Chapter 4

Genus value of m-polar fuzzy graphs*

4.1 Introduction

In developed countries, lot of cities are connected by the roads. Economically well developed country may have different vehicles for transport. These are not affordable for transport on all days due to heavy traffic. Crossing between the roads may lead to the accident or time consuming. To avoid such crossing, flyovers or tunnels are planted in the Highways. For these kinds of situations the planarity concept are used.

This chapter introduces embedding of mPFGs developed on the surface of the spheres. The m-polar fuzzy genus graph (mPFGG), strong and weak$m$PFGG is described withits genus value. There are discussed isomorphism characteristics on mPFGG. The relationship is established between the mPFG's planarity value and genus value.The Euler polyhedral equation is also defined with regard to thegenus value of the mPFGG. After this, the topological surface application of mPFGG will be given.

4.2 Embedding of m-polar fuzzy graph

The genus $g(G)$ of a crisp graph G is the least integer n, then G is S_n embeddable.

Definition 4.2.1. *Let G be an $mPFG$ and S be any surface. The graph G is defined as S-embedding of the $mPFG$ G if G can be drawn in S if there are no $mPFEs$ intersected.*

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We show that the embedding of mPFG in plane and sphere are equal. Since pressing or stretching will distort the sphere, its embedding property will remain unchangeable.

An mPFG drawn on the plane region can be drawn at the sphere surface since it is like a planesurface, which has an embedding property. The approach for mPFGs is identical to that of crisp graphs.

4.2.1 Relation between the plane and sphere of an m-polar fuzzy graph

A stereographic projection may clarify the relationship between the plane surface and the sphere surface. The sphere must be positioned on the surface of the plane and SP (South Pole) must be the point of contact between the sphere and the plane. Draw a perpendicular line from SP which is extending to the sphere's surface, naming it NP (NorthPole). Let G be an mPFG on the plane (See Fig. 4.1). We construct straight lines from NP to all the G vertices that cross the sphere. Afterwards, we get a unique mPF value of vertices and edges in the sphere, call the graph as G'. In the sphere, G' is an mPFG (See Fig. 4.1). Conversely, at any point of mPFG G on a sphere else like NP we can connect a vertex where the line from NP through the specified points intersects the plane. So, going to add all the points in the plane and trying to apply the same membership values to the vertexes and edges, we get a mPFG in this plane.(see Fig. 4.1).

Therefore there is one-to-one matching of mPF face values of both graphs (equal) between the two graphs. On the infinity region of the plane, the vertices in the plane are connected to the NP. As a result, NP is the infinity area in the sphere. We have the following theory from the above point of view.

Theorem 4.2.1. *Any mPFG embedded on the plane's surface can be embedded on the sphere's surface with the same mPFEs.*

4.3 Genus value of m-polar fuzzy graphs

The genus value of an mPFGs is strongly linked to the planarity value of the mPFGs. We are now defining a mPFGs genus value.

Definition 4.3.1. *An mPFG G is said to have a genus if a least positive integer n is available, then the mPFG G is S_n-embeddable. In other words, if the intersection*

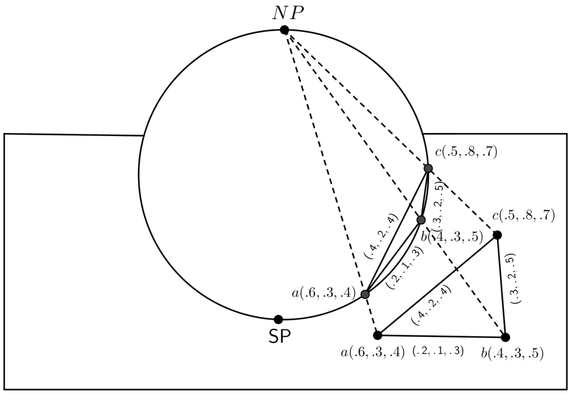

Figure 4.1: Embedding of a 3PFG in sphere

points between the edges for a geometrical representation are $P_1,\ P_2,\ \ldots,\ P_n$, *then* $\mathcal{P}^*(G)$ *is indicated as the genus value of the mPFG G or* $\mathcal{P}^* = (\mathcal{P}_1^*, \mathcal{P}_2^*, \ldots \mathcal{P}_m^*)$ *where,*

$$\mathcal{P}_i^* = \frac{\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i}{1 + \{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i\}},\ for\ i = 1, 2,\ \ \ldots,\ m$$

