Chapter 2

Types of vertices in m-polar graphs*

2.1 Introduction

The notion of mPF set as a generalisation of BFS was launched by Chen et al. [38] in 2014. The theory behind this concept is that "multipolar information" (not only bipolar knowledge that represents the two-valued logic) exists, since real world problems are often received from multiple agents. For example, mankind 's accurate telecom security level is one point in $[0,1]^n$ $(n \approx 7 \times 10^9)$ because different persons have been monitored at different times. A mPF model is useful in multi-agent, multi-attribute and multi-object network models that gives the system more precision, versatility and comparability than the standard, fuzzy, and bipolar fuzzy models. Chen et al. [38] first defined mPFGs. The fundamental target of this chapter is to present the idea of superstrong and strong mPF vertex of mPFGs using the concept of strong mPFE, strength of connectedness of path etc. Next we studied several properties on these paths. At the end, there is also an application of a strong path problem.

2.2 Generalized m-polar fuzzy graphs

The following described mPFG by Chen et al. [38]:

An mPFG is defined as a pair $G = (A, B)$ in which $A : V \to [0,1]^m$ and $B : E \to [0,1]^m$ satisfying $B(xy) \leq min\{A(x), A(y)\}$ for each $xy \in E$.

*A part of the work presented in this chapter is published in *Journal of Multivalued Logic and Soft Computing*, **34**, 263–282 (2020).

In addition, B is an mPFS in $E \subseteq V \times V$. By using the description, B is an mPFS defined in $\widetilde{V^2}$ satisfying $B(xy) = \mathbf{0} = (0, 0, \ldots, 0)$ $\forall$ $xy \in (\widetilde{V^2} - E)$. The definition above is difficult for the complement of mPFGs to be calculated. The generalized mPFGs are therefore described below.

We presume the following before defining generalized mPFGs:

For this set of V, describe a equivalence relation $\sim$ on $V \times V - \{(s, s) : s \in V\}$ as follows: $(s_1, t_1) \sim (s_2, t_2) \Leftrightarrow$ either $(s_1, t_1) = (s_2, t_2)$ or $s_1 = t_2$ and $t_1 = s_2$. The quotient set is marked with $\widetilde{V^2}$ and the equivalence class with the element (s, t) is marked with st or ts.

In the chapter, $G = (V, A, B)$ is an mPFG of a crisp graph $G^* = (V, E)$.

Definition 2.2.1. *An mPFG (or generalized mPFG) of G^* is a pair $G = (V, A, B)$ where $A : V \to [0, 1]^m$ is an mPFS in V and $B : \widetilde{V^2} \to [0, 1]^m$ is an mPFS in $\widetilde{V^2}$ s.t $p_i \circ B(st) \leq min\{p_i \circ A(s), p_i \circ A(t)\}$ $\forall$ $st \in \widetilde{V^2}$, $i = 1, 2, \ldots, m$ and $B(st) = \mathbf{0}$ $\forall$ $st \in (\widetilde{V^2} - E)$, $\big(\mathbf{0} = (0, 0, \ldots, 0)$ is the lowest element in $[0, 1]^m\big)$.*

Here, $p_i \circ A(s)$ is the ith degree of membership of the vertex s and $p_i \circ B(st)$ is the ith degree of membership of the edge st. A is called the mPFVS of G and B as the mPFES of G.

Example 2.2.1. *Here, A 3PFG G of the crisp graph $G' = (V, E)$ is displayed in Fig. 2.1, where $V = \{s'_1, s'_2, s'_3, s'_4\}$ and $E = \{s'_1s'_2, s'_2s'_3, s'_3s'_4, s'_2s'_4, s'_1s'_3\}$.*

