PG/IS/PHS-102/13

M.Sc. 1st Semester Examination, 2013

PHYSICS

PAPER—PHS-102 (Group A & B)

Full Marks : 40

Time : 2 hours

Candidates are required to give their answers in their own words as far as practicable

The figures in the right-hand margin indicate marks

GROUP – A

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* questions : 2 × 5

(*a*) Consider the minimum energy needed for a photon to turn into an electron-positron pair. Calculate how long such a virtual pair can exist.

(*b*) A particle is represented by the wave-function $A\left(e^{ikx}+2e^{-ikx}\right)$. Find the expectation value of momentum along x direction for the particle.

(*c*) Let ϕ_1 and ϕ_2 be two orthogonal eigen-functions with energy eigenvalues 1 eV and 3 eV. A particle is in a state ψ which is a superposition state of ϕ_1 and ϕ_2. The average value of energy is 2 eV. Find the probability of finding the particle in the states ϕ_1 and ϕ_2.

(*d*) Show that the norm of the state vector evolving from the Schrödinger equation remains constant with respect to time.

(*e*) If $\hat{A}$ and $\hat{B}$ are hermitian operators, show that $\hat{A}\hat{B}+\hat{B}\hat{A}$ is hermitian while $\hat{A}\hat{B}-\hat{B}\hat{A}$ is not hermitian.

(*f*) Write the energy eigenvalues and normalized eigenfunction for the two lowest states of a simple Harmonic Oscillator. Also draw these two eigenfunctions.

(*g*) Find the eigenvalues of $\hat{L}^2$ for the eigenfunction $\psi(r, \theta, \phi) = A\, e^{-\alpha r} \sin\theta\, e^{i\phi}$, where A is a constant.

$$A e^{-\alpha r} \sin\theta\, e^{i\phi}$$

(*h*) Show that the transformation matrix which changes from one basis set to another is unitary.

2. (*a*) Using quantum condition, find an expression for the raising and lowering operators (a^+ and a) respectively.

(*b*) Show that

$$H = \left(a^+ a + 1/2\right)\hbar w$$

and using this condition, find ground state energy.

(*c*) Obtain the matrices for a^+, a, x and p_x.

(*d*) Consider a particle of mass m moving in a half harmonic potential well given by

$$V(x) = \infty, \quad x \le 0$$

$$= \frac{1}{2} m\, w^2 x^2, \quad x > 0.$$

What will be the energy eigenvalues of the particle ? 4 + 2 + 2 + 2

3. (*a*) Find the equation of motion, both for the wave function and the dynamical operator in the interaction picture.

(*b*) For the ground state of Hydrogen atom, calculate the most probable radial distance of the electron from the nucleus. Also find the expectation value of the radial position *r*. Compare the results and comment.

(*c*) The wave function for a particle is represented as

$$\psi(\vec{r},\ t) = \sum_n a_n(t)\ u_n(\vec{r})$$

where $H\, u_n(\vec{r}) = \varepsilon_n\, u_n(\vec{r})$

show that

$$\langle \psi | H | \psi \rangle = \sum_n |a_n(t)|^2\, \varepsilon_n. \qquad 4 + 3 + 3$$

GROUP – B

Answer **Q. No. 1** & **2** and any **one** from the rest

1. Answer any *two* bits : 2×2

(*a*) The atomic radius of copper is 0·1278 nm. Calculate the interplanar spacing for III plane.

(*b*) Find the structure factor for a 'C-face Centered' crystal and hence find the condition for systematic absence.

(*c*) What is meant by Van-Hove singularity.

2. Answer any *two* bits : 3×2

(*a*) Prove that the scattered amplitude of X-ray from a crystal of volume element *dv* is the Fourier Transform of electron density.

(*b*) Prove that for a monoatomic linear lattice the number of optical branches is zero.

(*c*) Discuss in detail the symmetry elements associated with point group.

3. (*a*) Prove the equivalence between vibrational mode in a solid and a harmonic oscillator.

(*b*) Prove that Brillouin zone of a f.c.c. lattice is octahedran. $8 + 2$

4. (*a*) What is the physical origin of energy gap in a solid and prove that the magnitude of energy gap is determined by the strength of periodic potential.

(*b*) What is meant by normal process and Umklapp process ? 8 + 2
