frac{1}{m}(u_{1j} + u_{2j} + .. + u_{mj})}, \\ &\frac{1}{m}(q_{1j} + q_{2j} + .. + q_{mj}) e^{i \frac{1}{m}(v_{1j} + v_{2j} + .. + v_{mj})}, \\ &\frac{1}{m}(r_{1j} + r_{2j} + .. + r_{mj}) e^{i \frac{1}{m}(w_{1j} + w_{2j} + .. + w_{mj})}\big) \end{aligned}$$

Now to evaluate the exact weights of the parameters, we have followed the following steps.

6.2.1 After equalization of all the parameters (by using Step 2 of Algorithm I) associated with the CNSS $(f_{\tilde{C}_N}, E)$, determine the mean potentiality of each of the parameters by using Equation 8.6.

6.2.2 Evaluate the complex neutrosophic distance measure (d_{sj}) between the evaluation x_{sj} of an alternative x_s; $s = 1, 2, .., m$ with respect to the parameter e_j and the mean potentiality (m_{e_j}) of the parameter e_j as follows,

$$d_{sj} = \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj})$$

where, $\tilde{d}_{\text{CN}}$ is the complex neutrosophic distance measure as given in Section 8.2.

6.2.3 After that, derive the total complex neutrosophic distance of all the alternatives with respect to the parameter e_j. Mathematically, it is denoted by, $\tilde{d}_{e_j}$ and is defined as,

$$\tilde{d}_{e_j} = \sum_{s=1}^{m} d_{sj} = \sum_{s=1}^{m} \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj}) \tag{8.7}$$

6.2.4 Then, the weighted complex neutrosophic distance function of this complex neutrosophic soft set based decision-making problem is denoted by, $\tilde{D}(W)$ and is constructed as follows.

$$\tilde{D}(W) = \sum_{j=1}^{n} \tilde{d}_{e_j} W_j = \sum_{j=1}^{n}\sum_{s=1}^{m} d_{sj} W_j = \sum_{j=1}^{n}\sum_{s=1}^{m} \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj}) W_j$$

Case-I. *When weights of the parameters are incompletely known:*
Based on the reference [175], the possible types of incomplete information about the weights of the parameters are as follows. For $j \neq j^{'}$,
(i) *Weak ranking:* $P1 = \{W_j \geq W_{j'}\}$;
(ii) *Strict ranking:* $P2 = \{W_j - W_{j'} \geq \beta_{j'}, \beta_{j'} \geq 0\}$;
(iii) *Ranking of differences:* $P3 = \{W_j - W_{j'} \geq W_k - W_{k'}, j^{'} \neq k \neq k^{'}\}$;
(iv) *Ranking with multiples:* $P4 = \{W_j \geq \beta_{j'} W_{j'}, (0 \leq \beta_{j'} \leq 1)\}$;
(v) *Interval forms:* $P5 = \{\beta_j \leq W_j \leq \beta_j + \varepsilon_j, (0 \leq \beta_j \leq \beta_j + \varepsilon_j \leq 1)\}$.

Let P be the set of these five possible categories of information about the weights of the parameters such that, $P = P1 \cup P2 \cup P3 \cup P4 \cup P5$.

Now, we have established an optimization model to derive the exact weights of the parameters as following:

$$\left.\begin{array}{l} max\ \tilde{D}(W) = \sum\limits_{j=1}^{n}\sum\limits_{s=1}^{m} \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj}) W_j \\ s.t., W \in P; \\ 0 \leq W_j \leq 1\ and\ \sum\limits_{j=1}^{n} W_j = 1 \end{array}\right\} \tag{8.8}$$

By solving the above equation, we will obtain the exact weights of the parameters.

Case-II. *When weights of the parameters are completely unknown:*
If, the weights are completely unknown in a decision-making problem, then the weights of the parameters can be derived as follows:

$$W_j = \frac{\tilde{d}_{e_j}}{\sum\limits_{j=1}^{n} \tilde{d}_{e_j}} = \frac{\sum\limits_{s=1}^{m} \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj})}{\sum\limits_{j=1}^{n}\sum\limits_{s=1}^{m} \tilde{d}_{\text{CN}}(m_{e_j}, x_{sj})} \tag{8.9}$$

8.6.3 Complex neutrosophic soft decision-making for multiple decision makers based problems

Problem description

Let us consider that, $X = \{x_1, x_2, .., x_m\}$ be a set of m alternatives and $E = \{e_1, e_2, .., e_n\}$ be a set of n corresponding parameters. Now assume that, k decision makers, $D = \{d_1, d_2, .., d_k\}$, have been assigned to select the best alternative from m alternatives over n parameters. It is considered that, all the decision makers have given their opinions about the alternatives in terms of complex neutrosophic valued evaluations. So, we get k complex neutrosophic soft sets, $(f_{\tilde{C}_{N_1}}, E), (f_{\tilde{C}_{N_2}}, E), .., (f_{\tilde{C}_{N_k}}, E)$, where,

$$(f_{\tilde{C}_{N_l}}, E) = \{(e_1, f_{\tilde{C}_{N_l}}(e_1)), (e_2, f_{\tilde{C}_{N_l}}(e_2)), .., (e_n, f_{\tilde{C}_{N_l}}(e_n))\}$$
$$= \{(e_1, ((x_1, x_{(11,l)}), (x_2, x_{(21,l)}), .., (x_m, x_{(m1,l)}))), (e_2, ((x_1, x_{(12,l)}), (x_2, x_{(22,l)}), .., (x_m, x_{(m2,l)}))), .., (e_n, ((x_1, x_{(1n,l)}), (x_2, x_{(2n,l)}), .., (x_m, x_{(mn,l)})))\}.$$

Here, $x_{(sj,l)} = (p_{(sj,l)}e^{iu_{(sj,l)}}, q_{(sj,l)}e^{iv_{(sj,l)}}, r_{(sj,l)}e^{iw_{(sj,l)}})$ represents the complex neutrosophic valued evaluation of an alternative $x_s; s = 1, 2, .., m$ over a parameter $e_j; j = 1, 2, .., n$ given by the decision maker $d_l; l = 1, 2, .., k$. Tabular form of k CNSS has been given in Table 8.4.

Table 8.4: Tabular form of k complex neutrosophic soft sets

	$(f_{\tilde{C}_{N_1}}, E)$				$(f_{\tilde{C}_{N_2}}, E)$			
	e_1	e_2	$\cdots$	e_n	e_1	e_2	$\cdots$	e_n
x_1	$x_{(11,1)}$	$x_{(12,1)}$	$\cdots$	$x_{(1n,1)}$	$x_{(11,2)}$	$x_{(12,2)}$	$\cdots$	$x_{(1n,2)}$
x_2	$x_{(21,1)}$	$x_{(22,1)}$	$\cdots$	$x_{(2n,1)}$	$x_{(21,2)}$	$x_{(22,2)}$	$\cdots$	$x_{(2n,2)}$
			$\cdots$				$\cdots$	
x_m	$x_{(m1,1)}$	$x_{(m2,1)}$	$\cdots$	$x_{(mn,1)}$	$x_{(m1,2)}$	$x_{(m2,2)}$	$\cdots$	$x_{(mn,2)}$

	$(f_{\tilde{C}_{N_k}}, E)$			
	e_1	e_2	$\cdots$	e_n
..	$x_{(11,k)}$	$x_{(12,k)}$	$\cdots$	$x_{(1n,k)}$
..	$p_{(21,k)}$	$x_{(22,k)}$	$\cdots$	$x_{(2n,k)}$
..			$\cdots$	
..	$x_{(m1,k)}$	$x_{(m2,k)}$	$\cdots$	$x_{(mn,k)}$

- **Nature of the considered parameters.**

In a complex neutrosophic soft set based group decision-making problem, conflicting parameter may exist in the considered parameter set E. Therefore, we will equalize the sense of all the parameters by transforming the evaluations of all the alternatives over each of the cost parameter into benefit sense by using complex neutrosophic complement.

- **Weights of the parameters.**

In this group decision-making problem, weights of the parameters may or may not be given initially in the problem. Even sometimes, the information about the weights of the parameters may be incomplete. So, when the weights of the parameters are completely unknown or incompletely known in a problem, then we will derive the weights of the parameters.

If, $W = \{W_1, W_2, .., W_n\};\ 0 \leq W_j \leq 1$ be the weights of the parameters, then, $\sum_{j=1}^{n} W_j = 1$.

Now, based on the above discussion, our goals are as follows:

(i) *Equalize the sense of the all considered parameters for handling the conflicting criteria.*
(ii) *Construct a resultant CNSS* $(F_{\tilde{C}_N}, E)$ *from* k *CNSSs* $(f_{\tilde{C}_{N_l}}, E); l = 1, 2, .., k$.
(iii) *Use a normalization process for handling non commensurable parameters.*
(iv) *Derive the exact weights of the parameters if they are completely unknown or incompletely known.*
(v) *Ranking the alternatives based on the opinions of all the* k *decision makers.*

Complex neutrosophic soft VIKOR approach for multiple decision maker based problems

In this section, we have provided a decision-making approach to get a synchronized solution from this complex netrosophic soft set based group decision-making problem by using VIKOR method.

Algorithm II: Complex neutrosophic soft VIKOR approach for multiple decision makers (CNSVIMDM-approach).

Step 1. *Input the necessary substance.*
Input m alternatives ($X = \{x_1, x_2, .., x_m\}$), n parameters ($E = \{e_1, e_2, .., e_n\}$) and k complex neutrosophic soft sets $(f_{\tilde{C}_{N_1}}, E), (f_{\tilde{C}_{N_2}}, E), .., (f_{\tilde{C}_{N_k}}, E)$ as given in Table 8.4. If, the weights of the parameters are given initially in the problem then input them ($W = \{W_1, W_2, ..W_n\}$) also.