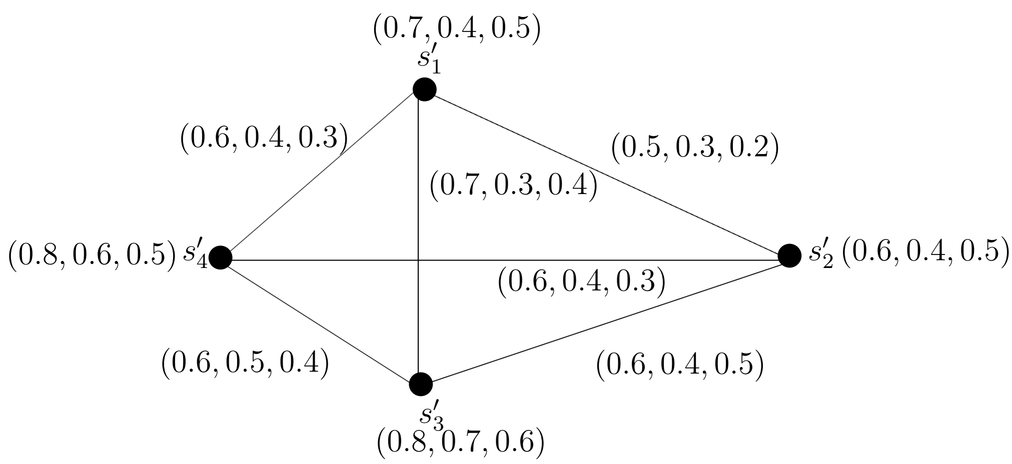

Figure 2.1: A 3PFG G

2.3 m-polar fuzzy path and connectedness

In this section, m-polar fuzzy path (mPFP) and m-polar fuzzy connectedness (mPFC) is described on mPFGs. Again, strong and strongest mPFP on mPFGs are described.

Definition 2.3.1. *An mPFP of an mPFG G is a sequence of separate nodes $s' = t_1, t_2, \ldots, t_n = t'$ s.t., $p_i \circ B(t_k, t_{k+1}) > 0 \ \forall \ k = 1, 2, \ldots, n-1$ and for at least one i and all the nodes are separate except t_1 may be the identical to t_n. Let $P : s' = t_0, t_1, \ldots, t_n = t'$ be an mPFP. If $n \geq 3$ and $t_0 = t_n$, then the mPFP is called an mPFC.*

Definition 2.3.2. *Let $P : s' = s_1, s_2, \ldots, s_n = t'$ be an mPFP in an mPFG G. Then the strength of the path P is defined as*

$$S(P) = (\min_{1\leq i<j\leq n}(p_1 \circ B(s_i, s_j)), \min_{1\leq i<j\leq n}(p_2 \circ B(s_i, s_j)), \ldots, \min_{1\leq i<j\leq n}(p_n \circ B(s_i, s_j)))$$

$$= (B_1^n(s', t'), B_2^n(s', t'), \ldots, B_m^n(s', t')).$$

The strength of connectedness between s' and t' is the maximum of the strengths of all mPFPs between those vertices and which is formulated below:

$$CONN_G(s', t') = \big(\big(p_1 \circ B(s_i, s_j)\big)^\infty, \big(p_2 \circ B(s_i, s_j)\big)^\infty, \ldots, \big(p_n \circ B(s_i, s_j)\big)^\infty\big),$$

where $\big(p_i \circ B(s', t')\big)^\infty = \max_{n\in N}(B_i^n(s', t'))$

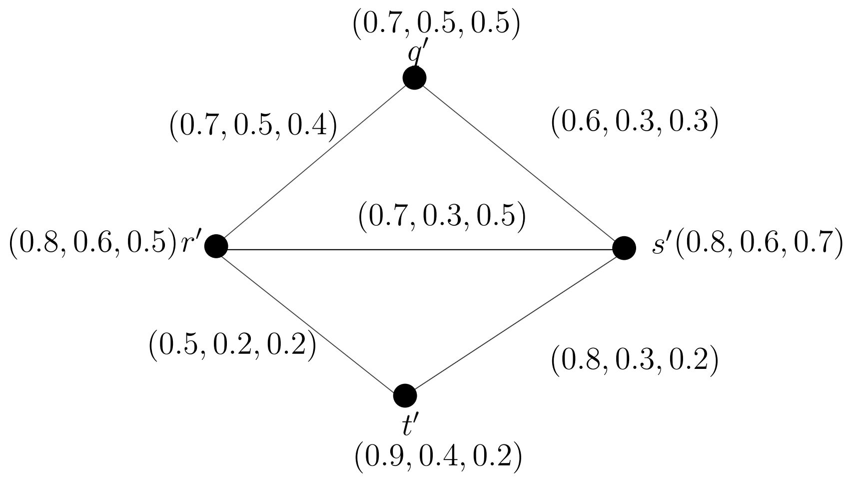

Figure 2.2: A 3PFG G.