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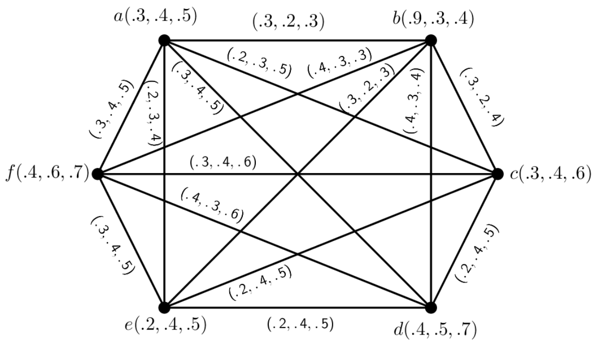

Figure 4.2: 3PFG

Example 4.3.1. *Consider, Fig 4.2 shows a 3PFG of* K_6. *Fig 4.3 shows the corresponding* 3*PFGG. Let* $P_1,\ P_2,\ P_3$ *be the three points of intersection between* 3 *polar fuzzy edges of these* 3*PFGG. Suppose* $P_3,\ P_2$ *and* P_1 *be the point of intersection between the edges* $((f, c)(0.3, 0.4, 0.6))$ *and* $((e, b), (0.3, 0.2, 0.3))$,$((b, a), (0.3, 0.2, 0.3))$, $((d, e), (0.2, 0.4, 0.5))$ *and* $((c, a), (0.2, 0.3, 0.5))$ *and* $((f, d), (0.4, 0.3, 0.6))$ *and respectively. The genus of the* 3*PFG is calculated as follows.*

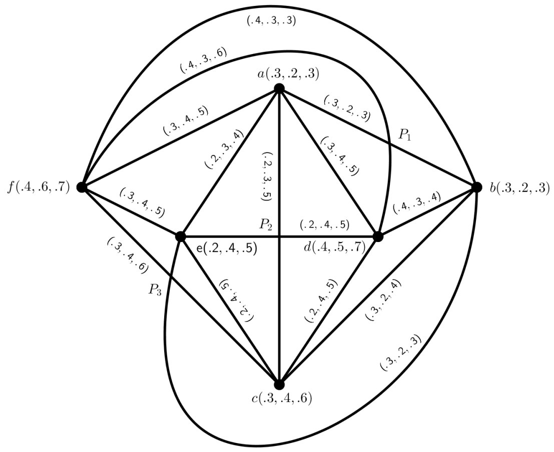

Figure 4.3: Corresponding 3PFGG

$I_{(a,b)} = (\frac{0.3}{min\{0.3,0.9\}}, \frac{0.2}{min\{0.4,0.3\}}, \frac{0.3}{min\{0.5,0.4\}}) = (1, 0.67, 0.75),$
$I_{(d,f)} = (\frac{0.4}{min\{0.4,0.4\}}, \frac{0.3}{min\{0.6,0.5\}}, \frac{0.6}{min\{0.7,0.7\}}) = (1, 0.6, 0.86),$
$I_{(e,d)} = (\frac{0.2}{min\{0.3,0.4\}}, \frac{0.4}{min\{0.5,0.5\}}, \frac{0.5}{min\{0.6,0.7\}}) = (0.67, 0.8, 0.83),$
$I_{(a,c)} = (\frac{0.2}{min\{0.3,0.3\}}, \frac{0.3}{min\{0.4,0.4\}}, \frac{0.5}{min\{0.5,0.6\}}) = (0.67, 0.75, 1),$
$I_{(c,f)} = (\frac{0.3}{min\{0.3,0.4\}}, \frac{0.4}{min\{0.4,0.6\}}, \frac{0.6}{min\{0.6,0.7\}}) = (1, 1, 1),$
$I_{(b,e)} = (\frac{0.3}{min\{0.9,0.3\}}, \frac{0.2}{min\{0.3,0.5\}}, \frac{0.3}{min\{0.4,0.6\}}) = (1, 0.67, 0.75).$