Step 2. *Equalization of the sense of all the parameters for handling conflicting criteria.*
Firstly, we have recognized the benefit parameters and cost parameters in the parameter set E. Suppose, A is the set of benefit parameters and B be the set cost parameters such that, $A \cup B = E$ and $A \cap B = \phi$. Then, we have transformed the evaluations of all the alternatives with respect to each of the cost parameters into benefit sense by using complex neutrosophic complement as follows:

$$\forall l = 1, 2, .., k\ ,\ f_{\tilde{C}_{N_l}}(e_j) = \{(x_1, x^c_{(1j,l)}), (x_2, x^c_{(2j,l)}), .., (x_m, x^c_{(mj,l)})\};\ \forall e_j \in B$$

Step 3. *Construction a resultant complex neutrosophic soft set* $(F_{\tilde{C}_N}, E)$.
By using complex neutrosophic soft intersection, we have constructed a resultant complex neutrosophic soft set $(F_{\tilde{C}_N}, E)$ from k complex neutrosophic soft sets $(f_{\tilde{C}_{N_1}}, E), (f_{\tilde{C}_{N_2}}, E), .., (f_{\tilde{C}_{N_k}}, E)$ as follows,

$$(F_{\tilde{C}_N}, E) = (f_{\tilde{C}_{N_1}}, E)\tilde{\cap}_N(f_{\tilde{C}_{N_2}}, E)\tilde{\cap}_N..\tilde{\cap}_N(f_{\tilde{C}_{N_k}}, E)$$

Then, from Definition 8.7, it is concluded that, $\forall j = 1, 2, .., n$

$$\begin{aligned} F_{\tilde{C}_N}(e_j) &= f_{\tilde{C}_{N_1}}(e_j)\tilde{\cap}_N f_{\tilde{C}_{N_2}}(e_j)\tilde{\cap}_N..\tilde{\cap}_N f_{\tilde{C}_{N_k}}(e_j) \\ &= \{x_1/(x_{(1j,1)}\tilde{\cap}_N x_{(1j,2)}\tilde{\cap}_N..\tilde{\cap}_N x_{(1j,k)}), x_2/(x_{(2j,1)}\tilde{\cap}_N x_{(2j,2)}\tilde{\cap}_N..\tilde{\cap}_N x_{(2j,k)}), .., \\ &\quad x_m/(x_{(mj,1)}\tilde{\cap}_N x_{(mj,2)}\tilde{\cap}_N..\tilde{\cap}_N x_{(mj,k)})\} \\ &= \{x_1/u_{1j}, x_2/u_{2j}, .., x_m/u_{mj}\} \end{aligned}$$

where, $\tilde{\cap}_N$ is the complex neutrosophic intersection and
$u_{sj} = (\bar{P}_{sj}e^{i\bar{U}_{sj}}, \bar{Q}_{sj}e^{i\bar{V}_{sj}}, \bar{R}_{sj}e^{i\bar{W}_{sj}});\ s = 1, 2, .., m.$

Step 4. *Derivation of the complex neutrosophic soft ideal evaluation of each of the parameters.*
Complex neutrosophic soft ideal evaluation $F^+_{\tilde{C}_N}(e_j)$ of a parameter e_j over the resultant complex neutrosophic soft set $(F_{\tilde{C}_N}, E)$ has been defined as follows:

$$\begin{aligned} F^+_{\tilde{C}_N}(e_j) = \quad & u_{1j}\tilde{\cup}^{max}_N u_{2j}\tilde{\cup}^{max}_N..\tilde{\cup}^{max}_N u_{mj} = (max(\bar{P}_{1j}, \bar{P}_{2j}, .., \bar{P}_{mj})e^{imax(\bar{U}_{1j},\bar{U}_{2j},..,\bar{U}_{mj})}, \\ & max(\bar{Q}_{1j}, \bar{Q}_{2j}, .., \bar{Q}_{mj})e^{imax(\bar{V}_{1j},\bar{V}_{2j},..,\bar{V}_{mj})}, \\ & max(\bar{R}_{1j}, \bar{R}_{2j}, .., \bar{R}_{mj})e^{imax(\bar{W}_{1j},\bar{W}_{2j},..,\bar{W}_{mj})}) = (P^+_j e^{iU^+_j}, Q^+_j e^{iV^+_j}, R^+_j e^{iW^+_j}) \end{aligned}$$

Step 5. *Determination of the bounded difference of* u_{sj} *form* $F^+_{\tilde{C}_N}(e_j)$.
We have evaluated the complex neutrosophic bounded difference $(\bar{d}^{\ominus}_{sj})$ of the evaluation u_{sj} of an alternative x_s over the parameter e_j from the associated ideal evaluation $F^+_{\tilde{C}_N}(e_j)$ over the soft set $(F_{\tilde{C}_N}, E)$ as follows:

$$\begin{aligned} \bar{d}^{\ominus}_{sj} &= F^+_{\tilde{C}_N}(e_j)\tilde{\ominus}_N u_{sj} = (max(0, P^+_j - \bar{P}_{sj})e^{imax(0,U^+_j - \bar{U}_{sj})}, \\ &\quad max(0, Q^+_j - \bar{Q}_{sj})e^{imax(0,V^+_j - \bar{V}_{sj})}, max(0, R^+_j - \bar{R}_{sj})e^{imax(0,W^+_j - \bar{W}_{sj})}) \\ &= (\check{P}_{sj}e^{i\check{U}_{sj}}, \check{Q}_{sj}e^{i\check{V}_{sj}}, \check{R}_{sj}e^{i\check{W}_{sj}}) \end{aligned}$$

The resultant Complex neutrosophic bounded difference soft set is denoted by,
$(F^{\ominus}_{\tilde{C}_N}, E) = (\bar{d}^{\ominus}_{sj})_{m\times n}$.

Step 6. *Evaluation of the score value of each of the entries in the soft set* $(F^{\ominus}_{\tilde{C}_N}, E)$.
We have derived the score value $(\hat{S}_{cr})$ of each of the entries $\bar{d}^{\ominus}_{sj}$ in the soft set $(F^{\ominus}_{\tilde{C}_N}, E)$. The

score values are presented in the score valued matrix $(\hat{S}_{cr}(\bar{d}^{\ominus}_{sj}))_{m\times n}$.

Step 7. *Normalization process to deal with non commensurable parameters.*
Firstly, we have determined the maximum score value $(\hat{S}^{+}_{cr}(\bar{d}^{\ominus}_{j}))$ over all the alternatives with respect to a parameter e_j associated with the matrix $(\hat{S}_{cr}(\bar{d}^{\ominus}_{sj}))_{m\times n}$ as,
$\hat{S}^{+}_{cr}(\bar{d}^{\ominus}_{j}) = max^{m}_{s=1}\hat{S}_{cr}(\bar{d}^{\ominus}_{sj})$.

Then, we have normalized the entry $\hat{S}_{cr}(\bar{d}^{\ominus}_{sj})$ in this matrix by the following:

$$\bar{\phi}_{sj} = \frac{\hat{S}_{cr}(\bar{d}^{\ominus}_{sj})}{\hat{S}^{+}_{cr}(\bar{d}^{\ominus}_{j})}$$

Step 8. *Evaluation of the group utility value of an alternative x_s (Utility measure).*
The group utility value $(\hat{S}^{W}_{cr}(x_s))$ of an alternative x_s is derived by adding the weighted values of $\bar{\phi}_{s1}$, $\bar{\phi}_{s2}$,..,$\bar{\phi}_{sn}$ over all the parameters as follows:

$$\hat{S}^{W}_{cr}(x_s) = \sum_{j=1}^{n} W_j\bar{\phi}_{sj} \tag{8.10}$$

where, W_j is the weight of the parameter e_j. If W_j is not given in the problem, then determine it by using Subsection 8.6.2 on the resultant complex neutrosophic soft set $(F_{\tilde{C}_N}, E)$.

Step 9. *Evaluation of the individual regret score of an alternative x_s (Regret measure).*
The individual regret $(\hat{S}^{max}_{cr}(x_s))$ of an alternative x_s is derived as follows:

$$\hat{S}^{max}_{cr}(x_s) = \max_{j}\{W_j\bar{\phi}_{sj}\} \tag{8.11}$$

Step 10. *Derivation of the compromising index $\hat{C}(x_s)$ of an alternative x_s.*
The compromising index of an alternative over all the parameters is evaluated by the following equation,

$$\hat{C}(x_s) = \theta\frac{\hat{S}^{W}_{cr}(x_s) - \overline{\hat{S}^{W}_{cr}}(x_s)}{\underline{\hat{S}^{W}_{cr}}(x_s) - \overline{\hat{S}^{W}_{cr}}(x_s)} + (1-\theta)\frac{\hat{S}^{max}_{cr}(x_s) - \overline{\hat{S}^{max}_{cr}}(x_s)}{\underline{\hat{S}^{max}_{cr}}(x_s) - \overline{\hat{S}^{max}_{cr}}(x_s)} \tag{8.12}$$

$\overline{\hat{S}^{W}_{cr}}(x_s) = \min_{s}\hat{S}^{W}_{cr}(x_s)$, $\underline{\hat{S}^{W}_{cr}}(x_s) = \max_{s}\hat{S}^{W}_{cr}(x_s)$, $\overline{\hat{S}^{max}_{cr}}(x_s) = \min_{s}\hat{S}^{max}_{cr}(x_s)$, $\underline{\hat{S}^{max}_{cr}}(x_s) = \max_{s}\hat{S}^{max}_{cr}(x_s)$
where, the parameter $\theta \in [0,1]$ indicates the weight of the strategy of maximum group of utility.

Step 11. *Determination of a compromise optimal solution.*
Now, we have obtained a compromise optimal alternative by using Step 10 of Algorithm I. (CNSVISDM-approach).
Flowchart of our proposed algorithm has been given in Figure 8.4.

