M.Sc 3rd Semester Examination 2010

PHYSICS

PAPER—PH-2101 (A + B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

PAPER—PH-2101 A

(*Relativistic Quantum Mechanics*)

[*Marks* : 20]

Answer **all** questions

1. Answer any *three* bits : 2×3

(*a*) Prove that

$$\alpha_x = \frac{1}{2}[\alpha_x \alpha_y, \alpha_y]$$

where $\vec{\alpha}$ is Dirac matrix.

(*b*) Find the velocity operator for the Dirac Hamiltonian.

(*c*) Define helicity operator.

(*Turn Over*)

(*d*) Obtain expressions for current density and probability density in the Dirac formalism.

(*e*) If $\alpha = i\hbar\psi^* \frac{\partial\psi}{\partial t} - \frac{\hbar^2}{2m}\vec{\nabla}\psi^* \cdot \vec{\nabla}\psi$

$$- V(\vec{x}, t)\psi^*\psi$$

be the Lagrangian density for Schrödinger field, prove that momentum density

$$\pi(\vec{x}, t) = i\hbar\psi^*(\vec{x}, t).$$

2. Answer any *one* bit : 4

(*a*) Prove that :

$$\text{Trace } (\vec{\alpha} \cdot \vec{B})(\vec{\alpha} \cdot \vec{C}) = 4\vec{B} \cdot \vec{C}.$$

(*b*) For a Dirac particle moving in a central potential show that the orbital angular momentum is not a constant of motion, rather the total angular momentum is conserved.

3. Answer any *one* bit : 10

(*a*) Find the eigenvalue for a spin zero particle in a Coulomb–field,

$$V(r) = \frac{ze^2}{-4\pi\varepsilon_0 r}$$

Find an estimate for the fine structure spread of the energy levels. 8 + 2

(*b*) (*i*) If radial momentum p_r and radial velocity α_r for an electron in a central potential are defined by,

$$p_r = \frac{\vec{r}\cdot\vec{p} - i\hbar}{r},\ \alpha_r = \frac{\vec{\alpha}\cdot\vec{r}}{r}.$$

Show that $\vec{\alpha}\cdot\vec{p} = \alpha_r p_r + \frac{i\hbar k\beta\alpha_r}{r}$

where $k = \frac{P(\vec{\sigma}^{\alpha}.\vec{L} + \hbar)}{\hbar}$.

(*ii*) Show that spin magnetic moment appears as a natural consequence for Dirac electron in an e.m. field. 5 + 5

PAPER — PH-2101 B

(*Statistical Mechanics*)

[*Marks* : 20]

Answer **Q.No.1** and any **one** from the rest

1. Answer any *five* bits : 2 x 5

(*a*) Prove that density of phase points is an explicit function of H only.

(*b*) Does equipartition of energy theorem is valid for quantum mechanical oscillator ? Explain your answer.

(*c*) Define density matrix $\hat{\varrho}$.

(*d*) Explain quantum mechanical average and ensemble average.

(*e*) Explain pure state and mixed state in the light of density matrix.

(*f*) If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, find the free energy of N such independent oscillators.

(*g*) The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3} a VT^4$, where a is a constant and T is the temperature of the gas. Find the chemical potential of the photon gas.

(*h*) Deduce Sackur–Tetrode equation using the expression of ground potential

$$\Omega = -kTVe^{\mu\beta}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

where $\beta = \frac{1}{kT}$.

2. (*a*) An ensemble of N three level systems with energies $\epsilon = -\epsilon_0,\ 0,\ +\epsilon_0$ is in thermal equilibrium at temperature T. Let $\beta = \frac{1}{kT}$ and if $\beta\epsilon_0 = 2$. Prove that the probability of finding the system in the level $\epsilon = 0$ is $P(0) = (1 + 2\cosh 2)^{-1}$. 4

(*b*) An ideal collection of N two level systems is in thermal equilibrium at temperature T. Each system has a ground state energy $-\in$ and an excited state energy $\in$. Prove that

$$<E> = -\in N \tan h \beta \in \text{ and } C_v = k \beta^2 \in^2 N \sec h^2 \beta \in.$$

3 + 3

3. (*a*) Prove that $Tr \; \hat{\varrho}^2 \leq 1$ where $\hat{\varrho}$ is the density matrix. 2

(*b*) Deduce an expression of Fermi distribution function from grand partition function. 4

(*c*) Prove that in co-ordinate representation the density matrix for a system of free particles in a box of volume V is given by 4

$$\varrho_{rr'} = \frac{1}{V} e^{-\frac{mkT}{2\hbar^2}|(\vec{r} - \vec{r}')|^2}$$