Therefore, the intersecting values are $\mathcal{I}_{P_1} = \frac{I_{(a,b)}+I_{(d,f)}}{2} = (1, 0.64, 0.81)$, $\mathcal{I}_{P_2} = \frac{I_{(e,d)}+I_{(a,c)}}{2} = (0.67, 0.775, 0.92)$ *and* $\mathcal{I}_{P_3} = \frac{I_{(c,f)}+I_{(b,e)}}{2} = (1, 0.835, 0.875)$. *Then,* $\mathcal{P}_1^* = \frac{1+.67+1}{1+[1+.67+1]} = \frac{2.67}{3.67} = 0.727$, $\mathcal{P}_2^* = \frac{0.64+0.775+0.835}{1+[0.64+0.775+0.835]} = \frac{2.25}{3.25} = 0.692$ *and*
$\mathcal{P}_3^* = \frac{0.81+0.92+0.875}{1+[0.81+0.92+0.875]} \frac{2.605}{3.605} = 0.722$

Hence, the genus of the 3*PFG of Fig 4.2 is* $(0.727, 0.692, 0.722)$.

Remark 4.3.1. *Similarly to that planarity value of an* m*PFG, the genus value of* m*PFG is bounded, i.e.* $0 < \mathcal{P}_i^* \leq 1 \ \forall \ i = 1, 2, \ldots, m$.

Theorem 4.3.1. *Any* m*PFG has a genus.*

Proof. Let G be an mPF multigraph. The mPF multigraph is geometrically represented to determine the number of crossings between mPFEs.

Case 1: Let $\sum\limits_{j=1}^{k} \mathcal{I}_{P_j}^i = 0, \forall \ i.$

Then the number of intersection of mPFEs in G is zero, and the given mPFG's planarity value is $(1,1,\ldots,1)$. Then the mPFG is obviously planar mPFG. Now,

$$\mathcal{P}_i^* = \frac{\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i}{1+\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i\}} = \frac{0}{1+0} = 0,\ i = 1,2,\ \ldots\ m.$$

Therefore, the given mPFG has genus value$(0,0,\ldots,0)$.

Case2: Let $\sum_{j=1}^{k} \mathcal{I}_{P_j}^i \neq 0$, $\forall\ i$.

Then, an intersection of edges exists in the mPFG G. The genus of an mPFG achieved by the number of intersection of the mPFEs in G. using the definition of genus value of the mPFG the we get it.

Hence, every mPFG has a genus. □

Theorem 4.3.2. *For every mPFG G, if it has mPF genus value* $(0,0,\ldots,0)$ *iff mPF planarity value is* $(1,1,\ldots,1)$.

Proof. Let G be an mPFG having mPF genus value $(0,0,\ldots,0)$. For a certain geometrical representation, let the intersection points between the edges be $P_1, P_2, \ldots, P_k$. Then using the definition of genus value, $\sum_{j=1}^{k} \mathcal{I}_{P_j}^i = 0$, $\forall\ i$. Then

$$\mathcal{P}_i^* = \frac{1}{1+\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i\}} = \frac{1}{1+0} = 1,\ for\ i = 1,2,\ \ldots\ m.$$

Hence, the given mPFG is mPF planar graph and $(1,1,\ldots,1)$ is the planarity value of that graph.

Conversely, let G be an mPFG having mPF planarity value $(1,1,\ldots,1)$. This implies, $\sum_{j=1}^{k} \mathcal{I}_{P_j}^i = 0$, $i = 1,2,\ldots,m$. Therefore,

$$\mathcal{P}_i^* = \frac{\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i}{1+\{\mathcal{I}_{P_1}^i + \mathcal{I}_{P_2}^i + \ldots + \mathcal{I}_{P_k}^i\}} = \frac{0}{1+0} = 0,\ i = 1,2,\ \ldots\ m.$$

Hence, the given mPFG is mPFGG with genus value $(0,0,\ldots,0)$.

□

The relationship between the genus and the degree of planarity of mPFG is indicated below from this Theorem.

Theorem 4.3.3. *For every mPFG,* $\mathcal{P}_i^* = 1 - \mathcal{P}_i$, $i = 1,2,\ldots,m$.