Example 2.3.1. *A 3PFG G of $G' = (V, E)$ is seen in Fig. 2.2, where $V = \{q', r', s', t'\}$ and $E = \{q'r', r's', r't', s't', q's'\}$. We find out the strength of connectedness between q' and t'. The paths from q' and t' are $q' - s' - r' - t'$, $q' - r' - s' - t'$, $q' - s' - t'$ and $q' - r' - t'$. The strength of the paths $q' - r' - t'$, $q' - s' - t'$, $q' - r' - s' - t'$ and $q' - s' - r' - t'$ are $(0.5, 0.2, 0.2)$, $(0.6, 0.3, 0.2)$, $(0.7, 0.3, 0.2)$ and $(0.5, 0.2, 0.2)$ respectively. So, $CONN_G(q', t') = (0.7, 0.3, 0.2)$ is the strength of connectedness between q' and t'.*

Definition 2.3.3. *Let G be an $mPFG$. When there is an edge between each vertex pair, G is called a mPF connected graph. In other words, an $mPFG$ will be an mPF connected graph if there is one at least i, $\big(p_i \circ B(s', t')\big)^\infty > 0$.*

Definition 2.3.4. *Let $P : s' = s_1, s_2, \ldots, s_n = t'$ be an $mPFP$ in an $mPFG$ G. Then, it is said that the path P is the strongest $mPFP$ if $B_i^n(s', t') = (p_i \circ B(s', t'))^\infty$ $\forall\ i = 1, 2, 3, \ldots, m$ i.e. $S(P) = CONN_G(s', t')$.*

Definition 2.3.5. *Let (s', t') be an arc in $mPFG$ G. Then, it is said that the arc (s', t') is a strong m-polar fuzzy edge (strong $mPFE$) if $p_i \circ B(s', t') \geq p_i \circ CONN_{G-(s',t')}(s', t')$ $\forall\ i = 1, 2, 3, \ldots, m$ i.e. if $B(s', t') \geq CONN_{G-(s',t')}(s', t')$. If $\forall\ i$, $p_i \circ B(s', t') > 0$ then we said that s' and t' are strong mPF neighbors.*

Definition 2.3.6. *Let $P : s = s_1, s_2, \ldots, s_n = t$ is a path from s to t. Then, it is said that the path P is strong $mPFP$ if (s_j, s_{j+1}) is strong $mPFE$ $\forall\ 1 \leq j \leq n - 1$.*

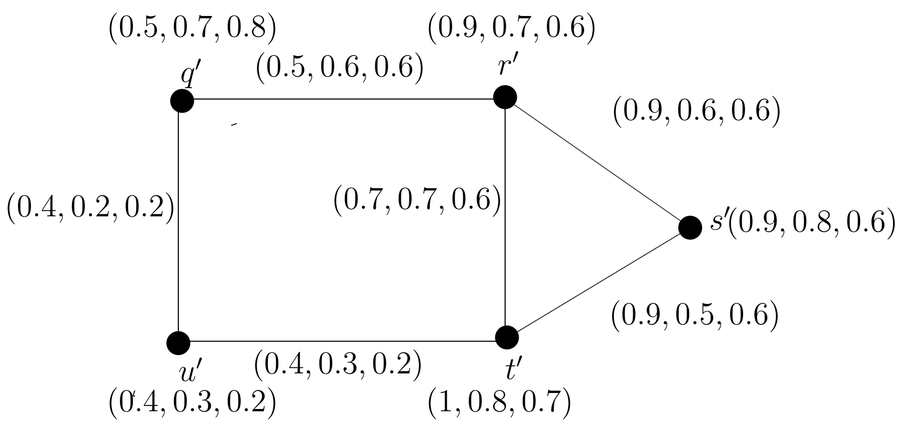

Figure 2.3: Illustration of example 2.3.2.

Example 2.3.2. *The Fig. 2.3 shows a 3PFG G of $G' = (V, E)$, where $V = \{q', r', s', t', u'\}$ and $E = \{q'r', r's', r't', s't', t'u', q'u'\}$. We consider all the paths from q' to t' for finding $CONN_G(q', t')$. They are $q' - r' - t'$, $q' - u' - t'$ and $q' - r' - s' - t'$ and the strength of those paths are $(0.5, 0.6, 0.6)$, $(0.4, 0.2, 0.2)$ and $(0.5, 0.5, 0.6)$ respectively. So, $CONN_G(q', t') = (0.5, 0.6, 0.6)$ is the strength of connectedness between q' and t'. Again, $q' - r' - t'$ path strength is $(0.5, 0.6, 0.6)$ and it is equal to $CONN_G(q', t')$. The $q' - r' - t'$ path therefore represents the strongest path.*

Definition 2.3.7. *A vertex of an mPFG is known as an mPF pendant vertex if its degree is one.*

2.4 Strong and superstrong m polar fuzzy vertices of mPF graphs

In this section, strong m-polar fuzzy vertices (strong mPFV) and superstrong m-polar fuzzy vertices (superstrong mPFV) is described on mPFGs.