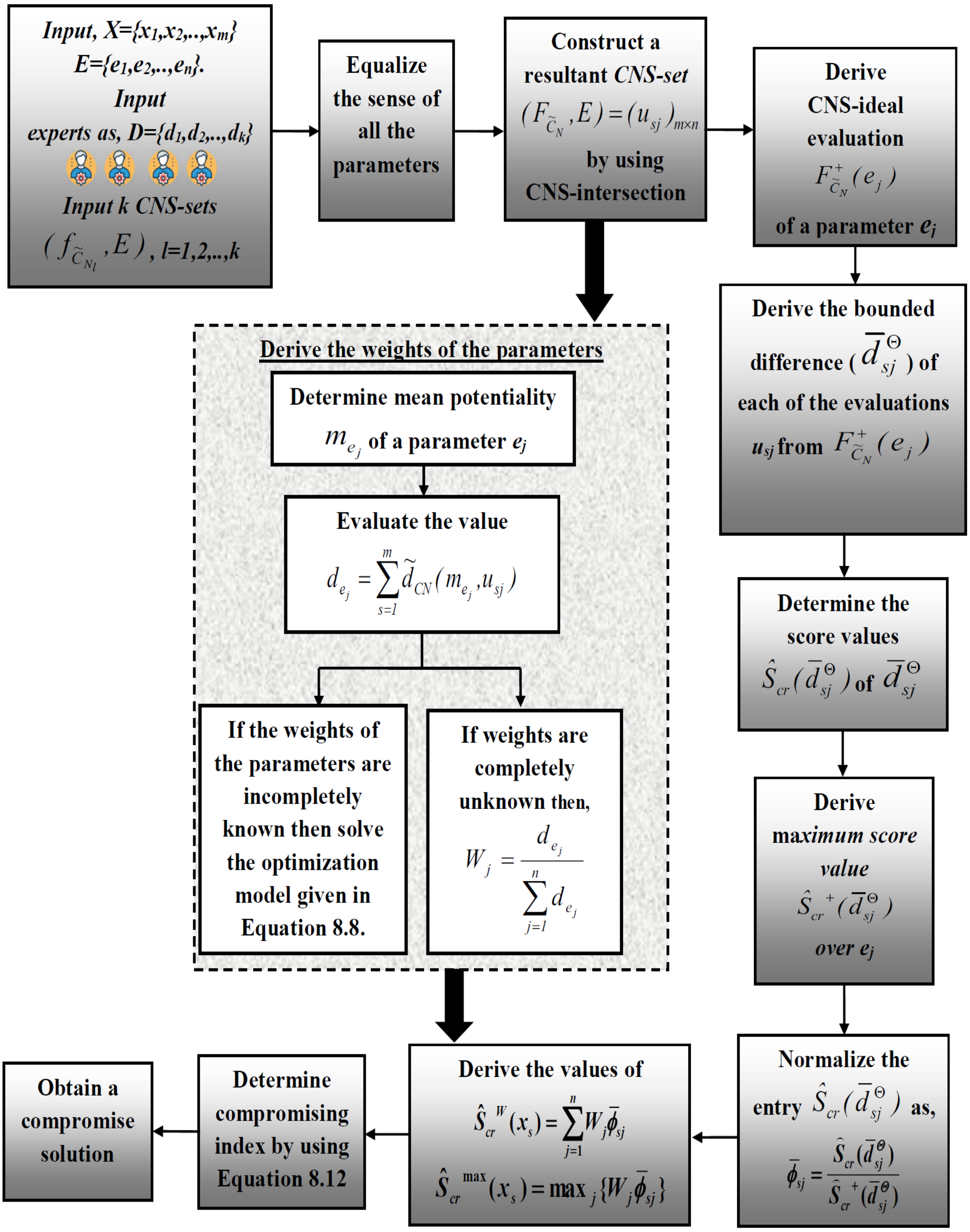

Figure 8.4: Flowchart of our proposed approach

8.7 Case study

8.7.1 An application of CNSVISDM-approach in manager selection of a company

At the present era, the number of competitors of a company is increasing rapidly in global market. Therefore, to preserve its position in global market and to increase its economic environment, proper manager selection is an important task to a company. Therefore, in this section, we have solved a manager selection problem of a company with the help of our proposed Algorithm I.

Example 8.4. Now consider that, four candidates have come to give an interview in the manager post of a company. The corresponding parameters over these four candidates are, $\{engineering\ background\ knowledge,\ ability\ in\ budge\ managing,$ $ability\ to\ listening\ and\ understanding\ the\ customer's\ comment,\ realiability\}$.

Now if a decision maker evaluates the satisfaction of the parameter '*engineering background knowledge*' over a candidate, then, the only information about the engineering result of the candidate may not be sufficient for best manager selection. Because, in that case, additionally, the information about the '*technical knowledge*' of the candidate is also necessary and should be known to the decision maker. So, these two information '*engineering background knowledge*' and '*technical knowledge*' should be considered together. Now, if we will use complex neutrosophic environment, then we can handle this problem by taking '*engineering background knowledge*' as the amplitude term and '*technical knowledge*' as the phase term in the evaluation of a candidate over the parameter '*engineering background knowledge*'. Again, for the parameter '*ability in budget managing*', the information about the '*supervising responsibility*' of a candidate is a necessary information. Therefore, here we have taken '*ability in budget managing*' as the amplitude term and '*supervising responsibility*' as the phase term in the evaluation of a candidate over the parameter '*ability in budget managing*'. Similarly, for the parameter '*ability to listening and understanding the customer's comment*', we have taken the information '*good behaving ability to the others*' additionally. Then, '*ability to listening and understanding the customer's comment*' has been considered as the amplitude term and '*good behaving ability to the others*' has been considered as the phase term. Lastly, for the parameter '*reliability*', the information about the '*political sensitivity*' has been considered as the phase term where, the information about the '*reliability*' of a candidate has been considered as the amplitude term.

Now, assume that, $X = \{C_1, C_2, C_3, C_4\}$ be the set of four candidates as the initial universal set and $E = \{Engineering\ background\ knowledge\ (e_1),\ Ability\ in\ budge$ $managing\ (e_2),\ Ability\ to\ listening\ and\ understanding\ customer's\ comment\ (e_3),$ $Realiability\ (e_4)\}$ be the set of corresponding parameters of the elements of X. The

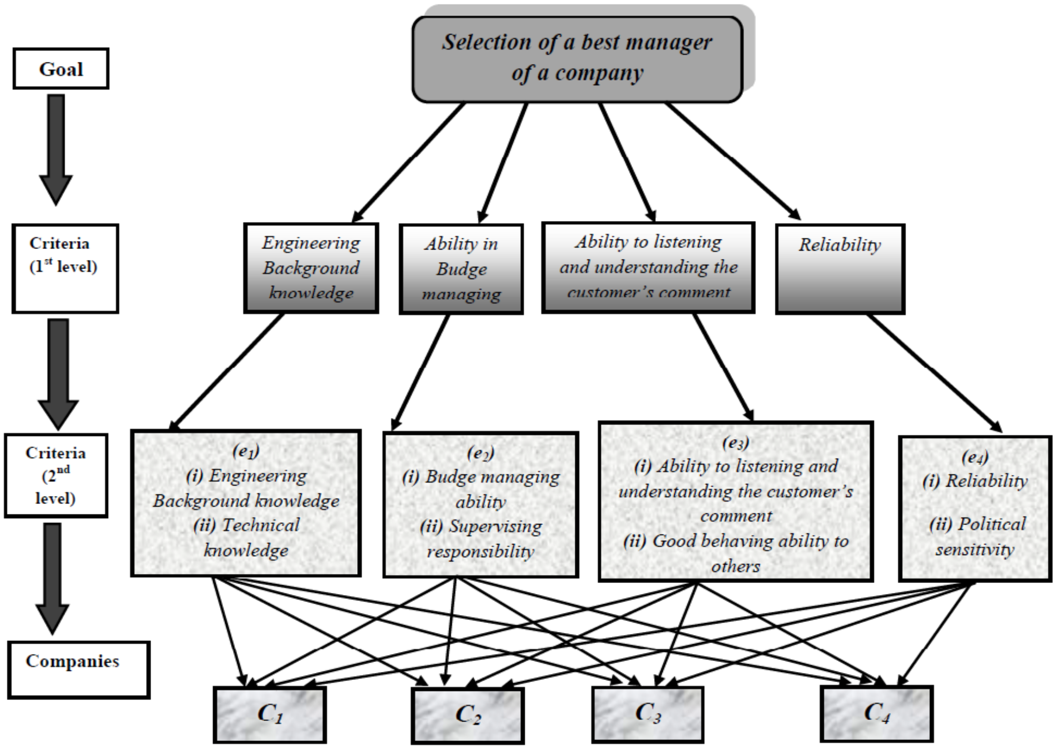

Figure 8.5: Modeling framework of case study 8.7.1

evaluations of the four candidates over these four parameters have been given in a CNSS $(f_{\tilde{C}_N}, E)$ (Table 8.5).

Table 8.5: CNSS $(f_{\tilde{C}_N}, E)$ (Example 8.4)

	e_1	e_2	e_3
C_1	$(0.3e^{i\pi/2}, 0.2e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.8e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.2e^{i2\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$
C_2	$(0.9e^{i\pi}, 0.2e^{i\pi}, 0.1e^{i\pi})$	$(0.2e^{i\pi/3}, 0.3e^{i\pi/3}, 0.9e^{i\pi/3})$	$(0.1e^{i\pi/6}, 0.4e^{i\pi/6}, 0.9e^{i\pi/6})$
C_3	$(0.6e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.5e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.1e^{i\pi/4}, 0.5e^{i\pi/4}, 0.7e^{i\pi/4})$
C_4	$(0.2e^{i\pi/2}, 0.6e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.6e^{i3\pi/4}, 0.4e^{i3\pi/4}, 0.3e^{i3\pi/4})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.7e^{i2\pi/3})$

	e_4
C_1	$(0.8e^{i\pi/3}, 0.1e^{i\pi/3}, 0.2e^{i\pi/3})$
C_2	$(0.3e^{i\pi/2}, 0.6e^{i\pi/2}, 0.7e^{i\pi/2})$
C_3	$(0.1e^{i2\pi}, 0.5e^{i2\pi}, 0.7e^{i2\pi})$
C_4	$(0.2e^{i\pi/2}, 0.3e^{i\pi/2}, 0.4e^{i\pi/2})$

Now the problem is 'to select the best candidate for the manager post who satisfies all the

considered parameters with maximum evaluation level'.
In Figure 8.5, graphical framework of this case study has been expressed.

Solution:
We have solved this manager selection problem by using our proposed Algorithm I.
Step 1,2. The corresponding CNSS $(f_{\tilde{C}_N}, E)$ has been given in Table 8.5.

In this decision problem, all the considering parameters are benefitted parameters. Therefore, equalization process is not needed here.
Step 3. Complex neutrosophic soft ideal evaluation of each of the parameters over the CNSS $(f_{\tilde{C}_N}, E)$ is as follows:
$f^+_{\tilde{C}_N}(e_1) = (0.9e^{i\pi}, 0.6e^{i\pi}, 0.8e^{i\pi})$; $f^+_{\tilde{C}_N}(e_2) = (0.8e^{i3\pi/4}, 0.4e^{i3\pi/4}, 0.9e^{i3\pi/4})$;
$f^+_{\tilde{C}_N}(e_3) = (0.4e^{i2\pi/3}, 0.5e^{i2\pi/3}, 0.9e^{i2\pi/3})$; $f^+_{\tilde{C}_N}(e_4) = (0.8e^{i2\pi}, 0.6e^{i2\pi}, 0.7e^{i2\pi})$.
Step 4. The bounded difference $(d^{\ominus}_{sj})$ of each of the evaluations of the alternatives with respect to a parameter e_j from the associated complex neutrosophic soft ideal evaluation $f^+_{\tilde{C}_N}(e_j)$ is given in the complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (Table 8.6).