Theorem 4.3.4. *Let G be an mPF complete multigraph. The mPF genus value* $\mathcal{P}^* = (\mathcal{P}_1^*, \mathcal{P}_2^*, \ldots, \mathcal{P}_m^*)$ *is given by* $\mathcal{P}_i^* = \frac{n_k}{1+n_k}$, *$i$ where the number of points at which the edges intersect is n_k.*

Proof. We have $p_i \circ B^j(xy) = min\{p_i \circ A(x), p_i \circ A(y)\}\ \forall\ x, y \in V,\ i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, p$, because G is complete. Let $P_1, P_2, \ldots, P_{n_k}$ be the points at which the edges intersect in G. For an arc (x, y) in G, $I^i_{(x,y)} = \frac{p_i \circ B^j(xy)}{min\{p_i \circ A(x), p_i \circ A(y)\}} = 1,\ i$. Therefore, for the point P_1 at which the edges (a, b) and (c, d) are intersect, $\mathcal{I}_{P_1} = (1, 1, \ldots, 1)$ is the intersecting value. So, $\mathcal{I}_{P_i} = (1, 1, \ldots, 1)$ for $i = 1, 2, \ldots, n_k$. Now for $i = 1, 2, \ldots, m$

$$\mathcal{P}^*_i = \frac{\{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2} + \ldots + \mathcal{I}^i_{P_k}}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2} + \ldots + \mathcal{I}^i_{P_k}\}} = \frac{1 + 1 + \ldots + 1}{1 + (1 + 1 + \ldots + 1)} = \frac{n_k}{1 + n_k}.$$

Therefore, the mPF genus $\mathcal{P}^* = (\mathcal{P}^*_1, \mathcal{P}^*_2, \ldots, \mathcal{P}^*_m)$ where $\mathcal{P}^*_i = \frac{n_k}{1+n_k}$, for $i = 1, 2, \ldots, m$. □

Theorem 4.3.5. *Let G be an mPFGG with mPF genus value $\mathcal{P}^* = (\mathcal{P}^*_1, \mathcal{P}^*_2, \ldots, \mathcal{P}^*_m)$ is s.t. $\mathcal{P}^*_i < 0.5$ for $i = 1, 2, \ldots, m$. Then the number of points at which the mPFSEs intersect is at most one in G.*

Proof. Whenever possible, let G has at least two points of intersection P_1 and P_2 between two mPFSEs. For any mPF strong edge $((s, t), B^j(s, t))$, $I^i_{(s,t)} \geq 0.5$ for $i = 1, 2, \ldots, m$. So for any two intersecting mPFSEs $((s, t), B^j(s, t))$ and $((w, x), B^k(w, x))$ and $\frac{I^i_{(s,t)} + I^i_{(w,x)}}{2} \geq 0.5$ i.e. $\mathcal{I}^i_{P_1} \geq 0.5$, for $i = 1, 2, \ldots, m,$. Eventually,$\mathcal{I}^i_{P_2} \geq 0.5$. Then $1 + \mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2} \geq 2$ i.e. $\frac{1}{1+\mathcal{I}^i_{P_1}+\mathcal{I}^i_{P_2}} \leq 0.5$ and

$$\begin{aligned}
\mathcal{P}^*_i &= \frac{\{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}} \\
&= \frac{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\} - 1}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}} \\
&= \frac{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}} - \frac{1}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}} \\
&= 1 - \frac{1}{1 + \{\mathcal{I}^i_{P_1} + \mathcal{I}^i_{P_2}\}} \\
&\geq 0.5.
\end{aligned}$$

This is a contradiction since for $i = 1, 2, \ldots, m$, $\mathcal{P}^*_i < 0.5$.

Hence,there can not be exist two points. □

Theorem 4.3.6. *Let G be an mPFGG with mPF genus value $\mathcal{P}^* = (\mathcal{P}^*_1, \mathcal{P}^*_2, \ldots, \mathcal{P}^*_m)$. If $\mathcal{P}^*_i \geq 0.33$ for $i = 1, 2, \ldots, m$, then G can not contain any point at which two mPFSEs intersect.*

Proof. If possible, let P be a point at which two mPFSEs $((s,t), B^j(s,t))$ and $((w,x), B^k(w,x))$ are intersect. For any mPFSEs $((s,t), B^j(u,v))$ we have $I^i_{(s,t)} \geq 0.5$, $i = 1, 2, \ldots, m$. For minimum value of $I^i_{(s,t)}$, $I^i_{(w,x)}$ and $\mathcal{I}^i_P = 0.5$, $i = 1, 2, \ldots, m$. Then $\mathcal{P}^*_i < 0.33$ for $i = 1, 2, \ldots, m$, a contradiction. So, no intersecting point between two mPFSEs can be found in G □

