PG/IS/PHS-102/10

M.Sc. 1st Semester Examination, 2011

QUANTUM MECHANICS AND PHYSICS

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER—PHS- 102 A

(*Quantum Mechanics*)

[*Marks* : 20]

Answer **Q.No.1** and any **one** from the rest

1. Answer any *five* questions : 2×5

(*a*) For a photonic wave, show that the uncertainty product $\Delta \varepsilon \Delta t \geq \hbar/2$ can be written as

$$\Delta \lambda \, \Delta x \geq \lambda^2/(4\pi c).$$

(*Turn Over*)

(b) If $\hat{A}$ and $\hat{B}$ are hermitian operators, show that $(\hat{A}\hat{B} + \hat{B}\hat{A})$ is hermitian whereas $(\hat{A}\hat{B} - \hat{B}\hat{A})$ is not hermitian.

(c) For what values of the constant C will be the function $f(x) = Ae^{-ax}$ be an eigen function of the operator $\hat{Q} = \frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx} + \frac{c}{x}$?

(d) Prove that the eigenvalues of a Hermitian operator are real.

(e) If $Axe^{-x^2/2}$ is an eigen function of

$$\hat{H} = \left(-\frac{d^2}{dx^2} + x^2\right),$$

what is the associated eigenvalue ?

(f) The normalizat on constant N, for

$$\psi(r) = Nre^{-ar} \quad \text{for } 0 < r < \infty$$

is ———. (with calculation).

(*g*) If a 3–dim quantum mechanical harmonic oscillator has an energy $3{\cdot}5\ \hbar\omega$ in a particular state, then find the degree of degeneracy.

(*h*) The ground state wave function for hydrogen in co–ordinate space

$$\psi(r) = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$

Calculate the momentum wave function.

2. A particle is confined in a one dimensional rigid box with walls at $x = \pm L/2$.

(*a*) Find the energy eigenvalues and corresponding eigen functions.

(*b*) Draw the first three eigen functions.

(*c*) For the ground state wave function find $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p_x \rangle$ and $\langle p_x^2 \rangle$.

(*d*) Use these results to determine the uncertainty product $\Delta x \Delta p_x$.

3 + 2 + 3 + 2

3. (*a*) Considering the operator

$$\hat{a}^{\dagger} = \frac{1}{(2m\omega\hbar)^{1/2}}(-i\hat{p} + m\omega\hat{x})$$

and its adjoint prove that for a linear oscillator of Hamiltonian

$$\hat{H}, \quad \hat{H}|n> = \left(n + \frac{1}{2}\right)^{\hbar\omega} |n>.$$

(*b*) Prove that $\hat{a}$ and $\hat{a}^{\dagger}$ has $(n-1)$ and $(n+1)$ eigenvalues respectively. 6 + 4

PAPER—PHS- 102 B

(*Physics*)

[*Marks* : 20]

Answer **Q.No.1 & 2** and any **one** from the rest

1. Answer any *two* bits : 2 x 2

(*a*) Clearly explain Zinc Blende structures and give an example.

(*b*) Explain why screw axis is an internal symmetry element.

(*c*) Calculate the expression for density of states in a three dimensional lattice.

2. Answer any *two* bits : 3×2

(*a*) Find the interplanar spacing of a hexagonal lattice.

(*b*) What is the order of normal to superconducting phase change (at T_c and below T_c) ? Justify your answer.

(*c*) Find the structure factor of a *C*-face centered crystal and hence find out the condition for systematic absence.

3. Prove the equivalence of a vibrational mode and a harmonic oscillator. Derive London equations for a superconductor and how its solution explain Meissner effect. $4 + 4 + 2$

4. Explain Debye-Waller effect and find an expression of Debye-Waller factor. What is a Brillouin Zone? How it can be constructed using Bragg's diffraction condition ?. $2 + 4 + 1 + 3$