Table 8.6: Complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (Example 8.4)

	e_1	e_2	e_3
C_1	$(0.6e^{i\pi/2}, 0.4e^{i\pi/2}, 0.1e^{i\pi/2})$	$(0e^{i\pi/12}, 0.1e^{i\pi/12}, 0.7e^{i\pi/12})$	$(0.2e^{i\pi/6}, 0.4e^{i\pi/6}, 0.2e^{i\pi/6})$
C_2	$(0e^{i0}, 0.2e^{i0}, 0.7e^{i0})$	$(0.6e^{i5\pi/12}, 0.1e^{i5\pi/12}, 0e^{i5\pi/12})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0e^{i\pi/2})$
C_3	$(0.3e^{i\pi/3}, 0.3e^{i\pi/3}, 0.3e^{i\pi/3})$	$(0.5e^{i\pi/4}, 0.3e^{i\pi/4}, 0.1e^{i\pi/4})$	$(0.3e^{i5\pi/12}, 0e^{i5\pi/12}, 0.2e^{i5\pi/12})$
C_4	$(0.7e^{i\pi/2}, 0e^{i\pi/2}, 0e^{i\pi/2})$	$(0.2e^{i0}, 0e^{i0}, 0.6e^{i0})$	$(0e^{i0}, 0.2e^{i0}, 0.2e^{i0})$

	e_4
C_1	$(0e^{i5\pi/3}, 0.5e^{i5\pi/3}, 0.5e^{i5\pi/3})$
C_2	$(0.5e^{i3\pi/2}, 0e^{i3\pi/2}, 0e^{i3\pi/2})$
C_3	$(0.7e^{i0}, 0.1e^{i0}, 0e^{i0})$
C_4	$(0.6e^{i3\pi/2}, 0.3e^{i3\pi/2}, 0.3e^{i3\pi/2})$

Step 5. By using Definition 8.5, the score value of each of the entries in the complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (given in Table 8.6) has been given in Table 8.7.
Step 6. *Normalization process.*
The maximum score value with respect to each of the parameters is s follows:
$\hat{S}^+_{cr}(d^{\ominus}_1) = 0.74$; $\hat{S}^+_{cr}(d^{\ominus}_2) = 0.72$; $\hat{S}^+_{cr}(d^{\ominus}_3) = 0.66$; $\hat{S}^+_{cr}(d^{\ominus}_4) = 0.77$.
Then, by using Equation 8.2, the normalized value of each of the entries in the score valued matrix has been given in Table 8.8.

Step 7. *Determination of the weights of the parameters.*
Since, in this problem, weights of the parameters are completely unknown, therefore, we

Table 8.7: Score valued matrix (Example 8.4)

	e_1	e_2	e_3	e_4
C_1	0.64	0.53	0.59	0.36
C_2	0.52	0.71	0.66	0.62
C_3	0.59	0.66	0.65	0.77
C_4	0.74	0.60	0.60	0.54

Table 8.8: Normalized score valued matrix (Example 8.4)

	e_1	e_2	e_3	e_4
C_1	0.86	0.75	0.89	0.47
C_2	0.70	1	1	0.80
C_3	0.79	0.93	0.98	1
C_4	1	0.86	0.91	0.70

have obtained the exact weights of the parameters by using subsection 8.6.2.
7.1 By using Definition 8.9, the mean potentiality of each of the parameters is as follows:
$m_{e_1} = (0.5e^{i2\pi/3}, 0.375e^{i2\pi/3}, 0.525e^{i2\pi/3})$; $m_{e_2} = (0.475e^{i9\pi/16}, 0.275e^{i9\pi/16}, 0.55e^{i9\pi/16})$;
$m_{e_3} = (0.2e^{i19\pi/48}, 0.375e^{i19\pi/48}, 0.75e^{i19\pi/48})$; $m_{e_4} = (0.35e^{i5\pi/6}, 0.375e^{i5\pi/6}, 0.5e^{i5\pi/6})$.
7.2 Then the complex neutrosophic distance (d_{sj}) of the evaluation (x_{sj}) of an alternative with respect to the parameter e_j and its corresponding mean potentiality m_{e_j} has given in Table 8.9.

Table 8.9: Tabular form of the distance values (Example 8.4)

	e_1	e_2	e_3	e_4
C_1	0.2	0.35	0.275	0.425
C_2	0.425	0.35	0.15	0.225
C_3	0.1	0.25	0.125	0.58
C_4	0.3	0.25	0.2	0.167

7.3 Now, the total complex neutrosophic distance of all the alternatives from the corresponding mean potentiality m_{e_j} associated with the parameter e_j is as follows:
$\tilde{d}_{e_1} = 1.02$; $\tilde{d}_{e_2} = 1.2$; $\tilde{d}_{e_3} = 0.75$; $\tilde{d}_{e_4} = 1.4$.
7.4 Now, By using Equation 8.9, the weights of the parameters are as following,
$W_1 = 0.2$; $W_2 = 0.3$; $W_3 = 0.2$; $W_4 = 0.3$.
Step 8. Then, by using Equation 8.3, group utility values of the alternatives are as follows:
$\hat{S}^W_{cr}(C_1) = 0.716$; $\hat{S}^W_{cr}(C_2) = 0.88$; $\hat{S}^W_{cr}(C_3) = 0.933$; $\hat{S}^W_{cr}(C_4) = 0.85$.
Step 8. Then, By using Equation 8.4, individual regret values of the alternatives are as follows:
$\hat{S}^{max}_{cr}(C_1) = 0.225$; $\hat{S}^{max}_{cr}(C_2) = 0.3$; $\hat{S}^{max}_{cr}(C_3) = 0.3$; $\hat{S}^{max}_{cr}(C_4) = 0.258$.
Step 9. Then, by utilizing Equation 8.5, the compromising index of the alternatives are,
$\hat{C}(C_1) = 0$; $\hat{C}(C_2) = 0.88$; $\hat{C}(C_3) = 1$; $\hat{C}(C_4) = 0.53$. Here, we have taken $\theta = 0.5$
Step 10. *Ranking of the alternatives:*
Now, based on $\hat{S}^W_{cr}$, the ranking order is, $C_1 > C_4 > C_2 > C_3$,
based on $\hat{S}^{max}_{cr}$, the ranking order is, $C_1 > C_4 > C_2 = C_3$ and based on $\hat{C}$, the order is,

$C_1 > C_4 > C_2 > C_3$.
Condition 1. Again, Condition 1 satisfies truly since,
$\hat{C}(C_1) - \hat{C}(C_4) = 0.53 - 0 = 0.53 > \frac{1}{4-1} = 0.33$.
Condition 2. Condition 2 also satisfies truly since, C_1 is the best each of the $\hat{C}$, $\hat{S}^{W}_{cr}$ and $\hat{S}^{max}_{cr}$.

Therefore according to our algorithm it is concluded that, the alternative C_1 is a compromise optimal solution for this decision problem.
Hence, the candidate C_1 is best for the manager post of this company.

8.7.2 An application of CNSVISDM-approach in sustainable manufacturing material selection of an automotive industry

In today's life, sustainable manufacturing in an automotive industry gains more attention because of the essentiality of balancing social, environmental and economic impacts so that, the development of the industry does not harmful to the society or environment. Now, we have used our proposed complex neutrosophic soft VIKOR approach to select the best sustainable automobile manufacturing material alternative in an automotive industry.

Example 8.5. Based on the reference [152], we have considered four automobile manufacturing material alternatives such as, $\{ferrous\ metal\ (iron\ or\ steel),\ organic\ composite,\ plastics, aluminum\}$ and three corresponding parameters including, $\{environmental\ impact,\ economic\ impact,\ social\ impact\}$. Now, if the decision maker gives his/her concentration only on the '*greenhouse gas emission*' of a manufacturing material alternative over the parameter '*environmental impact*', then this information is not sufficient to select the best sustainable manufacturing material alternative, because additionally, the information about the '*energy consumption level*' associated with the parameter '*environmental impact*' is also necessary for selecting the best sustainable manufacturing material. Therefore, by using complex neutrosophic environment, we have taken these two information '*greenhouse gas emission*' and '*energy consumption level*' together to evaluate the four manufacturing materials over this parameter '*environmental impact*' where,'*greenhouse gas emission*' has been considered as the amplitude term and '*energy consumption level*' has been considered as the phase term. Similarly, for the parameter '*economic impact*', we have taken two information together such as, '*product cost*' and '*maintenance cost*' where, '*product cost*' has been considered as the amplitude term and '*maintenance cost*' has been considered as the phase term. Lastly, for the parameter '*social impact*', we have taken two information together such as, '*health status of adjacent peoples*' and '*safety of the adjacent peoples*', where, '*health status of adjacent peoples*' has been taken as amplitude term and '*safety of adjacent peoples*' has been taken as the phase term.

Now let, $X = \{M_1\ (ferrous\ metal),\ M_2\ (organic\ composite),\ M_3\ (plastics),\ M_4\ (aluminum)\}$ be the initial universal set and $E = \{e_1\ (environmental\ impact),\ e_2\ (economic\ impact),\ e_3\ (social\ impact)\}$ be the set of parameters. The associated complex neutrosophic soft set, which represents the evaluations of the four alternatives over the three parameters, has been given in Table 8.10.

Table 8.10: CNSS $(f_{\tilde{C}_N}, E)$ (Example 8.5)

	e_1	e_2	e_3
M_1	$(0.7e^{i3\pi/2}, 0.8e^{i3\pi/2}, 0.3e^{i3\pi/2})$	$(0.2e^{i4\pi/3}, 0.7e^{i4\pi/3}, 0.8e^{i4\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$
M_2	$(0.1e^{i\pi}, 0.8e^{i\pi}, 0.9e^{i\pi})$	$(0.9e^{i5\pi/3}, 0.7e^{i5\pi/3}, 0.2e^{i5\pi/3})$	$(0.1e^{i\pi/6}, 0.4e^{i\pi/6}, 0.9e^{i\pi/6})$
M_3	$(0.5e^{i\pi/3}, 0.7e^{i\pi/3}, 0.6e^{i\pi/3})$	$(0.8e^{i3\pi/2}, 0.9e^{i3\pi/2}, 0.3e^{i3\pi/2})$	$(0.1e^{i\pi/4}, 0.5e^{i\pi/4}, 0.7e^{i\pi/4})$
M_4	$(0.8e^{i3\pi/2}, 0.6e^{i3\pi/2}, 0.2e^{i3\pi/2})$	$(0.3e^{i5\pi/4}, 0.6e^{i5\pi/4}, 0.6e^{i5\pi/4})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.7e^{i2\pi/3})$

Now, consider that, the information about the weights of the parameters is incompletely known.
If, $W = \{W_1, W_2, W_3\}$ be the weights of the three parameters , then the information is, $P = \{W_1 \geq 0.4;\ 0.1 \leq W_2 \leq 0.9;\ W_2 - W_3 < 0.3;\ W_3 \geq 0.2\}$.

Now, our problem is to select the best sustainable manufacturing material alternative over the three considered parameters.