Theorem 4.3.7. *Let G be an mPFGG with mPF genus value $\mathcal{P}^* = (\mathcal{P}^*_1, \mathcal{P}^*_2, \ldots, \mathcal{P}^*_m)$ be such that $\mathcal{P}^*_i < 0.5$, $i = 1, 2, \ldots, m$ and considerable number c. Then the number of point of intersection between considerable edges in G is at most $[\frac{1}{c}]$ or $\frac{1}{c} - 1$ according as $\frac{1}{c}$ is not an integer or an integer respectively.*

Proof. Let $G = (V, A, B)$ be an mPFGG where $B = \{((s,t), B^j(s,t)),$ $j = 1, 2, \ldots, p : (s,t) \in V \times V\}$. Let $0 < c < 0.5$ be the considerable number. Let $((s,t), B^j(s,t))$ be a considerable edge, then we have $p_i \circ B(qr) \geq c\ min\{p_i \circ A(q), p_i \circ A(r)\}$, $i = 1, 2, \ldots, m$. This means that, $I^i_{(s,t)} \geq c$ for $i = 1, 2, \ldots, m$. Let $P_1, P_2, \ldots, P_l$ be the l intersection points between considerable edges. Also let, P_1 be the point oat which considerable edges $((s_1,t_1), B^j(s_1,t_1))$ and $((s_2,t_2), B^j(s_2,t_2))$ are intersect. Then $\mathcal{I}^i_{P_1} = \frac{I^i_{(s_1,t_1)} + I^i_{(s_1,t_1)}}{2} \geq c$. So,

$$\sum_{n=1}^{l} \mathcal{I}^i_{P_n} \geq lc$$

$$i.e.\quad 1 + \sum_{n=1}^{l} \mathcal{I}^i_{P_n} \geq 1 + lc$$

$$i.e.\quad \frac{1}{1 + \sum_{n=1}^{l} \mathcal{I}^i_{P_n}} \leq \frac{1}{1 + lc}$$

$$i.e.\quad 1 - \frac{1}{1 + \sum_{n=1}^{l} \mathcal{I}^i_{P_n}} \geq 1 - \frac{1}{1 + lc}$$

$$i.e.\quad \frac{\sum_{n=1}^{l} \mathcal{I}^i_{P_n}}{1 + \sum_{n=1}^{l} \mathcal{I}^i_{P_n}} \geq \frac{lc}{1 + lc}.$$

This implies,

$$\mathcal{P}^*_i \geq \frac{lc}{1 + lc}. \tag{4.1}$$

Again we know,

$$\mathcal{P}^*_i \leq 0.5. \tag{4.2}$$

From 4.1 and 4.2 we get,

$$0.5 \geq \mathcal{P}_i^* \geq \frac{lc}{1+lc},\ i.e.\ l \geq \frac{1}{c}.$$

It gives the value of l

$$l = \begin{cases} [\frac{1}{c}], & if\ \frac{1}{c}\ not\ an\ integer \\ \frac{1}{c} - 1, & if\ \frac{1}{c}\ is\ an\ integer \end{cases}$$

Hence the proof. □

The following definition is given for the strong mPFGG.

Definition 4.3.2. *A mPFGG G is called strong if its genus value is $\mathcal{P}^* = (\mathcal{P}_1^*, \mathcal{P}_2^*, \ldots, \mathcal{P}_m^*)$ s.t. $\mathcal{P}_i^* \geq 0.33$, $i = 1, 2, \ldots, m$. If the statement is not true then it is weak.*

Theorem 4.3.8. *If G is weak mPF planar graph, then G is a strong mPFGG.*

Proof. Let G is weak mPF planar graph. Then $\mathcal{P}_i \leq .67$ for $i = 1, 2, \ldots, m$. Again, $\mathcal{P}_i^* = 1 - \mathcal{P}_i$, for $i = 1, 2, \ldots, m$, so $1 - \mathcal{P}_i^* = \mathcal{P}_i \leq .67$, i.e. $\mathcal{P}_i^* \geq 0.33$, $\forall\ i$. Hence, G is strong mPFGG. □

Theorem 4.3.9. *If G is weak mPFGG, then G is a strong mPF planar graph.*

Proof. Similarly we prove this Theorem using the above Theorem's procedure. □

4.3.1 Isomorphism of m-polar fuzzy genus graph

The isomorphism between the two mPFGGs in the topological surfaces is always the same. Suppose that there is an isomorphism between the two mPFGs and one is mPFGG, and another is mPFGG. The following results are now available.

