PG/IS/PHS - 101/13

M.Sc. 1st Semester Examination, 2013

PHYSICS

PAPER— PHS - 101(A & B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PHS — 101 A

[*Marks* : 20]

Time : 1 hours

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 × 5

(*a*) Find the coordinates of the polynomial

(*Turn Over*)

$(x - 3x^2)$ relative to the ordered basis $\{1 - x, 1 + x, 1 - x^2\}$ in a vector space of polynomials.

(*b*) Find the value of $\Gamma\left(-\frac{3}{2}\right)$

(*c*) Find the residue of the function

$$f(z) = \frac{z}{(z-a)(z-b)}$$

at infinity.

(*d*) Show that

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

where j_1 is the spherical Bessel's function of order 1.

(*e*) Show that

$$\int_0^1 \log \Gamma(x) dx = \frac{1}{2}\log(2\pi).$$

(*f*) If $f(z) = u(x, y) + iv(x, y)$ is analytic, show that $\vec{\nabla}u . \vec{\nabla}v = 0$.

(*g*) Prove that the product of two hermitian matrices is hermitian if and only if they commute.

(*h*) If $erf(x)$ be error function of x, show that

$$\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}\left[erf(b) - erf(a)\right]$$

2. (*a*) Evaluate

$$\int_0^\infty \frac{x^2 dx}{x^6 + 1}$$

by Cauchy's residue theorem.

(*b*) Find the value of

$$I = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$$

(c) Prove that

In $\hat{A}\vec{r} = \vec{\alpha} \times \vec{r}$, $\vec{\alpha}$ = constant vector

$\hat{A}$ is a linear operator. 4 + 4 + 2

3. (a) If

$$f(x) = 0 \text{ for } -1 < x < 0$$
$$= \frac{\sqrt{x}}{2} \text{ for } 0 < x < 1.$$

and $f(x) = \sum_{n=0}^{\infty} A_n P_n(x)$

Find the values of A_0, A_1, A_2, A_3.

(b) Show that

$$\int_0^{\infty} \frac{x^{n-1}}{e^x - 1} dx \quad \text{where} \quad n > 0$$
$$= \Gamma(n)\{(n).$$

where $\{(n) = \sum_{m=1}^{\infty} \frac{1}{m^n}$ is the Riemann's zeta function. 5 + 5

PHS – 101 B

[*Marks* : 20]

Time : 1 hour

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *four* of the followings : $2\frac{1}{2} \times 4$

(*a*) If L is the Lagrangian for a system of n degrees of freedom satisfying Hamilton's variational principle, show that

$$L^1 = L + \frac{dF}{dt}(q_1, q_2 \ldots q_n; t)$$

also satisfies Hamilton's principle where F is any arbitrary well-behaved function.

(*b*) Prove that if a generalised coordinate is cyclic in the Lagrangian it should be cyclic in the Hamiltonian also.

(*c*) Assuming the canonical invariance of Poisson bracket prove that

$$[F, G]_{q,p} = [F, G]_{G,P}.$$

(*d*) Show that the total angular momentum of a system of particles about a point is the sum of the angular momentum of the system about the centre of mass and angular momentum about the reference point of the system mass concentrated at the centre of mass.

(*e*) For what values of α and β

$$Q = q^{\alpha} \cos\beta p \quad P = q^{\alpha} \sin \beta p$$

represents a canonical transformation. Also find the generating function.

(*f*) If U be a generating function depends only on Q_α, p_α, t. Prove that

$$P_\alpha = -\frac{\partial U}{\partial Q_\alpha}; \, q_\alpha = -\frac{\partial U}{\partial p_\alpha}, \; K = H + \frac{\partial U}{\partial t}$$

2. Using Hamilton's equation of motion, show that the Hamiltonian

$$H = \frac{p^2}{2m} e^{-rt} + \frac{1}{2} m w^2 x^2 e^{rt}$$

leads to the equation of motion of a damped harmonic oscillator

$$\ddot{x} + r\dot{x} + w^2 x = 0$$

Express the kinetic and potential energies of a system in terms of normal co-ordinates. 4 + 6

3. What are action angle variable ? Find out the frequency of a linear harmonic oscillator using action-angle variable method. Starting from the time dependent Schrödinger equation obtain the Hamilton-Jacobi equation. 10