Solution: Now, we have solved this problem by using our proposed Algorithm I.
Step 1. The corresponding complex neutrosophic soft set has been given in Table 8.10.
Step 2. Here, '*environmental impact*' and '*economic impact*' are two cost parameters, because over these two parameters, small evaluation indicates the goodness of a manufacturing material. Therefore, we have taken complex neutrosophic complement of all the evaluations over these two parameters. Results are given in Table 8.11.

Table 8.11: CNSS $(f_{\tilde{C}_N}, E)$ (Example 8.5) (after equalization)

	e_1	e_2	e_3
M_1	$(0.3e^{i\pi/2}, 0.2e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.8e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.2e^{i2\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$
M_2	$(0.9e^{i\pi}, 0.2e^{i\pi}, 0.1e^{i\pi})$	$(0.2e^{i\pi/3}, 0.3e^{i\pi/3}, 0.9e^{i\pi/3})$	$(0.1e^{i\pi/6}, 0.4e^{i\pi/6}, 0.9e^{i\pi/6})$
M_3	$(0.6e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.5e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.1e^{i\pi/4}, 0.5e^{i\pi/4}, 0.7e^{i\pi/4})$
M_4	$(0.2e^{i\pi/2}, 0.6e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.6e^{i3\pi/4}, 0.4e^{i3\pi/4}, 0.3e^{i3\pi/4})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.7e^{i2\pi/3})$

Step 3. The complex neutrosophic soft ideal evaluation of each of the parameters is as follows:
$f^{+}_{\tilde{C}_N}(e_1) = (0.9e^{i\pi}, 0.6e^{i\pi}, 0.8e^{i\pi})$; $f^{+}_{\tilde{C}_N}(e_2) = (0.8e^{i\frac{3\pi}{4}}, 0.4e^{i\frac{3\pi}{4}}, 0.9e^{i\frac{3\pi}{4}})$;
$f^{+}_{\tilde{C}_N}(e_3) = (0.4e^{i\frac{2\pi}{3}}, 0.5e^{i\frac{2\pi}{3}}, 0.9e^{i\frac{2\pi}{3}})$.
Step 4. The bounded difference of each of the evaluations of the alternatives with respect to a parameter e_j from the corresponding $f^{+}_{\tilde{C}_N}(e_j)$ are given in Table 8.12.
Step 5. The score value of each of the entries in Table 8.12 has been given in Table 8.13.

Table 8.12: Complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (Example 8.5)

	e_1	e_2	e_3
M_1	$(0.6e^{i\pi/2}, 0.4e^{i\pi/2}, 0.1e^{i\pi/2})$	$(0e^{i\pi/12}, 0.1e^{i\pi/12}, 0.7e^{i\pi/12})$	$(0.2e^{i\pi/6}, 0.4e^{i\pi/6}, 0.2e^{i\pi/6})$
M_2	$(0e^{i0}, 0.2e^{i0}, 0.7e^{i0})$	$(0.6e^{i5\pi/12}, 0.1e^{i5\pi/12}, 0e^{i5\pi/12})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0e^{i\pi/2})$
M_3	$(0.3e^{i\pi/3}, 0.3e^{i\pi/3}, 0.3e^{i\pi/3})$	$(0.5e^{i\pi/4}, 0.3e^{i\pi/4}, 0.1e^{i\pi/4})$	$(0.3e^{i5\pi/12}, 0e^{i5\pi/12}, 0.2e^{i5\pi/12})$
M_4	$(0.7e^{i\pi/2}, 0e^{i\pi/2}, 0e^{i\pi/2})$	$(0.2e^{i0}, 0e^{i0}, 0.6e^{i0})$	$(0e^{i0}, 0.2e^{i0}, 0.2e^{i0})$

Step 6. Now by using linear max normalization process, the normalized evaluation of each of the entries of Table 8.13 has been provided in Table 8.14.

Table 8.13: Score valued matrix (Example 8.5)

	e_1	e_2	e_3
M_1	0.64	0.53	0.59
M_2	0.52	0.71	0.66
M_3	0.59	0.66	0.65
M_4	0.74	0.60	0.60

Table 8.14: Normalized score valued matrix (Example 8.5)

	e_1	e_2	e_3
M_1	0.86	0.75	0.89
M_2	0.70	1	1
M_3	0.79	0.93	0.98
M_4	1	0.86	0.91

Step 7. Now, we have determined the exact weights of the parameters by using Subsection 8.6.2.
The given incomplete information about the weights of the parameters is, $P = \{W_1 \geq 0.4;\ 0.1 \leq W_2 \leq 0.9;\ W_2 - W_3 < 0.3;\ W_3 \geq 0.2\}$.
Then, the mean potentiality of each of the parameters is as follows:
$m_{e_1} = (0.5e^{i2\pi/3}, 0.375e^{i2\pi/3}, 0.525e^{i2\pi/3})$; $m_{e_2} = (0.475e^{i9\pi/16}, 0.275e^{i9\pi/16}, 0.55e^{i9\pi/16})$;
$m_{e_3} = (0.2e^{i19\pi/48}, 0.375e^{i19\pi/48}, 0.75e^{i19\pi/48})$.
After that, the total complex neutrosophic distance of all the alternatives from the corresponding mean potentiality m_{e_j} associated with a parameter e_j is, $\tilde{d}_{e_1} = 1.02$; $\tilde{d}_{e_2} = 1.2$; $\tilde{d}_{e_3} = 0.75$.
Now, to obtain the weights of the parameters, we have constructed a optimization model by using the given incomplete information as follows:

$$\left.\begin{array}{l} max\ \tilde{D}(W) = 1.02W_1 + 1.2W_2 + 0.75W_3 \\ s.t., W \in P; \\ 0 \leq W_j \leq 1; j = 1, 2, .., n; \\ \sum\limits_{j=1}^{n} W_j = 1 \end{array}\right\} \tag{8.13}$$

Solving the above optimization model, the weights of the parameters are,
$W_1 = 0.4;\ W_2 = 0.4;\ W_3 = 0.2$.
Step 8. The group utility values of the alternatives are,

$\hat{S}^W_{cr}(M_1) = 0.822$; $\hat{S}^W_{cr}(M_2) = 0.88$; $\hat{S}^W_{cr}(M_3) = 0.884$; $\hat{S}^W_{cr}(M_4) = 0.926$.
Step 9. The individual regret values of the alternatives are,
$\hat{S}^{max}_{cr}(M_1) = 0.344$; $\hat{S}^{max}_{cr}(M_2) = 0.4$; $\hat{S}^{max}_{cr}(M_3) = 0.372$; $\hat{S}^{max}_{cr}(M_4) = 0.4$.
Step 10. Then the compromising index of the alternatives are,
$\hat{C}(M_1) = 0$; $\hat{C}(M_2) = 0.78$; $\hat{C}(M_3) = 0.55$; $\hat{C}(M_4) = 1$. Here, we have considered $\theta = 0.5$.
Step 11. *Ranking the alternatives.*
The ranking order of the alternatives based on $\hat{S}^W_{cr}$ is, $M_1 > M_2 > M_3 > M_4$, the ranking order of the alternatives based on $\hat{S}^{max}_{cr}$ is, $M_1 > M_3 > M_2 = M_4$ and the ranking order of the alternatives based on $\hat{C}$ is, $M_1 > M_3 > M_2 > M_4$.
Condition 1. Now we have seen that, condition 1 is satisfied as,
$\hat{C}(M_3) - \hat{C}(M_1) = 0.55 > \frac{1}{4-1} = 0.33$.
Condition 2. Again, M_1 is best for both the cases of $\hat{S}^W_{cr}$ and $\hat{S}^{max}_{cr}$.

Hence, M_1 is a compromise optimal solution for this decision-making problem. Therefore, the manufacturing material alternative M_1 can be selected in the automobile industry.

8.7.3 An application of CNSVIMDM-approach in medical science

Proper disease diagnosis of a patient is a significant task in medical science. Several researchers have proposed different mathematical models by using soft set theory to deal with disease diagnosis problems. For instance, Basu et al. [22] proposed a fuzzy soft set based balanced solution to detect the exact disease of a patient. Then, Das and Kar [50] proposed a group decision-making approach by using intuitionistic fuzzy soft set theory where, a set of four experts have been selected to detect the exact disease of a patient. But, in these soft set based models, they have only considered the information about the '*degree of belongingness of a symptom*' for disease diagnosis. But, the information about the '*time duration of a symptom*' to a patient is also emergent here and to be considered for proper diagnosis of the patient. Therefore, to solve these types of medical diagnosis problems, we have used our proposed complex neutrosophic soft set based approach by considering two information *'belongingness of a symptom' and 'time duration of a symptom'* together.

Example 8.6 ([22, 50]) Consider that, a patient has four symptoms such as, $\{abdominal\ pain,\ fever,\ headache,\ weightloss\}$ and the corresponding diseases related to these four symptoms are $\{typhoid,\ peptic\ ulcer,\ food\ poisoning,\ acute\ viral\ hepatitis\}$. Now assume that, a set of three experts have been selected to make a common decision about 'which disease is more likely to appear in a patient'.

Now, consider that, $X = \{typhoid\ (x_1),\ peptic\ ulcer\ (x_2),\ food\ poisoning\ (x_3),\ acute\ viral\ hepatitis\ (x_4)\}$ be the universal set and $E = \{abdominal\ pain\ (e_1),\ fever\ (e_2),\ headache\ (e_3),\ weightloss\ (e_4)\}$ be the set of corresponding parameters. The weights of the three parameters have been given initially in this group decision-making as, $W_1 = 0.27,\ W_2 = 0.22,\ W_3 = 0.17,\ W_4 = 0.34.$

Table 8.15: CNSS $(f_{\tilde{C}_{N_1}}, E)$ (Example 8.6)

	e_1	e_2	e_3
x_1	$(0.4e^{i\pi/2}, 0.2e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.6e^{i2\pi/3}, 0.4e^{i2\pi/3}, 0.2e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.4e^{i\pi/2}, 0.7e^{i\pi/2})$
x_2	$(1e^{i\pi}, 0.4e^{i\pi}, 0.3e^{i\pi})$	$(0.6e^{i\pi/3}, 0.3e^{i\pi/3}, 0.1e^{i\pi/3})$	$(0.8e^{i\pi/6}, 0.2e^{i\pi/6}, 0.3e^{i\pi/6})$
x_3	$(0.5e^{i2\pi/3}, 0.2e^{i2\pi/3}, 0.6e^{i2\pi/3})$	$(0.4e^{i\pi/2}, 0.3e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.5e^{i\pi/4}, 0.3e^{i\pi/4}, 0.7e^{i\pi/4})$
x_4	$(0.1e^{i\pi/2}, 0.5e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.5e^{i3\pi/4}, 0.2e^{i3\pi/4}, 0.6e^{i3\pi/4})$	$(0.2e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.8e^{i2\pi/3})$

	e_4
x_1	$(0.7e^{i\pi/3}, 0.1e^{i\pi/3}, 0.2e^{i\pi/3})$
x_2	$(0.8e^{i\pi/2}, 0.3e^{i\pi/2}, 0.5e^{i\pi/2})$
x_3	$(0.1e^{i2\pi}, 0.5e^{i2\pi}, 0.7e^{i2\pi})$
x_4	$(0.2e^{i\pi/2}, 0.6e^{i\pi/2}, 0.4e^{i\pi/2})$