Theorem 4.3.10. *Let G_1 be an mPFG and its corresponding mPFGG be G_1'. If there an isomorphism $k : G_1 \to G_2$ exists where, G_2 is an mPFG, then G_2' can be defined as mPFGG s.t. $k' : G_1' \to G_2'$ is an isomorphism.*

Proof. Let G_1' be the mPFGG of the given mPFG G_1 and there an isomorphism $k : G_1 \to G_2$ exists where G_2 is an mPFG. Since, there is an one-to-one and onto functions between two mPFGs have the same mPF values of edges and vertices. Then there a graph G_2' exists and this graph is the mPFGG of G_2 and there is one-to-one correspondence between the G_1' and G_2' have the same mPF values of edges and vertices. Hence $k' : G_1' \to G_2'$ is an isomorphism. □

Theorem 4.3.11. *Let G_1 and G_2 be two isomorphic mPFG with mPF genus values $(\mathcal{P}_1^*, \mathcal{P}_2^*, \ldots, \mathcal{P}_m^*)$ and $(\mathcal{K}_1^*, \mathcal{K}_2^*, \ldots, \mathcal{K}_m^*))$ respectively. Then $\mathcal{K}_i^* = \mathcal{P}_i^*$, $\forall\ i = 1, 2, \ldots, m$.*

Proof. Let two isomorphic mPFGs be G_1 and G_2. By Theorem 4.14 , there an isomorphism between two mPFGG G_1' and G_2' exists. The mPF genus values of G_1' and G_2' are $(\mathcal{P}_1^*, \mathcal{P}_2^*, \ldots, \mathcal{P}_m^*)$ and $(\mathcal{K}_1^*, \mathcal{K}_2^*, \ldots, \mathcal{K}_m^*))$ respectively. Again since, the mPF genus values are same as G_1' and G_2' are isomorphism.

Hence, $\mathcal{K}_i^* = \mathcal{P}_i^*$, $\forall\ i = 1, 2, \ldots, m$. □

4.3.2 Euler polyhedral equation for m-polar fuzzy genus graphs

In the field of graph theory, Euler clarified several ideas. Euler polyhedral equation makes it easier to find the genus value of a graph and to define the limitations on the genus values. We have proved here that the Euler polyhedral equation for an mPFGG.

Theorem 4.3.12. *Let G be a connected mPFGG with edge set E, vertex set V and the m-polar fuzzy faces value be S_F. Then the mPF genus value satisfies the inequality in below:*

$V_i^* - E_i^* + S_{F_i}^* \leq 2 - \mathcal{P}_i^*$, $i = 1, 2, \ldots, m$, *where*

$$V_i^* = \frac{\text{The sum of the } i- \text{th membership values of the vertices in } G}{\text{Total number of vertices in } G},$$

$$E_i^* = \frac{\text{The sum of the } i\text{–th membership values ofthe edges in } G}{\text{Total number of edges in } G},$$

$$S_{F_i}^* = \frac{\text{The sum ofthe } i\text{–th membership values of the } m\text{–polar fuzzy faces in } G}{\text{Total number of edges in } G}.$$

Proof. By the process of contradiction, we prove that theory. Suppose we have that $V_i^* - E_i^* + S_{F_i}^* > 2 - \mathcal{P}_i^*$, $\forall\ i = 1, 2, \ldots, m$.

Case 1: Let $\mathcal{P}_i^* = 0$, $\forall\ i$.

Then $(0, 0, \ldots, 0)$ is the mPF genus value. The inequalities mentioned above are strictly greater than 2. But, V_i^*, E_i^*, $S_{F_i}^*$ all have been specified to range from 0 to 1. Then the value $V_i^* - E_i^* + S_{F_i}^*$ would be in the range of $(0, 1)$. This inequality does not therefore occur. So, a contradiction arise and then, $V_i^* - E_i^* + S_{F_i}^* \leq 2 - \mathcal{P}_i^*\ \forall\ i$.