Definition 2.4.1. *Let G be an mPFG and a^* be an vertex in G. Then it is also said that a^* is the strong mPFV if (a^*, b^*) is strong mPFE for all vertices b^* incident with a^* in G.*

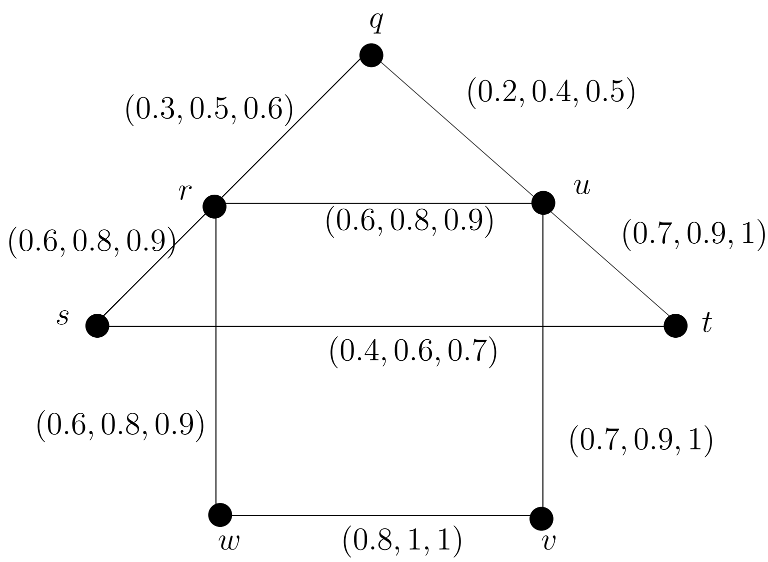

Figure 2.4: Strong mPFV in G.

Example 2.4.1. *A 3PFG G of a crisp graph G', where $V = \{q, r, s, t, u, v, w\}$ and $E = \{qr, rs, st, tu, qu, rw, wv, uv, ru\}$ is shown in Fig. 2.4. Here qr,rs, ru and rw are strong edges because* $(0.3, 0.5, 0.6) = B(q, r) > CONN_{G-(q,r)}(q, r) = (0.2, 0.4, 0.5)$, $(0.6, 0.8, 0.9) = B(r, s) = CONN_{G-(r,s)}(r, s) = (0.6, 0.8, 0.9)$, $(0.6, 0.8, 0.9) = B(r, u) = CONN_{G-(r,u)}(r, u) = (0.6, 0.8, 0.9)$ *and* $(0.6, 0.8, 0.9) = B(r, w) = CONN_{G-(r,w)}(r, w) = (0.6, 0.8, 0.9)$. *So all the edges incident with r are strong mPFEs. Here r is a strong mPFV.*

Definition 2.4.2. *Let G be an mPFG and $a^* \in V$ be an vertex in G. a^* is called superstrong mPFV if $\forall$ $i = 1, 2, 3, \ldots, m$, $p_i \circ CONN_G(s^*, t^*) = k_i$ for every $t^*(\neq s^*) \in V$ and for some $k_i \in (0, 1]$.*

Example 2.4.2. *A 3PFG G of a crisp graph G' , where $V = \{q, r, s, t, u\}$ and $E = \{qr, rs, st, qt, qu, ru\}$ is shown in Fig. 2.5. Here t is a superstrong mPFV because* $CONN_G(t, q) = CONN_G(t, r) = CONN_G(t, s) = CONN_G(t, u) = (0.3, 0.5, 0.7)$.

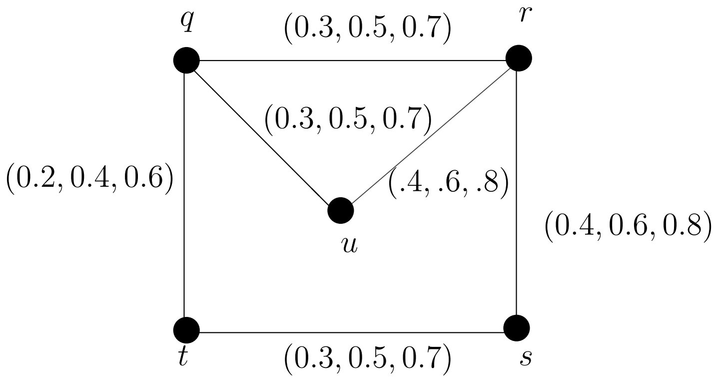

Figure 2.5: Superstrong mPFV in G.