Table 8.16: CNSS $(f_{\tilde{C}_{N_2}}, E)$ (Example 8.6)

	e_1	e_2	e_3
x_1	$(0.3e^{i\pi/2}, 0.3e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.5e^{i2\pi/3}, 0.1e^{i2\pi/3}, 0.4e^{i2\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.8e^{i\pi/2})$
x_2	$(0.8e^{i\pi}, 0.4e^{i\pi}, 0.2e^{i\pi})$	$(0.7e^{i\pi/3}, 0.4e^{i\pi/3}, 0.1e^{i\pi/3})$	$(0.5e^{i\pi/6}, 0.1e^{i\pi/6}, 0.4e^{i\pi/6})$
x_3	$(0.5e^{i2\pi/3}, 0.1e^{i2\pi/3}, 0.7e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.1e^{i\pi/4}, 0.5e^{i\pi/4}, 0.7e^{i\pi/4})$
x_4	$(0.5e^{i\pi/2}, 0.3e^{i\pi/2}, 0.6e^{i\pi/2})$	$(0.5e^{i3\pi/4}, 0.6e^{i3\pi/4}, 0.7e^{i3\pi/4})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.6e^{i2\pi/3})$

	e_4
x_1	$(0.4e^{i\pi/3}, 0.1e^{i\pi/3}, 0.2e^{i\pi/3})$
x_2	$(0.6e^{i\pi/2}, 0.3e^{i\pi/2}, 0.2e^{i\pi/2})$
x_3	$(0.1e^{i2\pi}, 0.4e^{i2\pi}, 0.7e^{i2\pi})$
x_4	$(0.2e^{i\pi/2}, 0.2e^{i\pi/2}, 0.6e^{i\pi/2})$

Table 8.17: CNSS $(f_{\tilde{C}_{N_3}}, E)$ (Example 8.6)

	e_1	e_2	e_3
x_1	$(0.2e^{i\pi/2}, 0.2e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.2e^{i2\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$
x_2	$(0.9e^{i\pi}, 0.2e^{i\pi}, 0.1e^{i\pi})$	$(0.7e^{i\pi/3}, 0.3e^{i\pi/3}, 0.4e^{i\pi/3})$	$(0.6e^{i\pi/6}, 0.4e^{i\pi/6}, 0.3e^{i\pi/6})$
x_3	$(0.5e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.5e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.3e^{i\pi/4}, 0.4e^{i\pi/4}, 0.6e^{i\pi/4})$
x_4	$(0.2e^{i\pi/2}, 0.4e^{i\pi/2}, 0.8e^{i\pi/2})$	$(0.5e^{i3\pi/4}, 0.4e^{i3\pi/4}, 0.3e^{i3\pi/4})$	$(0.4e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.8e^{i2\pi/3})$

	e_4
x_1	$(0.5e^{i\pi/3}, 0.1e^{i\pi/3}, 0.2e^{i\pi/3})$
x_2	$(0.8e^{i\pi/2}, 0.5e^{i\pi/2}, 0.3e^{i\pi/2})$
x_3	$(0.1e^{i2\pi}, 0.5e^{i2\pi}, 0.7e^{i2\pi})$
x_4	$(0.2e^{i\pi/2}, 0.3e^{i\pi/2}, 0.7e^{i\pi/2})$

Now, Three corresponding CNSSs given by three experts ($D = \{d_1, d_2, d_3\}$) are $(f_{\tilde{C}_{N_1}}, E), (f_{\tilde{C}_{N_2}}, E), (f_{\tilde{C}_{N_3}}, E)$ which have been given in Tables 8.15, 8.16, 8.17 respectively. In each of the membership magnitudes, amplitude term indicates the '*belongingness of a symptom*' and phase term indicates the '*time duration of the said symptom*'. All the data has been collected based on 10 consecutive days. The value 2π has been taken in the phase term instead of 10 days.

In Figure 8.6, the graphical framework of this decision-making problem has been given.

Solution. We have solved this group decision-making problem by using our proposed Algorithm II (**CNSVIMDM-approach**).

Step 1. Here, all the parameters are benefit parameters. Therefore, we do not need to equalize the sense of all the parameters.

Step 2. Now, we have constructed a resultant complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ form these three CNSSs $(f_{\tilde{C}_{N_1}}, E), (f_{\tilde{C}_{N_2}}, E)$ and $(f_{\tilde{C}_{N_3}}, E)$ by using complex neutrosophic soft intersection operator.

By using complex neutrosophic soft standard intersection, the resultant has been given in Table 8.18.

Table 8.18: Resultant CNSS $(f_{\tilde{C}_N}, E)$ (Example 8.6)

	e_1	e_2	e_3
x_1	$(0.2e^{i\pi/2}, 0.2e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.4e^{i2\pi/3}, 0.1e^{i2\pi/3}, 0.2e^{i2\pi/3})$	$(0.2e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$
x_2	$(0.8e^{i\pi}, 0.2e^{i\pi}, 0.1e^{i\pi})$	$(0.6e^{i\pi/3}, 0.3e^{i\pi/3}, 0.1e^{i\pi/3})$	$(0.5e^{i\pi/6}, 0.1e^{i\pi/6}, 0.3e^{i\pi/6})$
x_3	$(0.5e^{i2\pi/3}, 0.1e^{i2\pi/3}, 0.5e^{i2\pi/3})$	$(0.3e^{i\pi/2}, 0.1e^{i\pi/2}, 0.7e^{i\pi/2})$	$(0.1e^{i\pi/4}, 0.3e^{i\pi/4}, 0.6e^{i\pi/4})$
x_4	$(0.1e^{i\pi/2}, 0.3e^{i\pi/2}, 0.6e^{i\pi/2})$	$(0.5e^{i3\pi/4}, 0.2e^{i3\pi/4}, 0.3e^{i3\pi/4})$	$(0.2e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.6e^{i2\pi/3})$

	e_4
x_1	$(0.4e^{i\pi/3}, 0.1e^{i\pi/3}, 0.2e^{i\pi/3})$
x_2	$(0.6e^{i\pi/2}, 0.3e^{i\pi/2}, 0.2e^{i\pi/2})$
x_3	$(0.1e^{i2\pi}, 0.4e^{i2\pi}, 0.7e^{i2\pi})$
x_4	$(0.2e^{i\pi/2}, 0.2e^{i\pi/2}, 0.4e^{i\pi/2})$

Step 3. Then, the complex neutrosophic soft ideal evaluation of each of the parameters is as follows:

$f^+_{\tilde{C}_N}(e_1) = (0.8e^{i\pi}, 0.3e^{i\pi}, 0.7e^{i\pi})$; $f^+_{\tilde{C}_N}(e_2) = (0.6e^{i3\pi/4}, 0.3e^{i3\pi/4}, 0.7e^{i3\pi/4})$;
$f^+_{\tilde{C}_N}(e_3) = (0.5e^{i2\pi/3}, 0.3e^{i2\pi/3}, 0.7e^{i2\pi/3})$; $f^+_{\tilde{C}_N}(e_4) = (0.6e^{i2\pi}, 0.4e^{i2\pi}, 0.7e^{i2\pi})$;

Step 4. The bounded difference of each of the evaluations of the alternatives with respect to a parameter e_j from the corresponding $f^+_{\tilde{C}_N}(e_j)$ is given in Table 8.19.

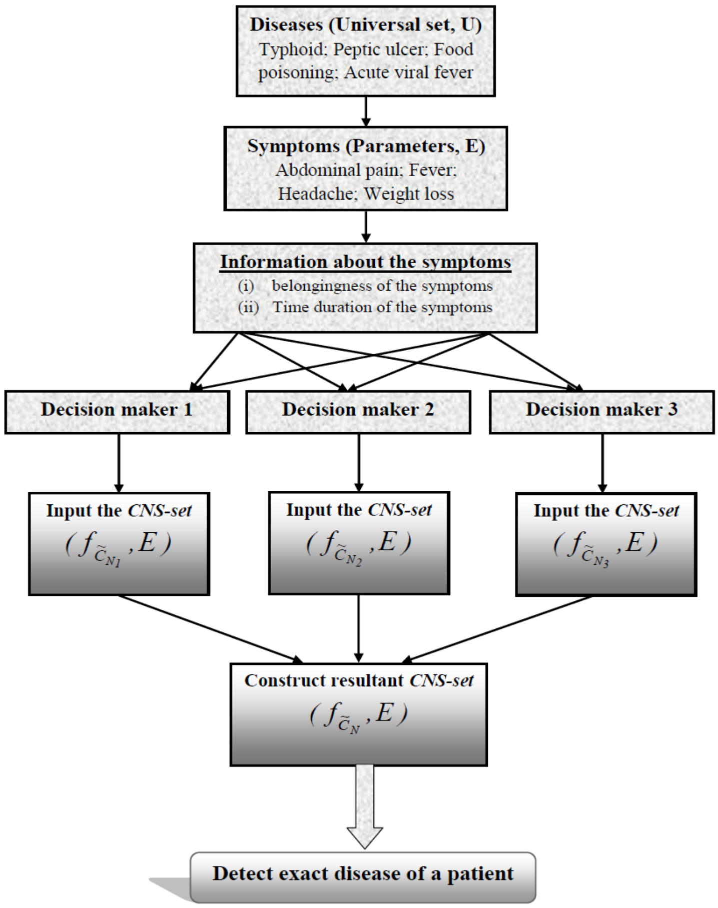

Figure 8.6: Modeling framework of case study 8.7.3

Table 8.19: Complex neutrosophic bounded difference soft set $(f^{\ominus}_{\tilde{C}_N}, E)$ (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	$(0.6e^{i\pi/2}, 0.1e^{i\pi/2}, 0e^{i\pi/2})$	$(0.2e^{i\pi/12}, 0.2e^{i\pi/12}, 0.5e^{i\pi/12})$	$(0.3e^{i\pi/6}, 0.2e^{i\pi/6}, 0e^{i\pi/6})$	$(0.2e^{i5\pi/3}, 0.3e^{i5\pi/3}, 0.5e^{i5\pi/3})$
x_2	$(0e^{i0}, 0.1e^{i0}, 0.6e^{i0})$	$(0e^{i5\pi/12}, 0e^{i5\pi/12}, 0.6e^{i5\pi/12})$	$(0e^{i\pi/2}, 0.2e^{i\pi/2}, 0.4e^{i\pi/2})$	$(0e^{i3\pi/2}, 0.1e^{i3\pi/2}, 0.5e^{i3\pi/2})$
x_3	$(0.3e^{i\pi/3}, 0.2e^{i\pi/3}, 0.2e^{i\pi/3})$	$(0.3e^{i\pi/4}, 0.2e^{i\pi/4}, 0e^{i\pi/4})$	$(0.4e^{i5\pi/12}, 0e^{i5\pi/12}, 0.1e^{i5\pi/12})$	$(0.5e^{i0}, 0e^{i0}, 0e^{i0})$
x_4	$(0.7e^{i\pi/2}, 0e^{i\pi/2}, 0.1e^{i\pi/2})$	$(0.1e^{i0}, 0.1e^{i0}, 0.4e^{i0})$	$(0.3e^{i0}, 0e^{i0}, 0.1e^{i0})$	$(0.4e^{i3\pi/2}, 0.2e^{i3\pi/2}, 0.3e^{i3\pi/2})$

Step 5. Now, the score values of the entries in Table 8.19 have been given in Table 8.20. The normalize score values of Table 8.20 have been provided in Table 8.21.

