PG/IS/PHS-101/10

M.Sc. 1st Semester Examination, 2011

METHODS OF MATHEMATICS AND PHYSICS

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER—PHS-101 A

(*Methods of Mathematics*)

[*Marks* : 20]

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 x 5

(*a*) If

$$\hat{L}\hat{A} = \left[\hat{H}, \hat{A}\right] = \hat{H}\hat{A} - \hat{A}\hat{H}$$

show that $\hat{L}, \hat{A}$ are linear operators.

(*b*) Consider the equation

$$\hat{L}y = 0 \quad \text{where} \quad \hat{L} \equiv \frac{d^2}{dx^2} + q(x).$$

If y_1 and y_2 are two linearly independent solutions of this equation, then show that their Wronskian is a constant.

(*c*) If $w = u + iv$ and $v = \frac{x}{x^2 + y^2} + \cos hx \cos y$ find u.

(*d*) $\int_{-1}^{+1} x\, P_n(x)\, P_{n-1}(x)\, dx = \ldots\ldots\ldots ?$

(with reasoning give the calculation).

(*e*) The residue of $\frac{z}{(z-a)(z-b)}$ at infinity is _____. (with calculation).

(*f*) State and prove Schwarz inequality.

(*g*) $A = \begin{pmatrix} 1 & 1 \\ 1 & i \end{pmatrix}$

Is it diagonalizable ? Explain it.

(*h*) Evaluate :

$$\int_C \frac{dz}{z^2 - 2z}; \; C : |z| = 1.$$

2. (*a*) If

$$\psi_1 = \begin{pmatrix} 1+i \\ 1 \\ i \end{pmatrix}; \; \psi_2 = \begin{pmatrix} i \\ 3 \\ 1 \end{pmatrix}; \; \psi_3 = \begin{pmatrix} 0 \\ 28 \\ 0 \end{pmatrix}$$

Find orthonormal set of vectors.

(*b*) Show that spherical bessel function of order 2 is

$$j_2(x) = \frac{3\sin x - 3x\cos x - x^2 \sin x}{x^3}.$$

(*c*) Prove that :

$$2x\, H_n(x) = 2n\, H_{n-1}(x) + H_{n+1}(x)$$

for Hermite polynomial $H_n(x)$. 4 + 3 + 3

3. (*a*) Evaluate :

$$\int_0^\infty \frac{dx}{1+x^6}$$

by the method of residue theorem.

(*b*) Evaluate :

$$\iiint x^{l-1} y^{m-1} z^{n-1} dxdydz$$

where x, y, z are all positive with condition

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1. \qquad 5 + 5$$

PAPER—PHS-101 B

(*Physics*)

[*Marks* : 20]

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* bits : 2 x 5

(*a*) Mention the conditions for the existence of plasma and explain.

(*b*) In plasma physics, usually temperature is given in unit of energy. Show that for 1eV the temperature is approximately 10^4 K.

(*c*) Elaborate the idea underlying mean free path (λ) and collision cross-section (σ) of partially ionized plasma.

(*d*) Draw a schematic circuit diagram of a dc discharge system.

(*e*) Define the mobility and diffusion coefficient of an ionized gas.

(*f*) What do you mean by "distribution function" in elements of plasma kinetic theory ?

(*g*) State and discuss the Boltzmann equation under the concepts of phase points.

(*h*) Write the equation which follows the collisionless Boltzmann's equation and explain.

2. What do you mean by electrical neutrality in a plasma ? How is electrical neutrality maintained with a sustained discharge ? Deduce the relation

$$\lambda_D = 69 \cdot 0 \left(\frac{T}{n_e} \right)^{1/2} \text{ in } m$$

where : λ_D ~ Debye length, T ~ Temperature of the particles and n_e ~ the particle number density. 2 + 2 + 6

3. Describe with schematic circuit exploding wire method used to produce ionization of a gas in the laboratory technique. Show and discuss current waveforms. Point out its applications. 5 + 4 + 1