Theorem 2.4.1. *Let G be a finite connected mPFG and a^* is a superstrong mPFV in G. Then for every $b^* \in V$, $CONN_G(a^*, b^*) \geq B(a^*, c^*)$ $\forall$ c^* adjacent to a^*.*

Proof. Let G be a finite and connected mPFG. Let s^* is a superstrong mPFV in G that means $\forall$ i, $p_i \circ CONN_G(s^*, t^*) = k_i$ for every $t^*(\neq s^*)$ and for some $k_i \in (0., 1]$ Then using the definition of superstrong mPFV we have, $\forall$ $i = 1, 2, 3, \ldots, m$

$$p_i \circ CONN_G(s^*, t^*) = p_i \circ CONN_G(s^*, u^*). \tag{2.1}$$

Again we know, $\forall\ i$

$$p_i \circ CONN_G(s^*, u^*) \geq p_i \circ B(s^*, u^*). \quad (2.2)$$

Now from equation 2.1 and 2.2 we get, $\forall\ u^*$ adjacent to s^* and for every $t^* \in V$

$$p_i \circ CONN_G(s^*, t^*) \geq p_i \circ B(s^*, u^*), \quad \forall\ i.$$

So, if $a^* \in V$ is a superstrong mPFV then for every $b^* \in V$, $CONN_G(s^*, t^*) \geq B(s^*, u^*)$ $\forall\ u^*$ adjacent to s^*. □

Corollary 2.4.1. *Suppose s^* is a superstrong mPFV in an mPFG G, then $CONN_G(s^*, t^*) = S(P)$ for at least one path P from s^* to t^*.*

Theorem 2.4.2. *Let G be a connected mPFG. If every edge incident at s^* has the same membership value, then s^* is a strong mPFV in G.*

Proof. Suppose the membership value of each edge incident at a^* has same i.e. $\forall\ i = 1, 2, \ldots, m$, $p_i \circ B(s^*, t^*) = \alpha_i$ for every t^* adjacent to s^*. Now we want to prove that s^* is a strong mPFV, which means we have to show that every edge incident at s^* is strong mPFE.

Case 1: If $s^* - t^*$ is only one mPFP from s^* to t^*, which is a trivial case. So clearly we see that the arc (s^*, t^*) is strong mPFE. Hence s^* is a strong mPFV.

Case 2: Suppose there exist more than one mPFP between s^* and t^*. Then the strength of all paths between s^* and t^* is less then or equal to $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ because every edge incident at s^* has the same membership value $(\alpha_1, \alpha_2, \ldots, \alpha_m)$, which implies the strength of connectedness between s^* and t^* is less then or equal to $(\alpha_1, \alpha_2, \ldots, \alpha_m)$. So we have, $p_i \circ B(s^*, t^*) \geq p_i \circ CONN_{G-(s^*,t^*)}(s^*, t^*)\ \forall\ i$. This implies that (s^*, t^*) is a strong mPFE for every s^* adjacent to t^*. Hence s^* is a strong mPFV. □

Theorem 2.4.3. *Let G be a connected mPFG. Every edge incident with $s^* \in V$ has the same membership value, which is minimum of all edges of G then s^* is a superstrong mPFV.*

Proof. Here $s^* \in V$ be a vertex in G. And every edge incident at s^* has the same membership value, say $(\alpha_1, \alpha_2, \ldots, \alpha_m)$. That means we say that $\forall\ i$, $p_i \circ B(x, s^*) = \alpha_i$ $\forall\ x$ adjacent to s^*. From the above Theorem we say that s^* is a strong mPFV. Next we want to prove that s^* is a superstrong mPFV. Here $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ is minimum among all edges of G. Then the ith component of strength of all paths from the vertex a^* to $x(\neq s^*) \in V$ is α_i $\forall\ x(\neq s^*)$ adjacent to s^*. Therefore, $\forall\ i$

$$p_i \circ CONN_G(s^*, x) = \alpha_i,\ for\ all\ x\ (\neq\ s^*)\ adjacent\ to\ s^*. \tag{2.3}$$

Let v^* be non-adjacent vertex to s^*. Then the strength of all paths from the vertex s^* to v^* is $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ because any path from s^* to v^* contain the edge whose membership value is $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ because every edge incident with $s^* \in$ have the same membership value $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ and these value is minimum among all edges in G.

Therefore, $\forall\ i$

$$p_i \circ CONN_G(s^*, v^*) = \alpha_i,\ for\ all\ v^*\ (\neq\ s^*)\ non-adjacent\ to\ s^*. \tag{2.4}$$

Then from equation 2.3 and 2.4 we get, s^* is a superstrong mPFV of G.