Table 8.20: Score valued matrix (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	0.71	0.58	0.67	0.43
x_2	0.55	0.53	0.52	0.44
x_3	0.62	0.66	0.68	0.75
x_4	0.72	0.60	0.7	0.52

Table 8.21: Normalize score valued matrix (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	0.99	0.88	0.96	0.57
x_2	0.76	0.80	0.74	0.59
x_3	0.86	1	0.97	1
x_4	1	0.91	1	0.69

Step 6. The group utility values of the alternatives over all the parameters are as follows: $\hat{S}^W_{cr}(x_1) = 0.818$; $\hat{S}^W_{cr}(x_2) = 0.708$; $\hat{S}^W_{cr}(x_3) = 0.957$; $\hat{S}^W_{cr}(x_4) = 0.875$.
Step 7. The individual regret values of the alternatives over all the parameters are as follows: $\hat{S}^{max}_{cr}(x_1) = 0.267$; $\hat{S}^{max}_{cr}(x_2) = 0.205$; $\hat{S}^{max}_{cr}(x_3) = 0.34$; $\hat{S}^{max}_{cr}(x_4) = 0.270$.
Step 8. The compromising index of the alternatives over all the parameters are as follows, $\hat{C}(x_1) = 0.45$; $\hat{C}(x_2) = 0$; $\hat{C}(x_3) = 1$; $\hat{C}(x_4) = 0.56$. Here, we have considered $\theta = 0.5$
Step 9. The ranking of the alternatives based on $\hat{S}^W_{cr}$, $\hat{S}^{max}_{cr}$ and $\hat{C}$ is same as,
$x_2 > x_1 > x_4 > x_3$.
Condition 1. Since, $\hat{C}(x_1) - \hat{C}(x_2) = 0.45 > \frac{1}{4-1} = 0.33$. So, Condition 1 is truly satisfied.
Condition 2. Again, x_2 is best for both the cases $\hat{S}^W_{cr}$ and $\hat{S}^{max}_{cr}$.
So, Condition 2 is also satisfied here.
Thus, the alternative x_2 is a compromise optimal solution for this decision-making problem.

Hence, we can conclude that, the patient is suffered by the disease peptic ulcer (x_2).

8.8 Comparative study and discussion

In this chapter, we have proposed a new approach to evaluate the best alternative from a complex neutrosophic soft set based decision-making problem by using VIKOR approach. In this section, we have examined the stability of the ranking results of our proposed approach through a sensitivity analysis and a comparative analysis. In sensitivity analysis part, we have examined the robustness of our approach for different values of the parameter θ which is employed in our algorithm and in comparative analysis part, we have compared our results with some well known exiting methodologies to verify the validity of our proposed approach.

8.8.1 Sensitivity analysis based on the parameter θ

In our methodology, the parameter θ has been assigned as a weight of the strategy of maximum group of utility and $(1 - \theta)$ has been assigned as a weight of the strategy of minimum individual regret to evaluate the compromising index of an alternative, as defined in Equation 8.5. Here, θ can adopt any value from 0 to 1. In the previous all examples, all the results have been taken by keeping the value of θ as 0.5 i.e., by giving equal importance on the both of maximum group of utility and minimum individual regret. But since, θ can take any value from 0 to 1, so, we have to verify the stability of the ranking results for different values of θ. Therefore, we have given a sensitivity analysis based on the parameter θ. The previous illustrated examples (Example 8.4, Example 8.5, Example 8.6) have been used here to conduct this process. In Figures 8.7, 8.8, 8.9, the variation of compromising index values of different alternatives for different values of θ have been provided based on Example 8.4, Example 8.5 and Example 8.6 respectively.

Now, from Figures 8.7 it is observed that, for any value of θ from 0.1 to 1, the ranking order of the four candidates is same as, $C_1 > C_4 > C_2 > C_3$ and when $\theta = 0$, the ranking order of the four candidates is, $C_1 > C_4 > C_2 = C_3$. So we can conclude that, the parameter θ did not affect on the ranking order of the alternatives in the interval $[0.1, 1]$. But when $\theta = 0$, i.e., when, one focuses only on the individual evaluations of the parameters (minimum individual regret) of the candidates, then, we can't say which one is at the third position and which one is at the fourth position between C_2 and C_3.

From Figure 8.8 we have seen that, the ranking order of the material alternatives is same for any value of the parameter θ from 0.1 to 0.9. But, when θ is tending to 1, then the ranking order of M_2 has been increased and the ranking order of M_3 has been decreased. i.e., when one focuses only on the group utility score (utility measure) of the material alternatives over all the parameters, then the risk level is high for being that, M_2 is best than M_3 and when $\theta = 0$, the ranking order of the alternatives has been slightly changed as, $M_1 > M_3 > M_2 = M_4$ i.e., when, one focuses only on the minimum individual regret (regret measure) of the material alternatives over all the parameters then, we can't say which one is placed at third position and which one is placed at the fourth position between M_2 and

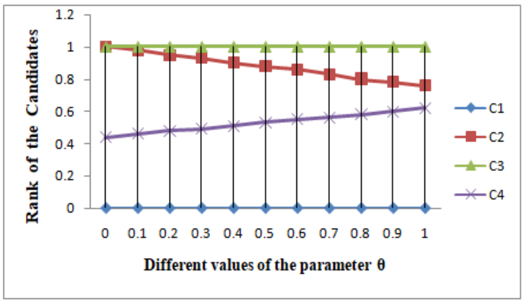

Figure 8.7: Sensitivity analysis based on Example 8.4

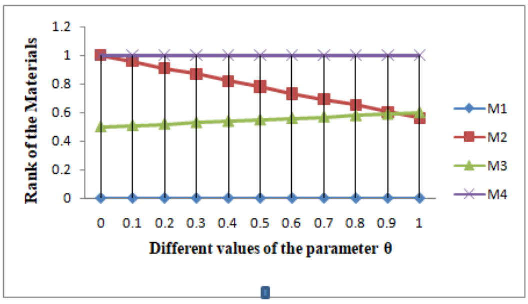

Figure 8.8: Sensitivity analysis based on Example 8.5

M_4.

Again, from Figure 8.9 we have seen that, different values of θ do not effect on the ranking order of the alternatives x_1, x_2, x_3, x_4.

So, from this discussion it is observed that, different values of the parameter θ may make an effect on the final ranking order of the alternatives in a problem. Since, in our method, decision-maker can give the importance to the strategy of maximum group of utility (utility measure) and minimum individual regret (regret measure) as he/she likes, so when, he/she changes his/her importance to the maximum group of utility and minimum individual regret, then obviously there may exist a different ranking result of the associated alternatives. So, changing of the parameter θ indicates the changing of the ranking order of the alternatives, which supports the results of the sensitivity analysis with respect to the parameter θ. Therefore, from this sensitivity analysis it is concluded that, our approach is more realistic.

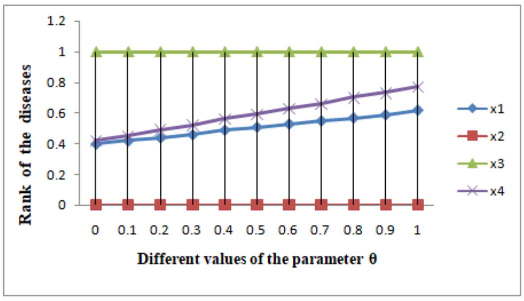

Figure 8.9: Sensitivity analysis based on Example 8.6

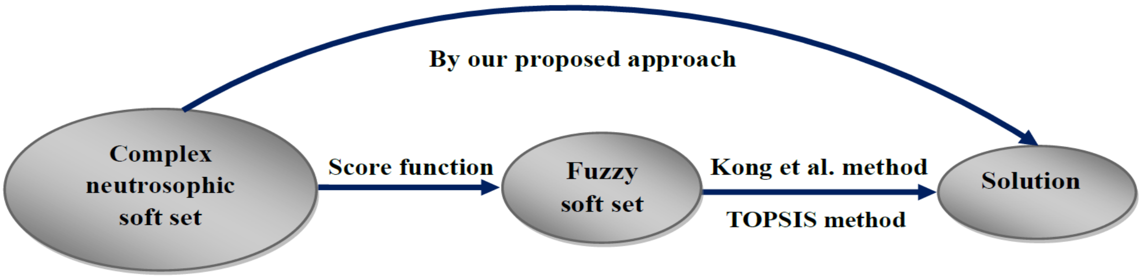

Figure 8.10: Comparative analysis

8.8.2 Comparative analysis of our proposed approach

Validation or confirmation of a new approach can be tested by comparing its resulting values with the existing methods. Now, it is seen that, complex neutrosophic framework is a super set with respect to fuzzy valued framework. But since, to solve our considered soft set based decision-making problems under the complex neutrosophic framework, there exists no method in literature till now. However, in fuzzy valued framework, some methodologies such as TOPSIS method [58], Kong et al. method [92] etc. have been saved in literature. So, to compare the results obtain from our method with the results using TOPSIS and Kong et al. method [92], we would transform all the complex neutrosophic values of a complex neutrosophic soft set into some real values (fuzzy soft set) by using score function (Definition 8.5). Because, the score value of a complex neutrosophic number by using score function is a reflection of the complex neutrosophic number in favor of truth degree. The schematically diagram of this discussion has been given in Figure 8.10. Now, in the following, we have discussed a comparative study by using some existing fuzzy valued (truth membership) methods to verify the validity of our proposed approach.