Case 2: Let $0 < \mathcal{P}_i^* < 1\ \forall\ i$. Then the mPF genus value lies within $(0, 1)$. Again, V_i^*, E_i^*, $S_{F_i}^*$, all have been specified to range from 0 to 1. Then, $V_i^* - E_i^* + S_{F_i}^*$ lie in

$(0,1)$. This inequality does not therefore occur. So, a contradiction arise and then, $V_i^* - E_i^* + S_{F_i}^* \leq 2 - \mathcal{P}_i^* \; \forall \; i$.

Therefore, by use of the relations, the mPF genus value is calculated: $V_i^* - E_i^* + S_{F_i}^* \leq 2 - \mathcal{P}_i^*$, for $i = 1, 2, \ldots, m$. □

Corollary 4.3.1. *Every mPF genus value of an mPFGG has supremum and infimum, i.e.* $0 \leq \mathcal{P}_i^* \leq \frac{V_i^* - E_i^* + S_{F_i}^*}{2}$, $i = 1, 2, \ldots, m$.

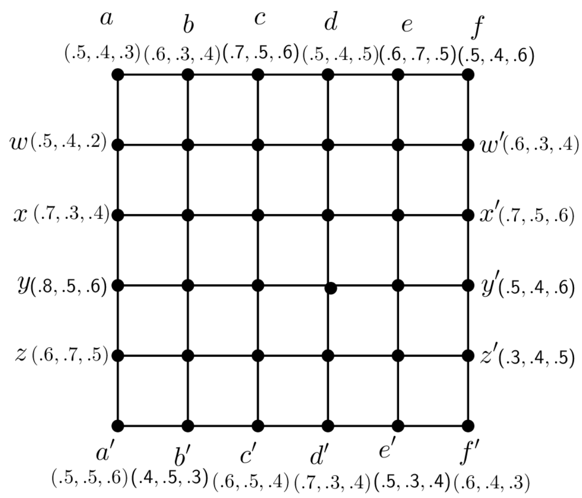

Figure 4.4: The grid network

4.4 m-polar fuzzy genus value in topological surface

Modification or modifying of the given graph in any other directions stands for topology. The graphs are embedded in topology in any surface or region. Topology is a large field to know the surface classification and These surfaces are available anywhere where we need them. the concept of the surface is commonly used in the field of physical science, aerodynamic and computer engineering.

The number of systems linked in the computer network is so large that no crossing is seen in a grid between systems. The network grid represents all structures that have to be taken as vertices and The link between them is the links that are considered as edges. Therefore, all edges of the network grid are connected in less time to enhance the communication between the two.

There are mPF values for the network grid. The vertices and edges have the mPF value. So it has to resembles like mPFG. The membership values of the vertices are

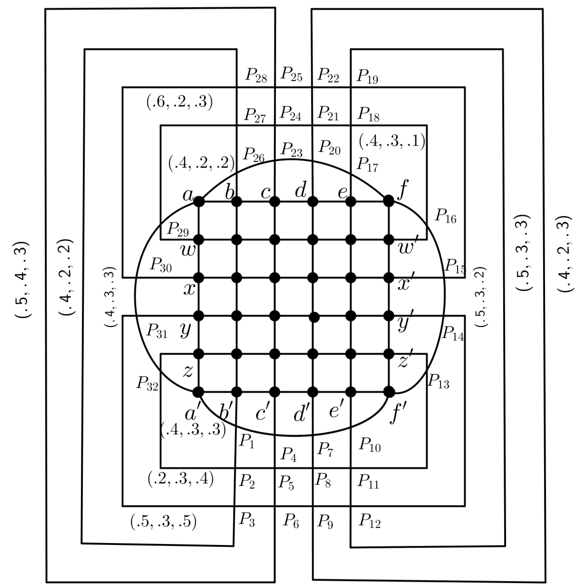

Figure 4.5: Intersection of edges in grid network

interpreted through the system's functioning and Communication pace is sensed on the membership value of the edges both systems pair.