□

Theorem 2.4.4. *Let G be a connected mPFG. An edge (s^*, t^*) is strong mPFE iff $\forall$ $i = 1, 2, \ldots, m$, $p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*)$.*

Proof. Let G be a connected mPFG and (s^*, t^*) be a strong mPFE. Then, $\forall\ i = 1, 2, \ldots, m$,

$$p_i \circ CONN_{G-(s^*,t^*)}(s^*, t^*) \leq p_i \circ B(s^*, t^*).$$

Again,

$$\begin{aligned} CONN_G(s^*, t^*) &= max\{S(P)\} \\ &= max\{CONN_{G-(s^*,t^*)}(s^*, t^*), B(s^*, t^*)\} \\ &= B(s^*, t^*)\ (since\ CONN_{G-(s^*,t^*)}(s^*, t^*) \leq B(s^*, t^*)). \end{aligned}$$

Conversely, Suppose $\forall\ i = 1, 2, \ldots, m$, $p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*)$. It is implies that $\forall\ i$, $p_i \circ CONN_{G-(s^*,t^*)}(s^*, t^*) \leq p_i \circ B(s^*, t^*)$. Hence the result. □

Theorem 2.4.5. *Let G be a connected $mPFG$. Every edge incident with $s^* \in V$ have the same membership value and these membership values are the minimum of all edges of G if and only if s^* is a strong and superstrong $mPFV$.*

Proof. Suppose s^* is a vertex in G. Membership value of each edge incident at s^* is the same. Now using the Theorem 2.4.2, we say that s^* is a strong mPFV. Again from Theorem 2.4.3, s^* is a superstrong mPFV. Hence, s^* is a strong and superstrong mPFV.

Conversely, let s^* be a strong and superstrong mPFV. Now, we show that each edge with $s^* \in V$ has the same membership value. Every edge incident with s^* is a strong edge because s^* is a strong mPFV. Then from Theorem 2.4.4 we get, if (s^*, t^*) is a strong edge then $p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*)\ \forall\ i$ where t^* is a adjacent vertex to s^*. Again s^* is also a superstrong mPFV, so $p_i \circ CONN_G(s^*, t^*) = k_i\ \forall\ i = 1, 2, \ldots, m$. We easily see that $\forall\ i = 1, 2, \ldots, m$, $k_i = p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*)$ for all t^* adjacent vertex to s^*. Therefore every edge incident with $s^* \in V$ have the same membership value.

□

Theorem 2.4.6. *Let G be a connected $mPFG$. Every edge incident with s^* has the same membership value and this membership value is minimum among all edges of E in G, then s^* is a unique superstrong $mPFV$.*

Proof. Here s^* be a node in G and the membership value of every arcs incident at s^* have the same say $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)$. So we say that $\forall\ i$, $p_i \circ B(t^*, s^*) = \alpha_i$ for all t^* adjacent to s^*. Therefore from the Theorem 2.4.3, we have s^* is a superstrong mPFV of G which implies that $p_i \circ CONN_G(t^*, s^*) = \alpha_i\ \forall\ i = 1, 2, \ldots, m$ and $\forall\ t^*(\neq s^*)$. Now we have to prove the uniqueness of s^*. Let q^* be another superstrong mPFV of G. Then,

$$p_i \circ CONN_G(s^*, q^*) = \alpha_i = p_i \circ CONN_G(q^*, s^*)\ for\ all\ i$$

Now we consider a vertex $r^* \in V$ in which $a^* \neq q^* \neq r^* \in V$, then

$$p_i \circ CONN_G(q^*, s^*) = \alpha_i \neq p_i \circ CONN_G(q^*, r^*)\ for\ all\ i$$

Since α is minimum and $\alpha < B(q^*, r^*)$.

$$p_i \circ CONN_G(q^*, s^*) \neq p_i \circ CONN_G(q^*, r^*) \ for \ all \ i \tag{2.5}$$

From equation 2.5 we get q^* is not a superstrong mPFV, which is a contradiction. Hence s^* is a unique superstrong mPFV in G. □

Theorem 2.4.7. *Let G be a connected mPFG. Let s^* be a pendant mPFV in V and the edge (s^*, t^*) is only one edge incident with a^* and the membership value of these edge $B(s^*, t^*)$ is minimum among all non incident edges with s^*, then s^* is both strong and superstrong mPFV in G.*