- *Comparison based on Example 8.4*

The solution of Example 8.4 through our proposed approach has been illustrated in Subsection 8.7.1. Then, after transforming all the given complex neutrosophic values (Table 8.5) into real values by using score function, the resultant score values are given in Table 8.22. So, Table 8.22 is the reflection of Table 8.5. Since, in this example, the final ranking result has been taken by using the weights of the parameters as, $W_1 = 0.2$; $W_2 = 0.3$; $W_3 = 0.2$; $W_4 = 0.3$, therefore, before applying any fuzzy valued methodology, we have constructed a weighted score matrix (Table 8.23) (weighted fuzzy soft set) by multiplying the score values (Table 8.22) of each of the alternatives over a parameter e_j with its associated parameter weight W_j.

Table 8.22: Score matrix based on Table 8.5 (Fuzzy soft set)

	e_1	e_2	e_3	e_4
C_1	0.52	0.66	0.52	0.72
C_2	0.68	0.47	0.45	0.46
C_3	0.56	0.52	0.46	0.32
C_4	0.46	0.59	0.51	0.54

Table 8.23: Weighted Score matrix (Weighted fuzzy soft set)

	e_1	e_2	e_3	e_4
C_1	0.104	0.198	0.104	0.216
C_2	0.136	0.141	0.09	0.138
C_3	0.112	0.156	0.092	0.096
C_4	0.092	0.177	0.102	0.162

Now, we have applied different decision-making methods to evaluate the best alternative from Table 8.23. The ranking results are given in Table 8.24.

Table 8.24: Comparison Table based on Example 8.4

	$C_1 > C_4 > C_2 > C_3$
Our approach ($\theta = 0.5$)	$\surd$
TOPSIS method [58]	$\surd$
Kong et al. ([92]) method	$\surd$

Since, for each of the methods, the ranking of the alternatives is, $C_1 > C_4 > C_2 > C_3$, therefore, this ranking order has been considered as the ideal ranking of this problem. Thus we have seen that, by using our proposed approach, the ranking result of Example 8.4 is consistent.

- *Comparison based on Example 8.5*

The solution of Example 8.5 by using our approach has been illustrated in Subsection 8.7.2. Now, after transforming all the complex neutrosophic values (Table 8.11) into score values by using score function, the resultants are given in Table 8.25. So, Table 8.25 is the reflection of Table 8.11. In this problem, the weights of the parameters are, $W_1 = 0.4$; $W_2 = 0.4$; $W_3 = 0.2$. Therefore, we have constructed weighted score matrix, by multiplying the score values of each of the alternatives over a parameter e_j with the associated parameter weight W_j, as given in Table 8.26.

Table 8.25: Score matrix based on Table 8.11 (Fuzzy soft set) (Example 8.5)

	e_1	e_2	e_3
M_1	0.525	0.661	0.523
M_2	0.686	0.472	0.453
M_3	0.578	0.525	0.462
M_4	0.453	0.588	0.511

Table 8.26: Weighted score matrix (Weighted fuzzy soft set)

	e_1	e_2	e_3
M_1	0.21	0.264	0.105
M_2	0.274	0.189	0.091
M_3	0.231	0.21	0.092
M_4	0.181	0.235	0.102

Now, by applying different fuzzy valued decision-making methods on Table 8.26, the ranking results are given in Table 8.27.

Table 8.27: Comparison Table based on Example 8.5

	$M_1 > M_3 > M_2 > M_4$	$M_1 > M_2 > M_3 > M_4$	$M_1 > M_3 > M_2 = M_4$
Our approach ($\theta = 0.5$)	$\surd$		
Our approach ($\theta = 0$)			$\surd$
Our approach ($\theta = 1$)		$\surd$	
TOPSIS method [58]		$\surd$	
Kong et al. ([92]) method		$\surd$	

From Table 8.27 we have seen that, by using our proposed approach, when, $\theta = 0.5$, i.e., when one gives equal importance on the both of maximum total group of utility and minimum individual regret, then the rank is, $M_1 > M_3 > M_2 > M_4$ and when, $\theta = 1$, i.e., when one gives importance only on the maximum total group of utility then, the rank as, $M_1 > M_2 > M_3 > M_4$. Moreover, we have seen that, TOPSIS method [58] and Kong et al. [92] method have given the same ranking order as, $M_1 > M_2 > M_3 > M_4$. Now, the background of TOPSIS method is, distance of an alternative from ideal solution and anti ideal solution over all the parameters and the background of Kong et al. [92] method is, the sum of the total evaluations over all the parameters (fuzzy choice value) of an alternative. So, the key idea of both the TOPSIS method [58] and Kong et al. [92] method is same (give the importance only on the total satisfaction over all the parameters not on the individual satisfaction of a parameter). This situation can also be controlled through our algorithm by taking $\theta = 1$. From Table 8.27 it is observed that, the ranking result of our method is same with TOPSIS method [58] and Kong et al. [92] method when $\theta = 1$. So, in this situation, the ranking result of our method is consistent. But when, in a decision-making problem, the satisfaction of an individual parameter is important not the total satisfaction of all the parameters, then TOPSIS method [58] and Kong et al. [92] method are unsuitable whereas, in that situation, our method can solve this problem by taking $\theta = 0$. So, by taking different values of the parameter θ we can get more accurate results of a decision-making problem. Therefore we can conclude that, the ranking result of our method is valid in this example.

- *Comparison based on Example 8.6*

Example 8.6 has been solved by our proposed approach at Subsection 8.7.3.

Now, by apply score function on Tables 8.15, 8.16, and 8.17, the score values are given in Tables 8.28, 8.29 and 8.30.

Table 8.28: Score matrix based on Table 8.15 (Fuzzy soft set) (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	0.525	0.661	0.492	0.706
x_2	0.634	0.672	0.703	0.625
x_3	0.561	0.525	0.562	0.317
x_4	0.442	0.554	0.461	0.492

Table 8.29: Score matrix based on Table 8.16 (Fuzzy soft set) (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	0.492	0.611	0.508	0.655
x_2	0.617	0.672	0.653	0.642
x_3	0.561	0.525	0.462	0.333
x_4	0.558	0.471	0.528	0.525

Table 8.30: Score matrix based on Table 8.17 (Fuzzy soft set) (Example 8.6)

	e_1	e_2	e_3	e_4
x_1	0.508	0.594	0.525	0.672
x_2	0.683	0.639	0.636	0.625
x_3	0.561	0.525	0.529	0.317
x_4	0.458	0.571	0.494	0.492

Then, by using fuzzy AND product the resultant score matrix from the three score matrices (Tables 8.28, 8.29 and 8.30) has been given in Table 8.31. Then by using the weights of the parameters ($W_1 = 0.27$; $W_2 = 0.22$; $W_3 = 0.17$; $W_4 = 0.34$), the resultant weighted score values are given in Table 8.32.
Now, the ranking of the alternatives by using different methods has been given Table 8.33. From Table 8.33 we have seen that, the ranking order of the alternatives through every method is same. so we can conclude that, the ranking result of our proposed approach is consistent in this example.
So, from the above illustration, what is the status of our method?
The answer of this question is that, our proposed approach is very effective because of the following reasons.

Table 8.31: Resultant score matrix (Fuzzy soft set)

	e_1	e_2	e_3	e_4
M_1	0.492	0.594	0.492	0.655
M_2	0.617	0.639	0.636	0.625
M_3	0.561	0.525	0.462	0.317
M_4	0.442	0.471	0.461	0.492

Table 8.32: Weighted resultant score matrix (Weighted fuzzy soft set)

	e_1	e_2	e_3	e_4
x_1	0.133	0.131	0.084	223
x_2	0.166	0.140	0.108	212
x_3	0.151	0.116	0.078	108
x_4	0.119	0.104	0.078	167

Table 8.33: Comparison Table based on Example 8.6

	$x_2 > x_1 > x_4 > x_3$
Our approach ($\theta = 0.5$)	$\surd$
TOPSIS method [58]	$\surd$
Kong et al. method [92]	$\surd$

- The domain of our considered decision-making problem is large than the existing one.
- There is no existing methodology to solve the real-life problems in this domain till now.
- In our method, two directions such as, ‘maximization of total group of utility’ and ‘minimization of individual regret’ have been considered together through a parameter θ by which a decision maker can get an exact direction by changing the value of θ according to his/her needs. But, in TOPSIS [58] method or Kong et al. method [92], there is no such facility.
- Further, in our approach, we have obtained a compromise optimal solution instead of a single optimal solution of a considered complex neutrosophic soft set based decision-making problem to overcome the problems in emergency situations. But, in TOPSIS-method [58] or Kong et al. [92] method, there is no such advantage.

Hence, from the above analysis it is concluded that, our proposed approach is valid and is very effective to solve decision-making problems involving a second dimension.

8.9 Conclusion

This chapter presents a complex neutrosophic soft set based decision-making by using VIKOR method. To reach this goal, few necessary operations and properties of complex neutrosophic sets and complex neutrosophic soft sets have been introduced.
The contributions of this chapter can be summarized as follows:

- Some new union operations (complex nuetrosophic standard union, complex neutrosphic algebraic sum, complex neutrosophic bold sum), intersection operations (complex neutrosophic standard intersection, complex neutrosophic product, complex neutrosophic bold intersection) and aggregation operations (complex neutrosophic arithmetic mean aggregation, complex neutrosophic geometric mean aggregation, complex neutrosophic harmonic mean aggregation) of complex neutrosophic sets have been proposed.

- The above union operations and intersection operations of complex neutrosophic sets have been generalized to complex neutrosophic soft sets.

- Score function of a complex neutrosophic number has been initiated.

- Two decision-making approaches have been developed by using VIKOR method to solve single decision-maker and multiple decision-makers based decision-making problems by using complex neutrosophic soft set theory.

- Moreover, to show the working progress of the proposed approaches, we have applied these algorithms in company's manager selection problem, medical diagnosis problem, sustainable manufacturing material selection problem etc.

- The sensitivity analysis of our proposed approach based on the parameter θ has also been illustrated.

- Also besides, we have discussed the validity and effectiveness of our proposed approach.

As further research, one can develop some algebraic structures of complex neutrosophic soft sets such as complex neutrosophic soft groups, complex neutrosophic soft rings etc. Moreover, distance measure is a useful tool that can solve many decision-making problems. So, one can propose several types of distance measures or similarity measures of complex neutrosophic soft sets. Further, single-valued neutrosophic 2-tuple linguistic environment ([166]), trapezoidal fuzzy environment ([165], [167]), hesitant fuzzy linguistic environment ([168]) etc. are very effective tools to solve real-life related decision-making problems. So, one can develop our proposed approach to these effective frameworks.