Now let us describe some of the grid network terms:

Let G be an mPFG. The end nodes are connected in a gird network so that new edges are generated and the membership value of all edges are known by the performance between them. The effects of the new edges are determined by the relation $p_i \circ B(st) \leq min\{p_i \circ A(s), p_i \circ A(t)\}$ for all $st \in \widetilde{V^2}$, $i = 1, 2, \ldots, m$. The definition of the 3PF genus value is shown by a simple example. In Fig. 4.4, the network grid of 6×6 is drawn and the connections between the 3PF vertices are shown. The crosses between the edges are shown clearly in the figure. 4.5.

The intersections between the 3PFEs is determined as follows::

Therefore, $\mathcal{P}_1^* = \frac{\sum_{i=1}^{32} \mathcal{I}_{P_i}^1}{1+\sum_{i=1}^{32} \mathcal{I}_{P_i}^1} = \frac{27.68}{1+27.68} = 0.965$, $\mathcal{P}_2^* = \frac{\sum_{i=1}^{32} \mathcal{I}_{P_i}^2}{1+\sum_{i=1}^{32} \mathcal{I}_{P_i}^2} = \frac{24.27}{1+24.27} = 0.960$ and $\mathcal{P}_3^* = \frac{\sum_{i=1}^{32} \mathcal{I}_{P_i}^3}{1+\sum_{i=1}^{32} \mathcal{I}_{P_i}^3} = \frac{27.08}{1+27.08} = 0.960$. The 3PFGG of the grid network is seen in Fig. 4.6. The Fig. 4.6 shows that the intersection of the 3PFEs in network grid can be reduced to a torus. When the network grid is fully connected with all the vertices and fulfills the 3PFG definition, then we get the 3PF toridal graph.

I_P	I_1	I_2	I_3	I_4	I_5
Intersecting value of P	(0.9, 0.71, 0.835)	(0.835, 0.71, 0.735)	(1, 0.71, 0.75)	(0.815, 0.775, 0.875)	(0.75, 0.775, 0.775)
I_P	I_6	I_7	I_8	I_9	I_{10}
Intersecting value of P	(0.915, 0.775, 0.79)	(0.8, 0.71, 0.875)	(0.735, 0.71, 0.775)	(0.9, 0.71, 0.79)	(0.9, 0.875, 0.875)
I_P	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
Intersecting value of P	(0.835, 0.875, 0.775)	(1, 0.875, 0.79)	(0.835, 0.75, 0.735)	(1, 0.75, 0.75)	(0.93, 0.71, 0.71)
I_P	I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
Intersecting value of P	(0.9, 0.875, .585)	(0.9, 0.75, 0.71)	(0.9, 1, 0.625)	(0.93, 0.835, 0.75)	(0.8, 0.585, 0.71)
I_P	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}
Intersecting value of P	(0.8, 0.835, 0.625)	(0.83, 0.67, 0.75)	(0.815, 0.65, 0.71)	(0.815, 0.9, 0.625)	(0.845, 0.735, 0.75)
I_P	I_{26}	I_{27}	I_{28}	I_{29}	I_{30}
Intersecting value of P	(0.9, 0.585, 0.67)	(0.9, 0.835, 0.585)	(0.93, 0.67, 0.71)	(0.8, 0.835, 0.75)	(0.83, 0.67, 0.875)
I_P	I_{31}	I_{32}			
Intersecting value of P	(0.9, 0.71, 0.915)	(0.735, 0.71, 0.9)			

Table 4.1: Intersection of 3PF edges of the grid network

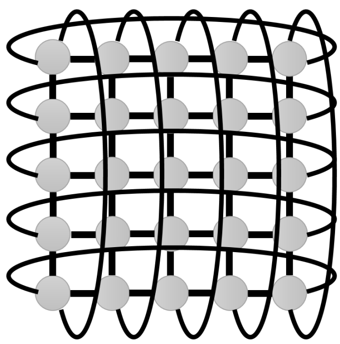

Figure 4.6: Torus graph

4.5 Summary

mPFGs theory are useful in real life situations such as computer science, image segmentation, including data mining, clustering, image capturing, networking, etc. To reduce the crossings between the edges choosing an alternate path so the edges become efficient and time consumable. Here, embedding of mPFGs which are developed on the surface of spheres is presented. The weak and strong mPFGG and mPFGG with its genus value are described. There were discussions about the isomorphism properties of mPFGG. Furthermore, a relationship has been identified between planarity value and genus value of the mPFG. The Euler polyhedral equation is also defined with respect to the mPFGG genus value. In the topological surface, a useful application of mPFGG is given.