Proof. Let G be a connected mPFG. Let $s^* \in V$ be a pendant mPFV. Suppose (s^*, t^*) be the only edge incident with s^* and the membership value of these edge $B(s^*, t^*)$ is minimum between all other edges (q^*, r^*) where (q^*, r^*) are non incident edge with s^*. Here the edge (s^*, t^*) is strong mPFE because there exist only one path $s^* - t^*$ between s^* and t^* and

$$p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*) \ \forall \ i \tag{2.6}$$

Thus, the pendant mPFV s^* is a strong mPFV. Next we want to prove that s^* is a superstrong mPFV. Suppose q^* be a vertex in G. Now the strength of connectedness between s^* and q^* is $p_i \circ CONN_G(s^*, q^*) = p_i \circ B(s^*, t^*) \ \forall \ q^* \in V$, because the edge (s^*, t^*) is only one edge incident with s^* and the membership value of these edge $B(s^*, t^*)$ is minimum among all non incident edges with s^*. Hence

$$p_i \circ CONN_G(s^*, q^*) = p_i \circ B(s^*, t^*) \ \forall \ i \tag{2.7}$$

From 2.6 and 2.7, we get $\forall \ q^* \in V$

$$p_i \circ CONN_G(s^*, t^*) = p_i \circ CONN_G(s^*, q^*) \ \forall \ i$$

This implies s^* is a superstrong mPFV. Hence the result.

□

Theorem 2.4.8. *Let G be a connected mPFG. Let a^* be a strong mPFV. If the membership value of each edge incident with s^* has distinct, then s^* is not superstrong mPFV.*

Proof. Let s^* be a strong mPFV. Then from the definition of strong mPFV we have each edge incident with s^* is strong mPFE. Now using Theorem 2.4.4 we have $\forall$ $i = 1, 2, \ldots, m$, $p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*)$ where for all vertex t^* adjacent with s^*. Given that the membership values of each edge incident with s^* is distinct. So if $q^*(\neq t^*)$ be another vertex adjacent to s^* then $\forall$ $i = 1, 2, \ldots, m$, $p_i \circ CONN_G(s^*, q^*) = p_i \circ B(s^*, q^*)$. Thus we easily say that $\forall$ $i = 1, 2, \ldots, m$, $p_i \circ B(s^*, t^*) \neq p_i \circ B(s^*, q^*)$ because the membership value of each edge incident with s^* has distinct. From the definition of superstrong mPFV, clearly we have if s^* is not a superstrong mPFV then $\forall$ $i = 1, 2, \ldots, m$, $p_i \circ CONN_G(s^*, t^*) \neq k_i$ for some $t^*(\neq s^*)$ and for some $k_i \in (0, 1]$. Since each edge incident with a^* are strong and membership value of these edges are distinct then we have $\forall$ $i = 1, 2, \ldots, m$,

$p_i \circ CONN_G(s^*, t^*) = p_i \circ B(s^*, t^*) \neq p_i \circ B(s^*, q^*) = p_i \circ CONN_G(s^*, q^*)$

$\Rightarrow p_i \circ CONN_G(s^*, t^*) \neq p_i \circ CONN_G(s^*, q^*)$.

Thus s^* is not a superstrong mPFV. □

2.5 An application

A fuzzy graph theory is now a few days essential to solve a lot of network-based problems, including networking of gas pipelines, social and road networks. Social networks are currently rising quite rapidly in human life. In models of road networking, a strong vertex problem is very important. People can exchange important goods very rapidly with the help of road networks that can be utilized for many purposes. These networks can be represented as a graph, where cities are considered as vertices and the relation between two cities is represented by an edge.

We consider a collection of cities and focus on finding that city which is best suitable to have a university or colleges. By suability we mean the city should be well connected with all the other cities of the collection and should be feasible to all in respect to communication, locality, ambiance etc. We model this problem through a 3PFG. The idea is to find out strong and superstrong mPFV and finally concluding the superstrong mPFV to be those cities most likely to have universities in them. And the strong vertices(cities) are suitable to have colleges.

We present the 3PFG model to find the strong and superstrong cities(vertices). In fig.2.6 each of the nodes represent a city and the arcs connecting the pair of vertices (cities) have membership values, all of which are characterized by three criteria :{Communication system, traffic, road condition }. Because all of the above features of an edge between two citations in real life are uncertain. We can measure edge membership values, using the relation $p_i \circ B(s,t) \leq min\{p_i \circ A(s), p_i \circ A(t)\}$ for each $(s,t) \in E$, $i = 1,2,3$ where these values together represent the interconnections of the good two cities.

