M.Sc. 3rd Semester Examination, 2011

PHYSICS

(*Statistical Mechanics/RQM*)

PAPER—PHS-301(A&B)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Use separate scripts for Gr. A and Gr. B.

(PHS-301A : *Statistical Mechanics*)

[*Marks* : 20]

Answer **Q. No. 1** and any **one** from the rest

1. Answer any *five* bits : 2×5

(*a*) Systems with finite number of micro-states give rise to concept of negative temperature. Explain.

(*Turn Over*)

(*b*) If the canonical partition function

$$Q_N\left(V,\ T\right)=\frac{V^N}{N!}\left(\frac{2\pi m K_B T}{h^2}\right)\frac{3N}{2}.$$

Find entropy.

(*c*) Prove that the magnetic susceptibility of a system obeying classical mechanics and classical statistics is strictly equal to zero.

(*d*) For a spin $\frac{1}{2}$ particle, prove that density matrix

$$\hat{\rho}=\frac{1}{2}\left(\hat{I}+\vec{P}\cdot\vec{\sigma}\right)$$

where $\vec{P}$ is the polarization vector.

(*e*) For non-interacting photons radiation pressure is $\frac{1}{3}u$; where u is the energy density. Why ?

(*f*) Prove that $T_r\hat{\rho}^r \leq 1$ where $\hat{\rho}$ is the density matrix.

(*g*) If free energy

$$F = -NK_BT \ln (2 \cosh (J_e/K_BT)).$$

What is the average energy and specific heat of the system ?

(*h*) Show that single particle occupation number for a fermion at T with an energy within $\pm K_BT$ of the Fermi energy is $\frac{e-1}{e+1}$.

2. (*a*) Deduce an expression of Bose-Einstein distribution function from grand partition function. 4

(*b*) By the method of density matrix formalism, prove that for spin $^1/_2$ particle : 6

$$\langle \mu_z \rangle = \mu_0 \tanh \left(\frac{\mu_0 H}{K_B T} \right).$$

3. (*a*) Show that the energy fluctuation in a canonical distribution is given by 4

$$\overline{(E - \overline{E})^2} = K_B T^2 C_v \cdot$$

(*b*) Prove that

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}, \hat{\rho} \right]$$

where $\hat{\rho}$ is the density matrix operator. 4

(*c*) Define pure and mixed state of polarization. 2

(PHS-301B : *RQM*)

[*Marks* : 20]

Answer **all** questions

1. Answer any *three* bits : 2 × 3

(*a*) Derive the continuity equation for a particle obeying Klein-Gordon equation and find expression for current density and probability density.

(*b*) Show that

$$(\vec{\alpha} \cdot \vec{A})(\vec{\alpha} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma}^{D} \cdot (\vec{A} \times \vec{B}).$$

(*c*) Show that the Dirac matrices can only be of even order and their eigenvalues are ± 1.

(*d*) Prove that $\overline{\Psi} r_5 \Psi$ is a pseudo-scalar.

(*e*) Show that $\not{A} = r^0 A^0 - \vec{r} \cdot \vec{A}$ ·

2. Answer any *one* bit :

(*a*) Write Dirac-Hamiltonian for an electron in a central potential $V(r)$ and show that the spin-orbit interaction comes automatically in the Dirac equation. 4

(*b*) What are negative energy states ? What is a hole ? What is Lamb-shift ? 2 + 1 + 1

3. Answer any *one* bit : 10

(*a*) Obtain the plane wave solution for the spin half particle which obey Dirac equation and comment on the negative energy states.

(*b*) Obtain the radial equation for the electron in a central potential in the Dirac formalism and hence obtain the energy eigenvalues for the hydrogen atom.
