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PG/IIS/PHS-201/13

M.Sc. 2nd Semester Examination, 2013

PHYSICS

PAPER – PHS- 201

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Use separate Scripts for Gr. A & Gr.B

GROUP – A

[*Marks* : 20]

Answer **Q.No.1 & 2** and any **one** from the rest

1. Answer any *two* bits : 2×2

(*a*) Obtain the eigenvalues and normalized eigenvectors of S_x and S_z for a spin-$\frac{1}{2}$ particle.

(*b*) The Hamiltonian H and the unnormalized wave function for a one dimensional harmonic oscillator are given as

$$H = \hbar\omega\left(a^{+}a + \frac{1}{2}\right) \text{ and } \psi = 2|1\rangle + |2\rangle$$

Find the energy of the oscillator.

(*c*) The eigenfunctions of parity operator belonging to the 'even' and 'odd' eigenstates are $\phi_{+}(\vec{r})$ and $\phi_{-}(\vec{r})$ respectively. Show that the eigenfunctions are orthogonal.

(*d*) Write the Hamiltonian operator of a helium atom using 'the concept of stationary perturbation.

2. Answer any *two* bits : 3 × 2

(*a*) An electron in an atom is in *p*-state. Find the possible values of total angular momentum which is the sum of the orbital angular momentum and spin momentum. Write the wave functions of the possible states of the total angular momentum from the component wave functions $|l, m_l\rangle$ and $|s, m_s\rangle$, where $l = 1$ and $s = 1/2$.

(*b*) Show that for a system with orbital angular momentum $\vec{L}$, the operator for the rotation of the system about the z-axis through an angle ϕ is given by

$$\hat{R}_z(\phi) = \exp\left(-i/_\hbar \, \phi \, L_z\right)$$

where L_z is the Z-component of $\vec{L}$.

(*c*) Find the ground state energy of a harmonic oscillator using the trial function $\psi = e^{-\alpha x^2}$.

(*d*) From space translation symmetry show that if Hamiltonian is invariant under space translation, momentum is conserved.

3. (*a*) Find the relation $[J_+, J_z]$. Using this relation, show that J_+ acts as a raising operator on the eigenvalues of J_z. 3

(*b*) Using a representation which is diagonal for both $\vec{J}^2$ and J_z and satisfy the eigenvalue equation $\vec{J}^2 |j,m\rangle = j(j+1)\hbar^2 |j,m\rangle$ and $J_z |j,m\rangle = m\hbar |j,m\rangle$ Find the relation for $\langle jm' | J_+ | j,m\rangle$ and $\langle jm' | J_- | j,m\rangle$. 5

(*c*) Using the relation for J_+ and J_- find the angular momentum matrices for J_x, J_y, J_z and $\vec{J}^2$ for $j = 1/2$. 2

4. (*a*) Establish the stationary perturbation theory for a doubly degenerate state. 4

(*b*) Find the energy corrections for the ground state $n = 1$ and the first excited state $n = 2$, of a hydrogen atom placed in a uniform electric field. Use the following expressions for the wave functions 6

$$\psi_{200}(r,\theta,\varphi) = \frac{1}{(32\pi a_0^3)^{1/2}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

$$\psi_{210}(r,\theta,\varphi) = \frac{1}{(32\pi a_0^3)^{1/2}} r\, e^{-r/2a_0} \cos\theta$$

GROUP – B

[*Marks* : 20]

Answer **Q.No.1** and any **one** from the rest

1. Answer any *five* from the following : 2 × 5

(*a*) Find the Fourier cosine transform of

$$f(x) = \frac{1}{1+x^2}.$$

(*b*) If $\theta(t-a)$ is the unit step function then prove that

$$\hat{L}\{f(t-a)\cdot\theta(t-a)\} = e^{-aS}F(s).$$

(*c*) State Dirichlet and Neuman boundary conditions.

(*d*) Find

$$L^{-1}\left\{\frac{8}{s^2(s-2)}\right\}.$$

(*e*) Solve $(D^2 + 6DD' + 9D'^2)z = 0$

where $D \equiv \frac{\partial}{\partial x}$; $D' \equiv \frac{\partial}{\partial y}$.

(*f*) State rearrangement theorem in Group.

(*g*) State Schur's Lemma.

(*h*) Show that $x' = ax + b$; $a \neq 0$ form a Lie group.

2. (*a*) Find the Fourier transform of

$$f(x) = (1-x^2), \text{ for } |x|<1$$
$$= 0 \quad , \text{ for } |x|>1$$

Hence evaluate

$$\int_0^{\infty}\left(\frac{x\cos x - \sin x}{x^3}\right)\cdot \cos x \, dx.$$ 5

(*b*) Construct the green's function :

$$\frac{d^2y}{dx^2} - y = f(x);\ y(0) = 0; y(2) = 0.$$ 5

3. (*a*) Find the particular integral of

$$u_{xx} - 3u_{xy} + 2u_{yy} = e^{x+y} + \sin(2x + y). \quad 5$$

(*b*) Obtain the Symmetry transformation of a square which form a group C_{4v}. 5