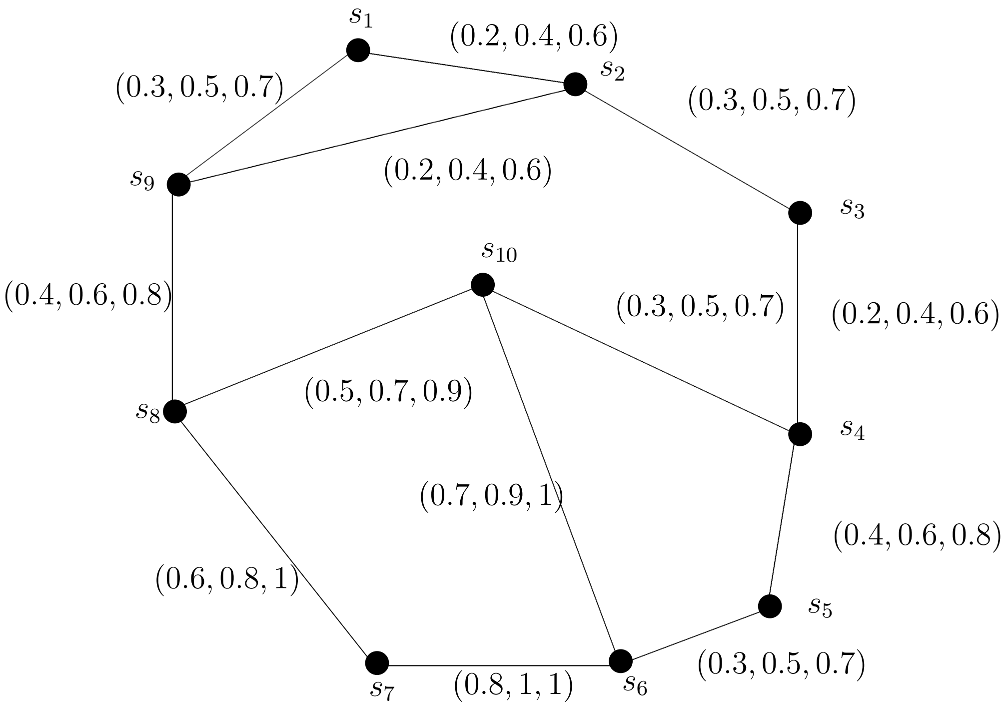

Figure 2.6: A 3PFG G of a road network.

Here the network contains 10 vertices and 13 edges. We take 10 cities of a country denoted as $V = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\}$ which can be seen from fig. 2.6. Here every city is related or connected to others through some paths. Here we want to check which one of the city is strong or superstrong. At first we consider the vertex s_2. All the edges incident at s_2 are (s_1, s_2), (s_2, s_9) and (s_2, s_3). Now we check that these edges are strong or not. Here $(0.2, 0.4, 0.6) = CONN_{G-(s_1,s_2)}(s_1, s_2) = B(s_1, s_2) = (0.2, 0.4, 0.6)$, $(0.2, 0.4, 0.6) = CONN_{G-(s_2,s_3)}(s_2, s_3) < B(s_2, s_3) = (0.3, 0.5, 0.7)$ and $(0.2, 0.4, 0.6) = CONN_{G-(s_2,s_9)}(s_2, s_9) = B(s_2, s_9) = (0.2, 0.4, 0.6)$, which means (s_1, s_2), (s_2, s_9) and (s_2, s_3) edges are strong edges. So, s_2 is a strong vertex. In this way, we check whether these other nodes are strong vertices or not. In this graph, s_2, s_4 and s_9 are strong vertices and these vertices are suitable to have a college. Similarly now we find out which city is superstrong. Here s_5 is a superstrong vertex because $CONN_G(s_5, s_i) = (0.3, 0.5, 0.7)$ for all $i = 1, 2, 3, 4, 6, 7, 8, 9, 10$. So only s_5 suitable to have a university.

According to our result we can conclude that s_5(corresponding city) can have university and s_2, s_4 and s_9 can have colleges in them best on the condition taken.

2.6 Summary

Fuzzy graph theory is widely used in research fields of computer science together with control theory, database theory and mining of data etc. In this chapter, we defined superstrong and strong mPFV of mPFGs using the concept of strong mPF arc, strength of connectedness of path etc. Next we discussed their related result. We are extending our search work to defined the concepts of superstrong and strong mPFVs along with distance and center of mPFGs on mPFG and its properties and its applications on real life problems etc.