M.Sc.

2011

4th Semester Examination

PHYSICS

PAPER—PH-2201

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Marks : 20*)

Answer *Q. No. 1* and any *one* from the rest.

1. Answer any *five* : 2×5

(a) Define scattering length. How is it related to zero-energy scattering cross-section?

(b) Prove that total scattering cross-section $\sigma = \frac{4\pi}{k} \operatorname{Im}[f(0)]$ where the symbols have usual meanings.

(c) Show that for small phase shift, scattering amplitude by partial wave analysis and Born approximation is same for central potential.

(d) Find the energy level and degeneracy with $\hat{H} = \lambda\hat{L}\cdot\hat{S}$ with $\ell = 2\hbar$ and $s = \hbar$.

(e) For low energy, prove that total scattering cross-section $\sigma = 4\pi a^2$ for hard sphere and it is $2\pi a^2$ for high energy, where a is the radius of the sphere.

(f) For a spin one particle $\hat{H} = AS_z + BS_x^2$ where A and B are real constants. Calculate the energy levels of this system.

(g) Find the zeroth order wave function for He-atom (i) in the ground state $1s^2$ and (ii) in the excited state 1s 2s.

(h) Write down the wave function for three electron system.

2\. (a) If $V(r) = A\bar{e}^{\mu r^2}$

Prove that total scattering cross-section

$$\sigma = \frac{\pi^2 m^2 A^2}{2\hbar^4 \mu^2 k^2}\left(1 - \bar{e}^{\frac{2k^2}{\mu}}\right) \quad \text{where } k^2 = \frac{2mE}{\hbar^2}$$ 5

(b) Write down the Hamiltonian for two non-interacting identical particles in the infinite square well in one dimension. Find the next two excited states-wave function and energies for each of the three cases (distinguishable, identical bosons, identical fermions).

3. (a) (i) In an excited state of alkali atom doublet structure show that $^2p_{3/2}$ state has higher energy than $^2p_{1/2}$ state.

(ii) Find the square of the dipole matrix elements for e_x, e_y and e_z between the states

$$\left|{}^2p_{3/2}, m = \frac{3}{2}\right\rangle \text{ to } \left|{}^2s_{1/2}, m = \frac{1}{2}\right\rangle.$$

Given $Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{i\phi}$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

Comment on the intensities of the lines

$2p_{3/2} - 2s_{1/2}$ and $2p_{1/2} - 2s_{1/2}$.

2+3

(b) If $\langle H'_z \rangle = (m_\ell + 2m_s)\mu_B B$ then construct perturbation matrix in H-atom for $\ell = 0$, 1 by considering intermediate field Zeeman effect.

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Group—B

(*Marks : 20*)

Answer *Q. No. 1* and any *one* from the rest.

1. Answer any *five* from the following : 2×

(a) Consider the Landau free energy $F_M = aM^2 + \frac{b}{2}M$ with $a = \alpha(\theta - \theta_c)$. Find specific heat at the curie poi θ_c.

(b) For a one-dimensional Debye solid, Find an expressi of lattice specific heat at low temperature.

(c) If f is FD distribution function, Find the value $\int_{-\infty}^{+\infty}\left(\frac{-\partial f}{\partial E}\right) dE$ and show that $\Delta f = 0.46$ within $\pm k_E$ around Fermi energy.

(d) The entropy of an ideal paramagnet in a magne field is given by $S = S_0 - aU^2$ where U is the ener of the spin system; 'a' is a constant. Sketch a gra of U VS. T

(e) Prove that for degenerate electron gas $PV = \frac{1}{2}U$

(f) Explain phase transition in the light of Lee an Yang's theory.

(g) Evaluate $\langle\hat{\sigma}_i . \hat{\sigma}_j\rangle$ for spin $\frac{1}{2}$ particle where i, j = 1, 2,

2. (a) In a uniform magnetic field B taken along the z-directions, the energy level, of an electron is given by

$$E(\ell, p_z) = \hbar\omega_c\left(\ell + \frac{1}{2}\right) + \frac{p_z^2}{2m}$$

where $\omega_c = \frac{eB}{mc}$; $\ell = 0, 1, 2, \ldots..$

(i) Find the degeneracy of the energy level.

(ii) Show that, the magnetization at high enough temperature is given as

$$M = -N\mu_B \propto\left(\frac{\mu_B B}{k_B T}\right) \quad ; \text{ where } \propto(x) = \coth x - \frac{1}{x} \text{ is}$$

the Langevin's function. 2+3

(b) (i) Write down the Bose-Einstein distribution function and hence make a comment on the sign of chemical potential μ.

(ii) Show that there is no off diagonal long range order for free bosons for dimension $D \leq 2$. 2+3

3. (a) Suppose the N Ising spins are arranged along a ring. Assume that the energy of this system is given by

$$H = -J_e \sum_{i=1}^{N} \sigma_i \sigma_{i+1}$$

If σ_i takes the value +1 or −1, what is the free en
of the system? What is the average energy
specific heat of the system? Sketch their behav
as a function of dimensionless variable J_e / k

2+

(b) The free energy for a photon gas is given

$$F = -\frac{a}{3} V T^4;$$ a is constant.

(i) Calculate the pressure of the photon gas and equation of state.

(ii) Calculate the energy of the photon gas.

(iii) What is the chemical potential of the ph gas?

(iv) Find the equation of the adiabaties of the ph gas.

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